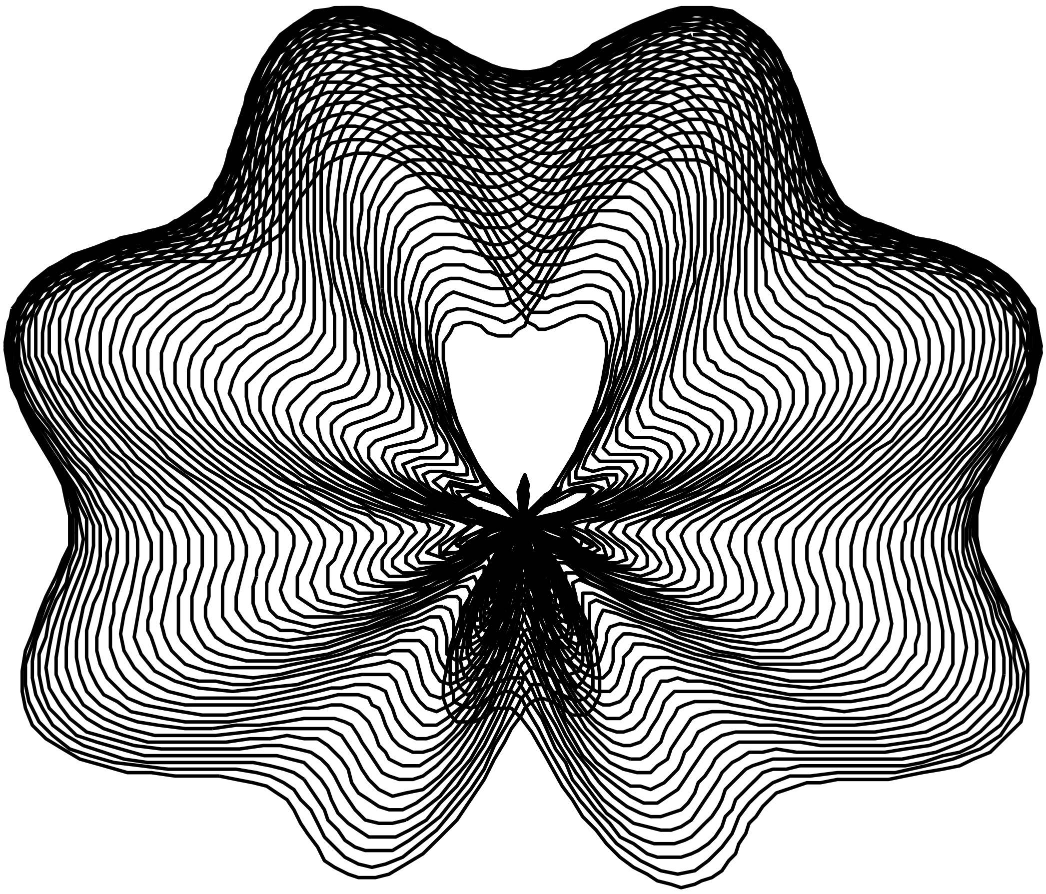
TEACHING MATHEMATICS

Volume 24 No. 1 March 1999

The Journal of the Queensland Association of Mathematics Teachers Inc



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Teaching Mathematics is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, August and November) and is provided as a right to all members of the association.

The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- and to speak and act on matters related to mathematical education.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the editor:

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Queensland School Curriculum Council	Phone (work):	(07) 3228 1501
PO Box 317	Fax (work):	(07) 3228 1555
Brisbane Albert Street	e-mail (home):	ilsley@cyberbiz.net.au
Qld 4002	e-mail (work):	none as yet

They can also be sent c/- the QAMT office (address inside back cover).

They can be sent by post (on floppy disk) or by e-mail (as an attachment). Short items (a few lines) can be faxed or e-mailed direct. Word for IBM (up to Word 97) is the preferred format. All receipts will be acknowledged – if you haven't heard within a week, ring David to check.

Copy dates are: mid-February; mid-May; mid-July; mid-October.

If you have any questions, contact David Ilsley by phone, fax or e-mail.

Lydia Commins handles books, software etc. for review; Sue Reid handles information to go in the QAMT Newsletter. These materials can be sent directly to Lydia and Sue.
Newsletter copy dates are a month earlier than those for the journal.

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Advertising Rates	Quarter page	$40 each for 1/2/3 issues	$150 for 4 issues
	Half page	$60 each for 1/2/3 issues	$200 for 4 issues
	Full Page	$120 each for 1/2/3 issues	$400 for 4 issues
	Insert (single A4 or folded A3)	$200 each for 1/2/3 issues	$700 for 4 issues
	Other	Contact QAMT	

QAMT Website: http://qamt.cqu.edu.au

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If undelivered, return to:

QUEENSLAND ASSOCIATION OF MATHEMATICS TEACHERS INC.
P.O. BOX 328 EVERTON PARK QLD 4053

TEACHING MATHEMATICS

Print Post Approved
No: PP451223/00246

March 1999 Volume 24 No 1

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Special Features

FROM THE EDITOR

David Ilsley
Queensland School Curriculum Council

Welcome to another year of QAMT activities – the last year of the millennium (depending on your point of view).

QAMT is expanding its list of publications. The journal, the newsletter and Tenrag have been published now for many years. 1997 saw the publication of the AToMIC Project and in 1999 work is continuing on the SubAToMIC Project (Subsequent Applications to Mathematics Incorporating Calculators) for lower secondary students. In addition, a book of Application assessment items for Maths A, B and C is being considered. It is hoped that QAMT will continue to produce valuable publications throughout the years to come.

Of course the big event coming up is the May Day Conference at the Brisbane Novotel. The May Day conferences have been held at the Bernard O'Shea Centre for some years now, but it was finally decided that we have outgrown the venue. We look forward to the new experience. Maree Mortlock is busy putting the final touches on the program and it looks like it will be an exciting and rewarding day.

The annual conference is of course in Rockhampton this year – our first venture outside of the south-east Queensland. Kiddy, Rex, Robert and others have been working hard securing a range of accommodation at very reasonable (even cheap) prices and a special $36 return fare from Brisbane on the tilt train. Bring the family for a bit of a getaway. There will be a three day conference program (Wednesday 29 September to Friday 1 October) and plenty of things for the family to do should they not want to attend the presentations.

Please note that I have moved from Windaroo SHS to QSCC. I now have an e-mail address at home and this is the best way to send contributions and articles for the journal. All the details are on the inside front cover.

Thanks to all those who have contributed to this issue of the journal. Hope you enjoy it.

FROM THE PRESIDENT

John McKinlay
Carmel College

In amongst all the inserts and fliers contained within this first edition of *Teaching Mathematics* for 1999 you will find the QAMT **Register of Interest**, a document closely modelled on that published by AAMT.

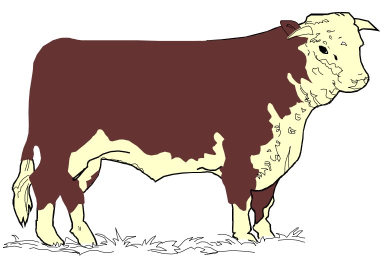
As our Association expands its Professional Development program and its Student Activities and as QAMT is approached more frequently to act as a Service Provider, we will need to have at our disposal a database of suitably qualified and enthusiastic members whom we can recommend for various activities.

Some of these activities will attract financial reimbursement.

The Register of Interest will assist QAMT in providing PD and Student Activities (and we acknowledge the great work already being done by many Local Branches in these areas) as well as ensuring that the good reputations of QAMT and our sponsors are upheld.

So if you have the skills and the motivation, complete the Register and return to our office ASAP.

I would also request all members to support the work being coordinated by the convenors of the various sub-committees. A special plug must be put for the QAMTAC 99 which is being held in Rockhampton. We took the plunge a number of years ago in offering a residential conference and it has proved highly successful. Now we take the plunge of offering a state conference outside the south-east corner. Help make it a success! Attend!



Toowoomba & District Mathematics Teachers' Association wins 1998 Australian College of Education Award for Excellence in Education.

Jeff Close, Centenary Heights SHS

Toowoomba & District Mathematics Teachers' Association (a QAMT Sub-branch) started its 25th year celebrations early when it was announced as 1998 winner in the Darling Downs Region Group of the Australian College of Education Awards for Excellence.

The Association was represented at a gala dinner where outstanding educators and educational projects were recognised. Amongst the projects nominated were reading intervention programs, electronic communication for junior primary schools, French immersion, catering and hospitality, senior English extension and an industry-school cooperative.

The nomination had to meet criteria which included: that the program was breaking new ground; that it was supported by outstanding testimony from educators, students and the community; and that the innovation was satisfying student, institutional and community needs.

The 42-page application highlighted the following aspects of the T&DMTA:

- wide support from across the various education systems
- the general meetings and the workshops, monthly discussions and guest speakers
- the sub-groups of the association such as
  - the gifted and talented after-school activities for Years 8–12, with problem solving, social activities and competitions
  - Maths Week activities which include a close liaison with the local newspaper and activities for schools and for students;
  - the annual three-day camp for Years 9–12
  - the Darling Downs Year 8 Pacific Coal Maths Competition
  - the Tournament of the Towns competition run by the University of Moscow and promoted by the T&DMTA
  - the Maths Team Challenge
  - promotional activities such as maths in the mall and articles in the local newspapers promoting mathematics
  - working with the University of Southern Queensland, for instance with sponsorship and providing a speaker at the beginning teachers' conference, where the role of the professional associations is promoted by the T&DMTA
  - the provision of professional development.

A special feature of the submission was the testimonials from students. One group wrote:

> "*We are happy to say that this extra-curricular maths activity has given us a(n) extreme advantage in our knowledge of mathematical applications*".

Another wrote :

> "*Thankyou for arranging the Maths Week Competition. I found it great fun. My classroom has a display of all the things you had to find. I really liked the cheque and I put it in the bank".*

About the maths camp, one year 11 student wrote :

> "*It was also an excellent experience to meet students of similar ability ... I made many new friends and at the same time had a chance to explore new and interesting branches of mathematics*".

1999 also sees the T&DMTA celebrating 25 years of professional activity. A dinner, multi-draw probability experiment (raffle) and social activities will be held on Wednesday 24 March. A commemorative coffee mug has been produced and a short history booklet is being compiled.

The T&DMTA invites all past members and interested persons to contact the secretary for the dinner details and mug or booklet purchases and invites interested persons and groups to obtain a copy of our winning nominations submission. Contact: Secretary, T&DMTA Harristown SHS, South Street, Toowoomba, Qld 4350.

What's happening?

in mathematics education

This will be a semi-regular feature of the journal. It is designed to help readers keep up with developments in mathematics education in Queensland and elsewhere. In this issue it contains four items on Queensland Mathematics syllabus development and an article on the new Raybould Fellow.

SYLLABUS UPDATE

Review of the Mathematics A, B and C syllabuses

A review of the syllabuses for Mathematics A,B and C is under way. A survey was sent to schools late in 1998 and these have now been analysed. The Subject Advisory Committee will now undertake a review of the present syllabuses and new syllabuses will be developed during 2000. They are expected to be available to schools early in 2001 for implementation with Year 11's in 2002 and Year 12's in 2003.

The changes are expected to be minor and, as such, there will not be a trial or pilot of the revised syllabuses.

Paul Sutton of the Subject Advisory Committee has made a call on the QAMT mailing list for comments and suggestions in relation to the syllabus review. You can still make suggestions or respond to other people's suggestions.

Trade and Business Maths Subject Area Specifications

Trade and Business Mathematics is presently undergoing an open trial. An evaluation was conducted at the end of 1998. Recommendations from that evaluation are being considered by the Trade and Business Sub-committee of the Mathematics Subject Advisory Committee. One proposal that is being debated is the inclusion of a Certificate 1 outcome in addition to the present Certificate 2. It is proposed that general implementation will take place in 2000.

Numeracy and Literacy SAS – Strand B: Consumer Maths

Like Trade and Business Maths, Consumer Maths is presently undergoing an open trial. Preliminary evaluation occurred at the end of 1998. Recommendations will be considered by a sub-committee. It is anticipated that minor revisions will be required and that general implementation will occur in 2001.

Review of the Years 1-10 Mathematics syllabus

The Queensland School Curriculum Council has just commenced work on a review of the Years 1-10 Mathematics syllabus. This is the first review since the present syllabus was produced in 1987.

QSCC began work on the Science and HPE syllabuses in 1996, on the LOTE and SOSE syllabuses in 1997 and on the Arts and Technology syllabuses in 1998. English and Mathematics are the last of the key learning areas, both commencing development in 1999.

As with the other seven key learning area syllabuses, the new 1-10 Mathematics syllabus will be written in terms of student learning outcomes at eight levels. In this respect it will have some similarities to Student Performance Standards. The project team will be striving, however, to make the new syllabus more user friendly.

Graham Meiklejohn is leading the project team. David Ilsley is one of the project officers. It is hoped that a second officer will be appointed in early March.

Timelines have not as yet been finalised, but it is likely that the syllabus will be available for implementation in 2003.

The next few months will be a period for major public consultation. Teachers and other interested people may make submissions or join the Consultative Network (see below). More informal discussion is taking place on the QAMT Mailing List.

Ed.

QUEENSLAND SCHOOL CURRICULUM COUNCIL

Call for Public Submissions

COMMENTS ON THE DESIGN OF THE NEW QUEENSLAND YEARS 1-10 MATHEMATICS SYLLABUS

In January 1999, the Office of the Queensland School Curriculum Council commenced a project involving the review of the Years 1-10 Mathematics syllabus and the development of sourcebook support material and an initial in-service package.

Brief written submissions are invited from interested individuals, groups and organisations who wish to offer suggestions to assist with this development.

Comments are invited on the philosophy, structure and content of the syllabus, on the nature and content of support materials and on any other aspects of the curriculum.

Invitation to join the Consultative Network

An invitation is also offered for individuals to join the project Consultative Network. Members of the Consultative Network will be kept informed of developments as they occur and, in return, will provide feedback to the project team in regard to particular suggestions and questions which arise during syllabus development.

Submissions and expressions of interest in membership of the Consultative Network should be sent to Graham Meiklejohn, Principal Project Officer, by mail or fax (see below) and should arrive by Friday 9 April.

Graham Meiklejohn
Office of the Queensland School Curriculum Council
PO Box 317
Brisbane Albert Street Q 4002
Fax: (07) 3228 1555

1999 Raybould Tutorial Fellow

The Raybould Tutorial Fellowship provides an opportunity for graduate teachers in mathematics to:

- update their mathematical skills;
- renew or continue involvement in mathematics at a tertiary level;
- assist in the development of quality learning and teaching practices in mathematics, particularly for students in Years 11 and 12.

The fellowship lasts for one year. Fellows spend the first semester in the Mathematics Department of the University of Queensland furthering their study and providing tutorial assistance for undergraduates. They then spend the second semester, working in the Education Services Directorate of Education Queensland at Central Office to produce a resource of value to teachers of senior mathematics.

The fellowship is funded jointly by Education Queensland and a bequest by Ethel Harriett Raybould, a University Medallist and the first female mathematics lecturer at the University of Queensland.

The fellowship was established in 1988. The fellows to date and their projects have been:

Year	Fellow	Project
1989	Lorrie Jardine	
1990	Gary Carter	School-Tertiary interface
1991	Mal Hartwig	Number Theory
1992	Lydia Commins	Networks & Linear Programming.
1993	Noela Duncan	Numeracy in post-compulsory education
1994	Bill Simpson	Graphics Calculators
1995	Leah Liddell	Hypothesis Testing
1996	Michael Craven	Making Sense of Data
1997	Rex Boggs	Exploring Data
1998	Peter Cooper	Financial Mathematics

The Raybould Tutorial Fellow for 1999 is Helen Weil. Helen was a mathematics and science teacher at Beerwah State High School until this year.

Helen Weil, 1999 Raybould Fellow

The project topic Helen is proposing to develop is Navigation and Land Measurement, primarily of interest in the Mathematics A syllabus, and of some practical value in Mathematics B when covering the topic of Applied Geometry.

Helen says that she will need a lot of help to produce relevant materials for teachers which are easily accessible, useful and practical. She would appreciate any contributions from schools and/or teachers to assist in determining what is most needed, what will be most effective and in what format resources would be most useful. She feels sure that there are already some great ideas out there which could be shared with others and developed further.

Helen is keen to establish links with schools and teachers to share ideas and interests as the project develops. She can be contacted as follows:

Semester 1: University of Queensland
Ph. (07) 3365 2315
e-mail: hmw@maths.uq.edu.au

Semester 2: Education Queensland
Ph. (07) 3237 0411 fax (07) 3210 0227
e-mail: helen.weil@qed.qld.gov.au

Home Ph: (07) 5494 7496 mobile: 0407 764 272

Ed.

Using Information Texts

to integrate numeracy and literacy learning

Rachel Griffiths

Numeracy, mathematics and context

Numeracy and mathematics are not identical, but obviously are intertwined. Numeracy implies using mathematics in everyday situations at home and at work, and is thus linked to mathematics in context.

We often present mathematical exercises and problems to students in a rarefied form – we give them exactly the information they need to reach an answer, and we ignore the context and purpose for the calculation. For example:

> *6 birds were on a tree. 3 more came. How many birds were there then?*

At a higher level, you may remember the 'stories' about A, B and C emptying and filling baths at different rates. We never know why they were obsessed with bath water!

Of course there are times when children need to concentrate on the mathematical processes without distraction by the context. However, we need to present mathematics more often in 'real' contexts, in which students need to think about the purpose of the exercise, what information is needed to solve a problem, whether the answer they reach is reasonable, or has practical value. It is the ability to use mathematics in context that defines a numerate person.

One source for these contexts is the wide range and variety of information texts that are now available, and that capture the interest of students either because of the topics they address, or because they are presented in exciting ways, or, in many cases, both. Another reason for using information texts in this way is the opportunity to integrate numeracy and literacy learning, which makes for a more efficient classroom, as well as giving a purpose to both the literacy and numeracy activities.

Information texts and literacy and numeracy skills

Learning to read or reading to learn? With information texts, the two processes are integrated. Narratives are traditionally the mainstay of reading programs, but increasing importance is being given to information texts within the literacy curriculum. The implications of this shift have not generally been worked out or made explicit. For example, information texts require different literacy skills from narratives. This is because information texts often

- have different language structures
- have different layouts
- include unfamiliar vocabulary
- use longer words
- include unfamiliar concepts and ideas
- need not be read sequentially or completely
- include book elements such as index, glossary, bibliography

Information texts may contain mathematical data

- in the body of the text
- in tables, diagrams, graphs, maps, captions

They may

- be focused on mathematical concepts, or
- use mathematical data to support or illustrate the main ideas

Information texts provide

- contexts for mathematical concepts
- contexts for developing and applying mathematical skills and knowledge

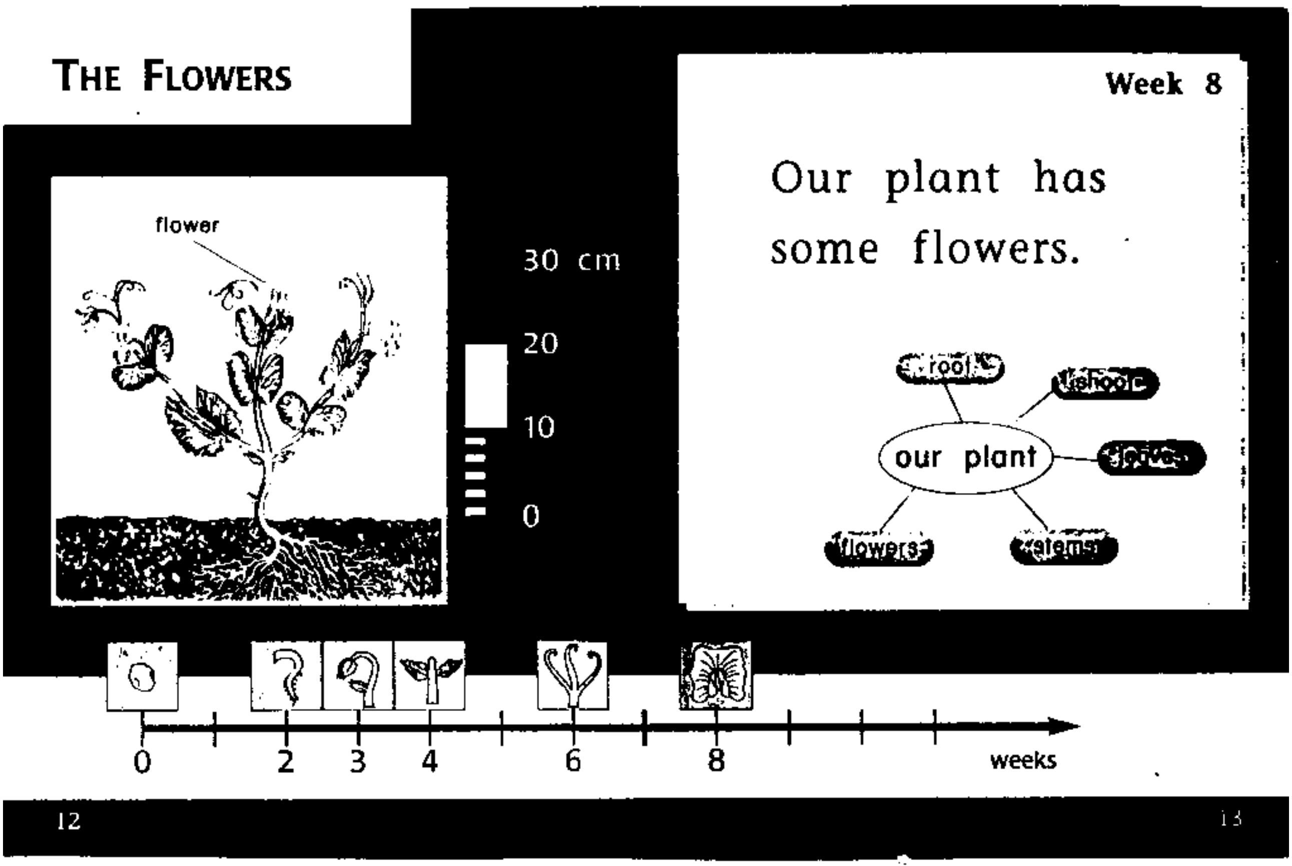
- opportunities for solving and posing problems
- opportunities for investigations and projects.

How can all this be implemented? Some examples follow.

Information texts in the early years

InfoActive, an innovative early literacy program of information texts, provides many opportunities for integrating literacy and numeracy learning. (Drew, 1997, 1998; Clyne & Griffiths, 1997, 1998)

On a first class reading of *Our Plant Diary*, which details the growth of a pea plant from seed to fruiting, children predict what may happen next, match words across each double-page spread, read the time line, interpret the scale that appears for the first time on page 8, and discuss the different ways the information is presented. This is not a process to be hurried; there is much to observe and talk about.



Further readings of the text, in a whole class or group situation, can focus on diary writing, on measurement, on plant parts, and on word wheels. Then children can grow plants from seed in the classroom, observe their growth, measure them, and record what happens using the same format as *Our Plant Diary*.

As they work with this text children are developing numeracy skills through

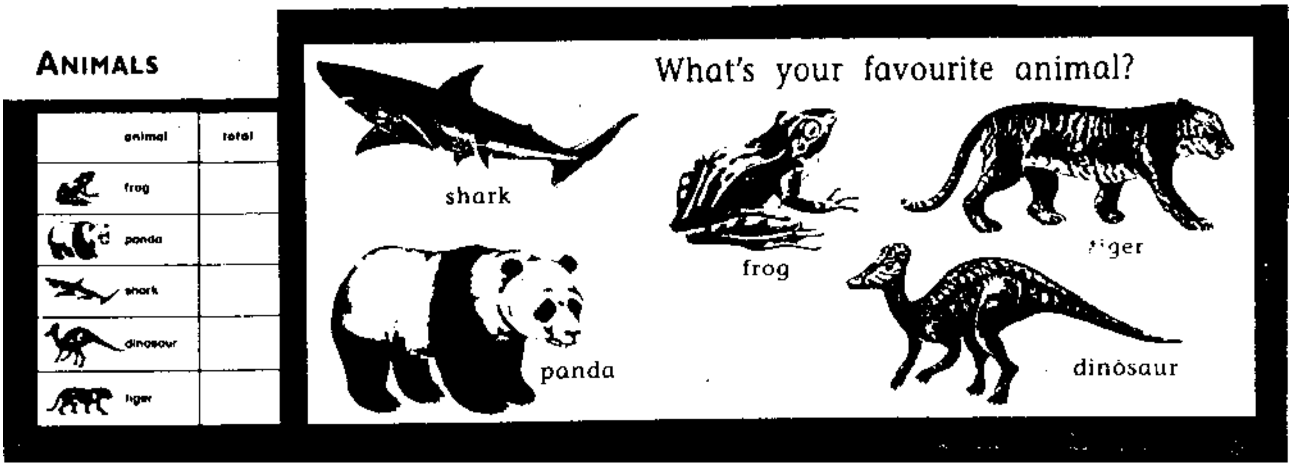
- interpreting and making scale drawings
- interpreting and making a time line
- estimating and measuring lengths.

They will be acquiring literacy skills through:

- matching words in headings, body text, diagrams, time line, and word wheel
- comparing information in headings, body text, diagrams, time line
- writing a report with the same structure
- creating word wheels
- writing a diary

What's Your Favourite? shows children how to make a classroom survey, and provides opportunities for:

- collecting, organising and displaying numerical data
- counting and tallying
- making and interpreting graphs
- comparing different data representations



Children will be acquiring literacy skills through:

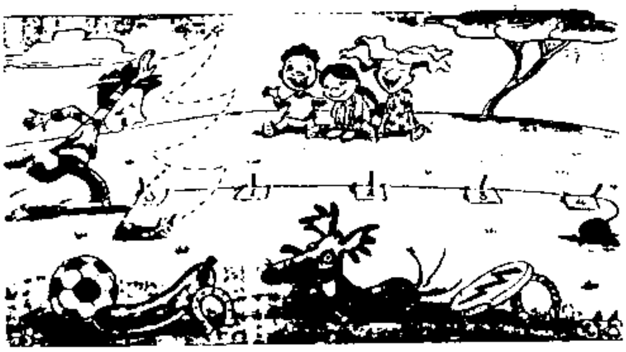
- matching words in tally sheets and captions
- writing about favourite things
- comparing contents and index

The third *InfoActive* example is *The Ball, The Stick, the Plane and the Feather*. The illustrations on each left hand page show children throwing different objects, and measuring the distances thrown, while on each right hand page a graph is built up through the book.

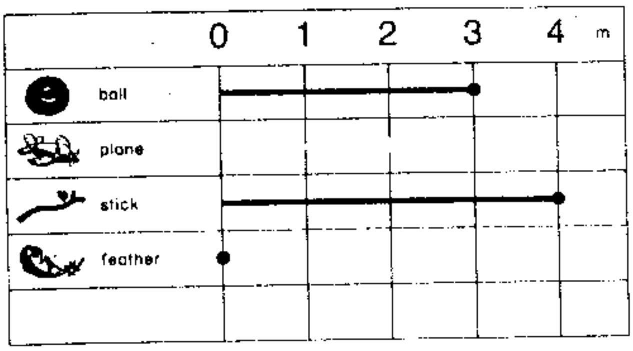
Children will develop numeracy skills through:

- comparing, estimating and measuring lengths
- reading and making graphs
- reading scale diagrams

but I threw the feather . . .



zero metres.



They will be acquiring literacy skills through:

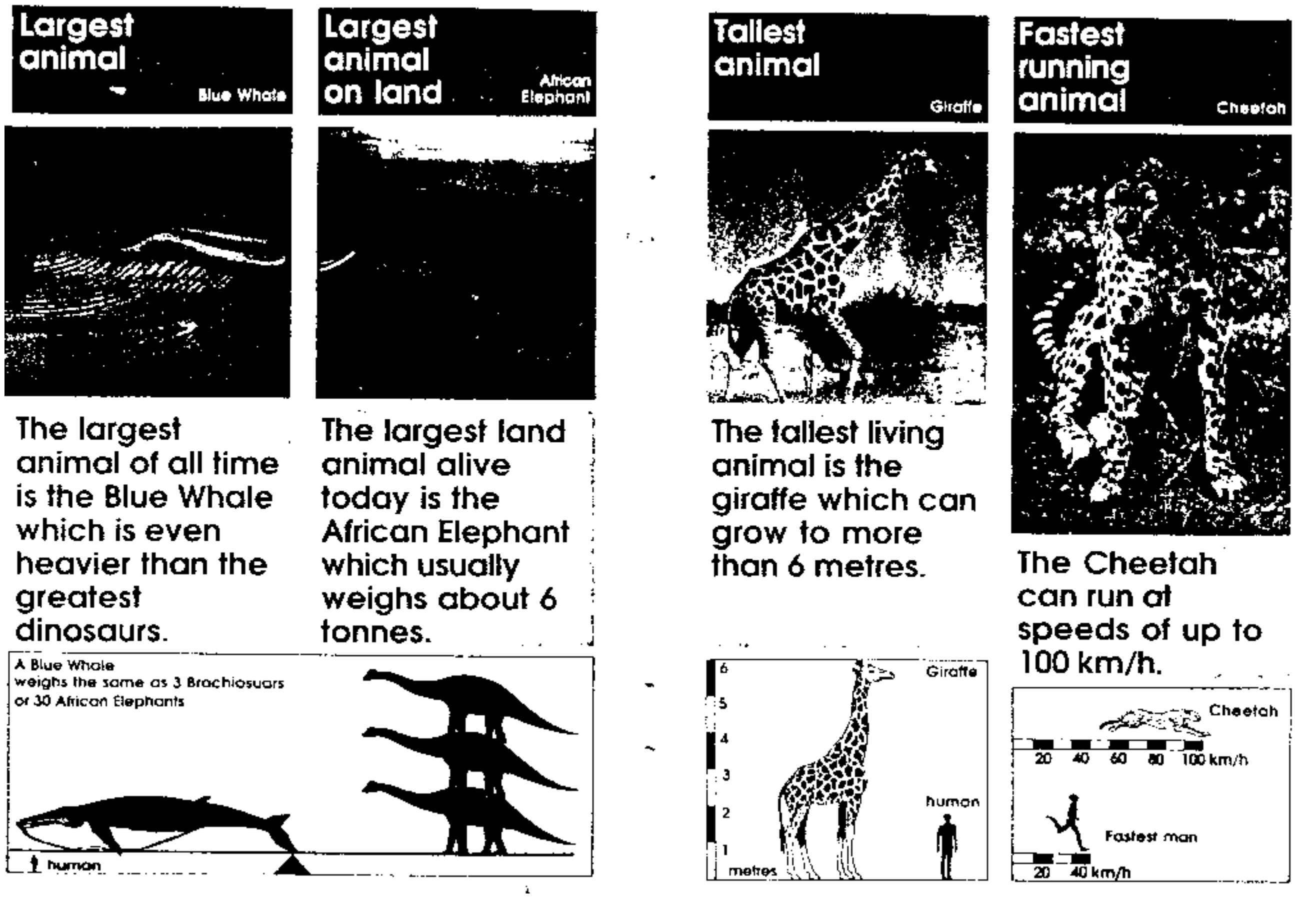
- comparing information in body text, illustration and graph
- writing a new text

As well as the *InfoActive* series, other examples of information texts for the early years that provide opportunities for numeracy learning can be found in the *Informazing* (Nelson), *Realization* (Rigby), and *Reading Discovery* (Scholastic) series. *Comet* magazine is another useful source, and junk mail such as catalogues also provides information which can be used for numeracy and literacy learning.

Information texts in the middle primary years

Learning about animals always interests children. The two *Informazing* titles *The Book of Animal Records* and *Animal Acrobats* present information that fascinates children, and present it imaginatively and creatively. (Drew 1987, 1990; Clyne & Griffiths 1991; Griffiths & Clyne 1996a) As children work with these books, they will be:

- interpreting measurements in text and diagrams
- estimating, measuring and comparing length, mass and time
- making scale and life-size drawings
- solving and posing measurement problems
- interpreting a map



They will be acquiring literacy skills by

- comparing information in headings, body text and diagrams
- using a glossary
- using an index
- researching and writing other records
- researching and presenting information on other 'amazing animals'.

Other examples of information texts for the middle primary years that provide opportunities for numeracy learning can be found in the *Informazing* (Nelson), *Realization* (Rigby), and *Mathshelf Middle Level* (Scholastic) series, as well as in simple reference books, *Explore* magazine, junk mail and other environmental print.

Information texts in the upper primary and lower secondary years

At this stage, understanding topics in science, technology and studies of society and environment requires understanding of and skills in number, measurement, space, and data handling. The range of information texts that can be used expands, as newspapers, magazines, and reference books (for example *The Guinness Book of Records*, atlases and encyclopaedias) become more accessible to children whose literacy skills are now more developed. Series such as *Look Inside Cross sections* (Penguin), *Mathshelf Upper Level* (Scholastic), *Magic School Bus* (Scholastic), *Realization* (Rigby) and *Informazing* (Nelson) will stimulate children's thinking.

The example I have chosen came originally from *Silver Kris*, the Singapore Airlines in-flight magazine. 'A High Old Time' (Griffiths & Clyne, 1996b) describes Big Ben, the famous London landmark, and its history. Mathematical information in the article includes number, time, length, mass, angle, shape and location. Children can be involved in

- reading and writing Roman numerals
- investigating the motion of a pendulum
- investigating gears
- estimating and comparing heights
- measuring and comparing mass
- interpreting maps
- making a time line
- investigating time around the world
- making life-size and scale drawings of the clock face

At the same time, children are developing literacy skills such as:

- summarising information
- locating and extracting information
- recording and reporting an experiment
- using a dictionary
- making a glossary

Summary

Numeracy and literacy can be defined as the ability to apply mathematics and to read and write in a range of everyday contexts and practical situations. Using information texts as described in this paper will assist students to develop literacy and numeracy skills, while at the same time learning about the world around them.

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David Drew (1997, 1998) *InfoActive* series, Melbourne, Longman

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Rachel Griffiths & Margaret Clyne (1996b) *Read Your Way to Maths* Book 3. Melbourne, Longman

Acknowledgments

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Financial Education For Our Children

by David Rosenberg

When I completed senior, my knowledge of finances generally was very poor.

I studied an academic course, including maths and science, but there were no practical studies, even in simple subjects, such as how to open a bank account or do a budget - nothing.

Our parents help us out, as best they can, and it is a moot point about whose responsibility it is to teach children financial management. The problem is, most parents need help themselves and do not necessarily have the skills to teach their children financial management.

So off we went into the big bad world, earning money and doing the best we could. Life was simple when I first started work, earning $32 a week. I paid $10 board, $12 for food and a few extras, and saved $10 to buy a stereo and put a deposit on my first car.

Then life got more complicated. I paid a deposit on a car and the balance was on hire purchase. By that time, I was earning $75 per week but living in a unit and going "out on the town". I often ran out of money before the next pay day. Still, bills were paid and, if I needed to eat, I could go to my parents.

Times have certainly changed and our financial lives are far more complicated. When I first started work in the early 70s superannuation was not what it is today. You could have any type of life insurance you wanted as long as it was whole of life or endowment. An investment was a term deposit and shares were misunderstood, and used generally by the wealthy.

"Most of us, I suppose, learnt as we went along..."

Most of us, I suppose, learnt as we went along and, in times of high inflation, the mere fact of home ownership increased our net worth.

Times have changed and therefore our children's financial education should not be left to chance.

How can we help?

Financial Technology has more than 100 clients in the teaching profession. They include university professors, lecturers, secondary and primary teachers, both private and public. Our advisers tell us that many of these educators suggest financial planning should be taught in schools.

We agree. We now participate in a schools program called "Mastering Your Money". The program is an eight-section lecture program aimed at teaching the basics of personal financial planning. Even though the program is aimed at year 11 and 12 students, it has been well received by TAFE colleges and even those in retirement.

The reason the program is so popular is that so many of us have forgotten the basics.

The program can be tailored to a one-hour lecture, or a full, two-day course, depending on the situation.

The program is not limited to schools and can be used by companies as a benefit to employees.

If you think we can be of assistance at the school your children attend, the school at which you teach, or the company at which you work, please give us or your adviser a call so we can provide further details.

Teaching e without calculus

David Ilsley
Queensland School Curriculum Council

In most Maths B courses, e is introduced as the value of a for which the derivative of a^x is a^x. But e can be introduced without the use of calculus. Introducing it without calculus leads to a simpler method of solving many problems involving exponential growth and decay.

Consider \$100 in an account earning interest at 100% p.a. paid annually. After 1 year there will be \$200 in the account. If the interest is paid half-yearly, there will be $\$100 \times 1.5^2 = \$225$. The list below shows the amounts for various frequencies of interest payment.

Yearly	\$200
Half-yearly	\$225
Quarterly	\$244.14
Monthly	\$261.30
Weekly	\$269.26
Daily	\$271.46
Hourly	\$271.81
Minutely	\$271.82

Let's generalise this. Consider \$$P$ in an account earning interest at rate r paid annually. The amount in the account after t whole years is given by $A = P(1+r)^t$. This is an example of discrete exponential growth.

Now suppose the interest is paid half-yearly or quarterly – in fact n times per year. The formula now becomes $A = P(1+\frac{r}{n})^{nt}$ or $A = P[(1+\frac{r}{n})^n]^t$.

Now let n become very large until we get effectively continuous growth. The formula then becomes $A = \lim_{n\to\infty} P[(1+\frac{r}{n})^n]^t$.

Let us now look at the inside part of this function: $(1+\frac{r}{n})^n$. For a given value of n, this is a function of r, but what type of function? We can find out by setting n to something suitably large, say 1000 000, and graphing the function against r on a graphics calculator. The result looks distinctly like an exponential function. It crosses the y-axis at $y = 1$ and therefore would be of the form $y = a^r$. By making this assumption, we can calculate the value of a. Pick a value for r, say 1, and substitute it into $(1+\frac{r}{n})^n = a^r$. This gives $a = (1 + .000\,001)^{1000\,000}$, which gives $a = 2.71828$. We can now plot 2.71828^r on the same axes as $(1+\frac{r}{n})^n$. The two graphs will be almost exactly superimposed. Moving the window to the left or right or zooming in will confirm the identity.

Students will be prepared at this point to accept that $(1+\frac{r}{n})^n$ is an exponential function of r with base 2.71828. It can be pointed out that the actual base is an irrational number 2.718281828… and that we call this number e.

Thus $(1+\frac{r}{n})^n = e^r$ and for continuous growth,

$$\boldsymbol{A = Pe^{rt}}$$

This formula describes continuous exponential growth. A is the amount present, P is the amount present when $t = 0$ and r is the fractional instantaneous rate of growth. In the bank deposit example, if the initial deposit is \$100 and the interest rate (nominal) is 100%, then the amount after 1 year is $100 \times e^{1\times 1}$, which comes to \$271.82. If the initial deposit is \$100 and the interest rate (nominal) is 7%, then the amount after 4.29 years is $100 \times e^{0.07 \times 4.29}$, which comes to \$135.02.

Many problems, which otherwise require a differential equation for their solution, can be solved using this formula. Here is an example.

Q. *A pool contains 50 kL of water. Water passes through the filter at the rate of 5 kL per hour. How long will it take for 90% of the water in the pool to have passed through the filter?*

A. Let the amount of unfiltered water be A.

$$A = Pe^{rt}$$

The fractional rate of increase in A is -0.1 per hour. $P = 50$. We wish to know t when $A = 5$. So

$$5 = 50e^{-0.1t}$$

$$t = 10 \ln 10 \approx 23 \text{ hours.}$$

Maths Teams Challenge

In the days of regions and maths advisers, regional Maths Teams Challenges were organised right across the state. After the change to districts and demise of the advisers, this became harder, but some smaller events have continued to be organised by classroom teachers for districts and smaller groups of schools. Generally, an entry fee of about $20 per team is charged and this covers the purchase of materials, hire of halls, labour involved in setting up and some TRS for the organising teachers.

Also, some schools have held their own intra-school event, taking half a day out of the normal program so that all students can take part. The benefits of this are at least as great as the benefits of taking a day for a swimming or athletics carnival.

As for previous years, the Toowoomba Education Centre is producing question papers for the 1999 Maths Teams Challenge. These are at four levels – Primary, Year 8, Years 9-10 and Years 11-12. The Centre also have past papers available for practice. Anyone interested in running an event for their own school or a group of schools might like to contact John Handley, director of the Toowoomba Ed. Centre (Ph. 0746 301 722). You might also like to contact Andy Boswell. Andy has many years of experience in organising Maths Teams Challenges on a regional, district and school scale and can tell you more about the events and how to get them off the ground. Andy's mobile phone number is 0419 680 861. This number has a message bank so Andy will be able to return your call if he is not available at the time.

Did you know?

The lowest number not expressible in eight words can be expressed as

'the lowest number not expressible in eight words'

– which is eight words.

From this it can be deduced that all numbers can be expressed in eight words. Here is the (somewhat dubious) proof.

It is possible to express 16 004 901 in eight words – sixteen million four thousand nine hundred and one. It is possible to express 2 257 390 144 in eight words – forty seven thousand five hundred and twelve squared. Is it possible to express 69 026 744 319 540 892 671 518 722 in eight words? 'Probably not' you might say. But, in fact, it is possible to express any number you choose in eight words.

Suppose we were to test each of the whole numbers, starting with 1, to see whether it can be expressed in eight words. We would go through several thousand without any problem, but eventually we would reach the first number not expressible in eight words. Let's call this number λ_1. Now λ_1 can be called 'the lowest number not expressible in eight words', which is eight words. So λ_1 is part of the set of numbers which are expressible in eight words.

Now we continue searching until we find the next one which is not expressible in eight words. Let's call this λ_2. Because λ_1 was in fact expressible in eight words, λ_2 can be called 'the lowest number not expressible in eight words', which is eight words. So λ_2 is part of the set of numbers which are expressible in eight words.

This process can be repeated ad infinitum thus proving that all whole numbers can be expressed in eight words!

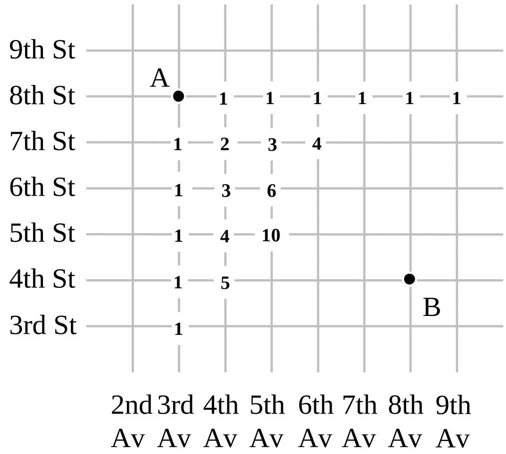
Page 15 Problem

Solution to 'Love in the big city'

This can be solved by counting all the possible routes, but this is very cumbersome. The following is a quicker method.

Suppose Anne lived at an intersection on 3rd Avenue. Irrespective of which intersection, there would be only one route for Jerry to take. Likewise if she lived at an intersection on 8th Street. Mark the number of routes to each of these intersections.

How many routes would there be if Anne lived at the intersection of 7th Street and 4th Avenue? 2. Mark this in.



How many to 6th Street and 4th Avenue? Well, Jerry could arrive at that intersection from the intersection of 7th Street and 4th Avenue or from the intersection of 6th Street and 3rd Avenue. As there are 2 ways to arrive at the former and one way to arrive at the latter, there are 3 ways to get to 6th Street and 4th Avenue. In fact, the number of ways of getting to any intersection is given by the sum of the number of ways to get to the one above and the number of ways to get to the one to the left. This gives us a way to build the numbers out from A until we reach B. The result is 126.

In doing this we generate Pascal's triangle. But Pascal's triangle suggests a combinations approach. To get from A to B, Jerry has to walk 9 blocks. 4 of these have to be north-south. The number of routes is the number of ways of selecting 4 blocks from 9, ie. $^9C_4$, which is 126.

I would like to thank Don Kerr of Zeno Educational Consultancy for introducing me to this next problem.

Game show

In the USA, there was a game show where a number of contestants competed to decide a winner for that show. At the end of the show, the winner was shown three curtains and told that there was a car behind one of them and much lesser prizes behind the other two.

The contestant chose a curtain. Every week, the compere then ordered one of the other curtains to be opened. The compare obviously knew where the car was because he never opened the curtain to reveal the major prize. The contestant was then given the choice of staying with their original selection or changing to the other unopened curtain.

What should the contestant do to maximise the chance of winning the car? Stay with the first choice, change to the other, or does it make no difference?

Level of mathematical knowledge required: Year 8

Ed.

Technology Education

for Pre-service Mathematics Teachers

Merrilyn Goos
The University of Queensland

A brief glance through recent journal issues and mathematics education conference programs should be enough to convince even the most casual observer that the impact of technology on mathematics teaching and learning is an issue attracting considerable interest. Of most immediate concern to classroom teachers is how to make the best use of these new technologies, whether the purpose is to free students from the burden of tedious calculations, to promote exploration of abstract mathematical ideas, or to tackle complex real life problems.

Professional development programs focussing on the integration of technology into mathematics teaching are becoming more widely available, and initiatives such as QAMT's AToMIC and Sub-AToMIC projects and their associated workshops provide a welcome opportunity for practising teachers to gain "hands on" experience with the hardware and to try out and share activities. However, it is also incumbent upon preservice teacher education providers to ensure that beginning teachers are familiar with the capabilities of mathematical software and graphing calculators, and have gained some experience in preparing technology based activities and materials for classroom use.

Students taking mathematics as their major teaching subject in The University of Queensland's Postgraduate Diploma in Education course have ample opportunities to develop and demonstrate these skills. As part of the assessment program for the subject, students work in pairs to prepare a technology based activity designed to teach some aspect of a topic chosen from the Senior Mathematics A, B or C syllabus. The activity need not be original, but could be one which they used or observed during practice teaching, or one which was obtained from another source. Students then present the activity to an audience of their peers in the form of a professional development seminar lasting 30 minutes. To accompany their oral presentation, students provide class members with a handout containing information which would enable another teacher to implement the activity, including the year level and syllabus topic, the source of the activity, a description of any modifications made to the original form of the activity, a statement of the problem to be solved or task to be completed, possible solutions, and teaching notes in the form of "instructions for use". Both the written handout and the oral presentation are assessed, using the following criteria:

- understanding of the use of technology in teaching mathematics
- relevance of the selected technology-based activity
- structure and organisation of the seminar presentation
- quality of oral communication
- quality of written communication

These seminars have proven to be a valuable experience for presenters and audience alike, resulting in positive engagement with the activities and stimulating post-activity discussion. Three activities prepared by students in the 1998 class will be presented on the following pages and in the next two issues of *Teaching Mathematics*. They are based on tasks found in mathematics textbooks, teaching journals and Internet sites. These resources are offered in the hope that readers will try them with their own students, so that the work of the teacher education students can be tested in authentic classroom settings.

Fibonacci Sequences on the Graphing Calculator

Jeremy Sullivan and Matthew Daniels
The University of Queensland

Background

This activity is appropriate for investigating sequences as part of the Structures and Patterns topic in the Senior Mathematics C syllabus (Board of Senior Secondary School Studies, 1992). The activity (adapted from Dion, 1995) was originally designed for the Texas Instruments TI-82 graphing calculator, but it is suitable also for the TI-83. Its purpose is to investigate the Fibonacci sequence and illustrate its connection with the Golden Ratio.

The origin of the Fibonacci sequence lies in the answer to a question posed by Leonardo of Pisa (also known as Fibonacci) in *Liber abaci*, an arithmetic book published in 1202. His question can be summarised as follows:

> *Suppose a male/female pair of adult rabbits is placed in an enclosed pen to breed. The rabbits and each pair of offspring produce one additional male/female pair of rabbits every month, beginning at two months of age. Assuming none of the rabbits die, how many pairs will result by the end of a year?*

The answer to this question leads to the familiar recursive definition of the Fibonacci sequence:

$$t_1 = 1$$
$$t_2 = 1$$
$$t_n = t_{n-1} + t_{n-2} \text{ for } n \geq 3$$

which generates the sequence {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ...}. We can see that this translates into "each term is the sum of the previous two terms".

Fibonacci sequences and the Golden Ratio

The ratio of consecutive terms of the Fibonacci sequence approaches the value $(1+\sqrt{5})/2$ as the index of these terms approaches infinity; that is, the limit of the sequence

1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55, 144/89, 233/144, ... is $(1+\sqrt{5})/2$.

This limit is known as the Golden Ratio. It can also be obtained by dividing a line segment of unit length into two parts such that the ratio of the greater part to the smaller is equal to the ratio of the whole segment to the larger part. In addition, any rectangle whose sides are in the proportion

$$\frac{\text{length}}{\text{width}} = \frac{1+\sqrt{5}}{2}$$

is called a Golden Rectangle. Such rectangles seem to have proportions which are pleasing to the eye, a fact which has been exploited by artists and architects throughout history (for example, the Golden Rectangle forms the basis of the design of the Parthenon in Athens).

Instructions for generating the Fibonacci sequence on the TI-83

The TI-83 can only handle sequences where each term is defined in terms of the previous term. However, each term in the Fibonacci sequence is defined in terms of the previous *two* terms. We can circumvent this problem by using two sequences for our definition:

$$u_1 = 1$$
$$v_1 = 1$$
$$u_n = v_{n-1}$$
$$v_n = u_{n-1} + v_{n-1}$$

The first six terms of both sequences are given in Table 1, which demonstrates that the Fibonacci sequence has been defined as intended.

u_n	v_n
$u_1 = 1$	$v_1 = 1$
$u_2 = 1$	$v_2 = 2$
$u_3 = 2$	$v_3 = 3$
$u_4 = 3$	$v_4 = 5$
$u_5 = 5$	$v_5 = 8$
$u_6 = 8$	$v_6 = 13$

Table 1. Defining the Fibonacci sequence as the sum of two sequences

On the calculator this table can be created by first selecting Seq(uence) and Dot via the MODE menu, and then defining the table via TBLSET, with Tblstart = 1 and ΔTbl = 1. The sequences are then entered through the Y= editor, by typing in u(n) = v(n-1) with u(nMin) = 1 and v(n) = u(n-1) + v(n-1) with v(nMin) = 1 (see Figure 1). On pressing TABLE, the terms of the two sequences are generated (see Figure 2).

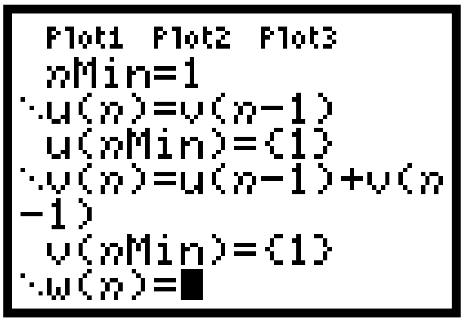


Figure 1. Defining two sequences

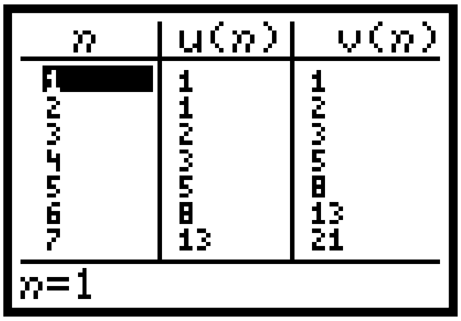


Figure 2. Generating the two sequences

Investigating the ratio of consecutive terms

A third sequence needs to be defined as the ratio of consecutive terms in the sequences u(n) and v(n). In the Y= editor, set w(n) = v(n-1)/u(n-1), with w(nMin) = 1 (Figure 3). The table of terms for this sequence can be scrolled to observe how the values approach the limit of 1.618 (see Figure 4).

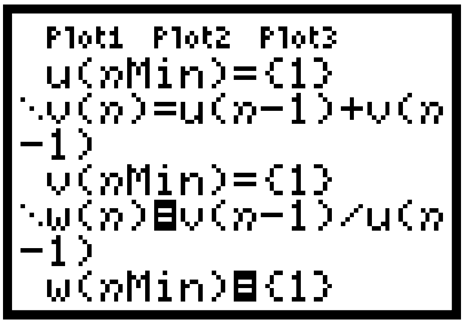


Figure 3. Defining a new sequence as the ratio of consecutive terms of a Fibonacci sequence

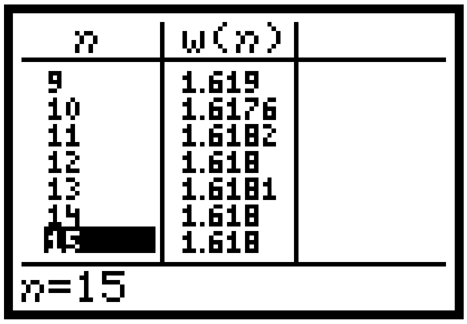


Figure 4. Ratio of consecutive terms approaches a limit of 1.618

A graphical representation of the convergence of the ratio of consecutive terms to the Golden Ratio can be also be produced. First ensure that the sequences u(n) and v(n) are turned off (through the Y= editor). A suitable window is defined by nMin = 1, nMax = 15, Xmin = 0, Xmax = 15, Ymin = 0 and Ymax = 3. Pressing GRAPH gives the result shown in Figure 5.

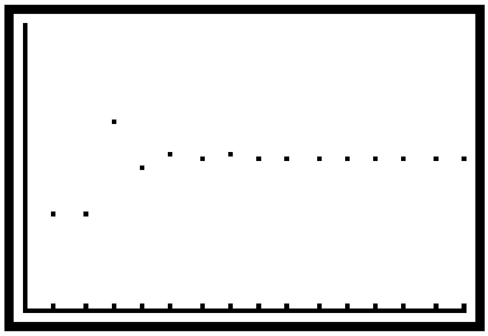


Figure 5. Graphical representation of convergence to the Golden Ratio

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Board of Senior Secondary School Studies (1992). *Senior Syllabus in Mathematics C*. Brisbane: Board of Senior Secondary School Studies.

Dion, G. S. (1995). Fibonacci meets the TI-82. *The Mathematics Teacher*, *88*(2), 101-105.

REVIEWS

Mathematics for Queensland

Year 8 496 pp 1996
Year 9 496 pp 1996
Year 10 528 pp 1996
Teacher Resource Book 72 pp 1998

Debra Manché, Barbara Wasley, Robyn Winter

Oxford University Press, Melbourne

Oxford have put out a series of junior mathematics textbooks which will suit many schools in Queensland and should be adopted as a textbook.

The content covered is in line with the 1-10 syllabus and addresses the National Statement on Mathematics for Australian Schools.

These texts can be used equally well by teachers who like to take a traditional approach to teaching and by those who like their students to undergo a discovery learning approach.

As an example, take the Year 9 text, Chapter 7 on Length, Area and Volume. The chapter opens with a page about Archimedes and information on how a metre is measured exactly. The content covered is as follows: lengths and perimeters, area, surface area of solids with plane faces, surface area of solids with curved surfaces, volume, capacity and measurement applications. Interspersed through this chapter are eleven explorations with topics which many teachers will recognise such as ‘Posting parcels’, ‘Sprinkler system’, ‘Points and areas’, ‘Don’t be a drip, save every drop’ and ‘The shade house’. The end of the chapter has a checklist of topics in the chapter which could be used by students to record what they have learnt and what they need to do more work on. This is followed by a Review Test then not one, but two Review Sets – nice for revision! Finally there are often a couple of extension activities which would make nice investigations for the students – one called ‘Sam the Sheep’ in which Sam is tethered and his roaming area is to be discovered in various situations and the other is a packing problem relating to aircraft.

Each text in the series has a large section at the end entitled ‘Activities Section’. These are a great strength of the books. They contain puzzles, logic problems, games and problem-solving sets. This section could be used in many ways by teachers and so adds to the flexible nature of usage for the tests. The Year 9 text has no less than 168 questions in this section. For example:

- Five friends exchange presents at a Christmas party. How many presents are exchanged?
- How many times do I have to multiply 1.03 by itself until I get an answer which exceeds 2?

These questions would make nice starters for investigation activities or for pieces of alternative assessment. They could also be used to keep a fast working student busy and thinking while the class completed set work.

Looking now at the Teacher Resource Book for Year 8-10, the grid pages at the rear of this book will prove useful – there are even two pages of blank graph paper that I often find hard to source as bought graph paper is done in blue and doesn’t photocopy! Most of the blackline masters cover activities in the Year 8 and 9 texts with the Year 10 section being very small. The activities included are referenced back to specific pages in the text. The activities are OK but could have been more inspiring.

Overall, this is a well-constructed series of texts that would work efficiently in the classrooms of Queensland schools.

Lydia Commins
Ipswich Girls Grammar School

Trade & Business Maths 1

Sue Thomson and Ian Foster

Longman

This is the first of a series of two student textbooks written for the Queensland Study Area Specification, Trade and Business Mathematics.

The book is attractively presented with well-spaced text and black and white line drawings. Some use is made of a second colour for headings and diagrams. The format and language of the book are appropriate and appealing to students. Unfortunately, Trade & Business Maths 1, in common with many mathematics textbooks, lacks an index.

The content is organised thematically. For example, Chapter 1 is titled *Maths in Hospital* and includes exercises (which the authors term worksheets) on *reading scales, rounding off measurements, rounding decimals,* etc. The worksheets can stand alone with respect to content, so teachers can pick and choose if they do not wish to follow the authors’ thematic approach.

Each chapter has a common overall structure, beginning with a chapter overview, followed by syllabus objectives and key terms. The syllabus objectives include a list of core units, core unit options and modules addressed within the chapter. Within the body of the chapter, these core units, core unit options and modules from which the content is derived are identified with icons. Teachers will welcome this feature of the text.

Each chapter includes worked examples, most of which provide plenty of guidance for the students and sometimes include key sequences for a calculator. Some examples, however, would benefit from further explanation. For example, students are advised to

"Check these examples carefully ...

a $2 \times 3^2 = 18$
b $9 + 2^3 \times 4 = 35$" (p.15)

Word explanations are provided on order of operations, but there is no preceding review of the use of index notation and radical signs.

The authors provide, where relevant, suitably concise, highlighted rules and summaries. Liberally scattered through the chapters are snippets headed *Did you know?* designed to provide interest and relevance.

The text features module practice tests. The practice tests claim to cover all the skills in the topic (ie. module). They are designed to enable students to check their grasp of the relevant material before attempting the module competency test provided in the companion teacher's resource on CD-ROM. Individual items in the practice tests are cross-referenced back to the related worksheet within the body of the text to enable the student to revisit the skill. Additional worksheets will also be available from the teacher's resource.

Teachers will welcome the final package of two texts and CD-ROM as it provides a much-needed set of resources for this Study Area Specification.

Sue Reid
Redbank Plains SHS

Topics in Senior Maths Books 1 and 2

Barbara Lynch

Published by Emerald City Books 1998

Pages per Book: 48

RRP: $55 each or $45.50 each for schools

Both books have been written to provide teachers across Australia with ready to use worksheets for within their classroom. Book 1 covers the topics of: Number; Data and Analysis; Algebra; Trigonometry; Measurement; Money and Finance; Review of Skills; and Mathematical Activities. Book 2 incorporates: Probability; Graphs and Relationships; Maths in Space; Land and Time Measurement; Mathematics in Construction; Review of Skills; and Mathematical Activities. While not directly written for Queensland, both books are a valid resource for teachers of Mathematics A.

The worksheets expose students to a variety of question types including: multiple choice; short and extended answer; investigation; modelling; practical problems; and puzzles. Each question is clearly worded, with easy to follow instructions and an appropriate use of vocabulary. The questions selected for each worksheet follow a well balanced mix of pure mathematical and life related problems.

Each worksheet is very well presented with: a title; page border; easy to read text, diagrams and tables; and formulae in italics. The same page layout is maintained in the answer section. Answers are provided to all questions including those that require the construction of tables, graphs and diagrams etc. All worksheets have been designed to be photocopier friendly. For example the shading used in linear programming section (algebra) is dark enough to maintain the same quality when photocopied.

The time taken for students to complete each A4 (single sided) worksheet suits the length of any single period within our school system. Each worksheet starts with content/recall and progresses sequently to more difficult process orientated questions. The depth of difficulty of Senior Maths Books 1 and 2 does not meet the requirements of our complex CIII application section. However, the worksheets would prove useful for student consolidation and revision with or without their regular classroom teacher.

I would recommend Senior Maths Books 1 and 2 for their intended purpose of providing worksheets to supplement your existing resources.

Joseph Abeya
Lowood State High School

Empowering students

to accept responsibility for their own learning

Lyn Nothdurft
St Patrick's College, Gympie

This paper discusses a study seeking to empower students to become autonomous (or self-directed) learners who accept the responsibility for their own learning.

Rationale for change

The study was conceived in response to concerns about students' lack of independence: many students expect the teacher to provide them with sufficient information for assessment purposes, and make little effort themselves to understand the concepts. Research has shown that many students believe that *doing* mathematics means following the rules given by the teacher and *knowing* mathematics means remembering and applying the correct rule (Lampert, 1990). They then expect learning to occur through the teacher presenting fixed information and instructions, which they record for recalling later (Baird & Mitchell, 1991). Inappropriate behaviours, such as passive learning behaviours and the use of superficial learning strategies, have been related to these beliefs and expectations. Passive learning behaviours include not engaging in nor persisting with mathematical activities such as problem solving. Students who use superficial learning strategies can manipulate knowledge by juggling formulae and reproducing memorised textbook knowledge, but are unable to apply that knowledge because they do not understand what they are doing or why they are doing it (Ramsden, 1992). This results in such students being unable to apply known facts and formulae, interpret information, develop models or evaluate solutions when solving problems.

Modern curriculum documents in mathematics education, including *The National Statement on Mathematics for Australian Schools* (Australian Educational Council, 1991), and *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989), as well as the *Queensland Senior Syllabus in Mathematics B* (Board of Senior Secondary School Studies, 1992), have recommended emphasising the understanding of mathematics, especially problem solving, investigation and inquiry and applications of knowledge. This involves a shift from the traditional teacher-centred approach, underpinned by the concept of transmitting approved knowledge from the teacher to students, to a student-centred approach, requiring students' active engagement in learning and acceptance of responsibility for their learning.

Purpose of the study

This study investigates the empowering of students to become autonomous learners. The approach focuses on:

- the roles of student and teacher as learner and facilitator of learning respectively, and
- learning as requiring the active involvement of the student to develop deep understanding.

The term *empowerment* refers to feelings of individual self-worth and self-reliance and to the capacity to exercise power (Smith, 1993). This study seeks to enhance students' self-awareness and commitment to taking greater responsibility for their own lives. It also aims to make the students more aware of their understandings and expectations about teaching and learning, and willing to change to improve their learning, through their sharing of perceptions and reflections, and involvement in the planning of activities or teaching/learning approaches (Smith, 1993, p. 81). Autonomous learners are considered to have: a concept of themselves as independent and self-directing learners, able to set goals and accept the responsibility for their own learning, and to learn effectively; the ability to select and use effective learning strategies, and an awareness of their learning; and the ability to work cooperatively and individually, and to relate to teachers as facilitators of their learning (Higgs, 1988; Candy, 1991).

Context

The study involves the Year 12 Mathematics B class at a coeducational Catholic College. There are fifteen students in the class, all currently achieving at Sound Achievement level or better. The students' previous experiences of school mathematics have generally been in classes based on the following traditional teaching sequence: the teacher reviews homework; the teacher introduces the new concept, giving a rule or procedure, which the students copy into their workbooks; the teacher works through an example, which the students copy; and the students then work individually through related examples and complete these for homework.

The changed teaching approach involves the use of an inquiry-oriented approach, collaborative learning and reflection on learning. New concepts are introduced through a problem which groups of students seek to solve using their current knowledge. After class discussions focused on the concepts, the students are provided with worksheets detailing one to three weeks' work, including what they are required to know and be able to do, as well as exercises to work. This allows the students the freedom to plan their use of time, deciding how much should be done in class, and how much for homework. The students are required to develop their own procedures and rules, although I provide scaffolding by checking these, especially early in the year.

The approach is a partnership between the students and me aimed at improving their learning outcomes. The students are becoming aware of their learning: how they go about it, and what strategies are proving effective for them. Class discussions and reflective journal writing are used to determine, and if necessary modify, approaches to teaching and learning. Thus, the students' insights into the teaching and learning process can change practices to enhance their learning.

Setting the scene for a changed approach

At the beginning of the year, the class discussed why they had experienced difficulties in learning mathematics, and developed the following categories of reasons:

- Attitude and effort – Many students found it hard to be sufficiently motivated to make the effort they considered necessary and to persist in maintaining concentration in class and at home. They felt they needed to give a lot of time to learning mathematics, but often did not because of frustration with their lack of success.
- Difficulties in learning – Students indicated that they found it hard to learn because they needed to remember a lot of rules, but did not know why or when to use them. Some believed they were required to work through questions following specified steps, even if this was not the way they thought would solve those problems. They also found a conflict between following prescribed procedures for class work, and then needing to think creatively to solve problems.
- Difficulties due to teaching – Concerns in this category included learning a procedure and then being expected to answer questions using it, but not knowing why they were doing so. They mentioned that they generally could not see the point of doing much of the work, because it did not appear to have any application or use out of school. New work was generally not perceived as being related to previous knowledge.
- Content – Some found the formal language and symbolism alienating. Most believed that mathematics allows only one correct answer and one solution path for problems, requires the exact following of procedures and memorisation of formulae, is a collection of unrelated concepts, and would never be of any use to them.

The students discussed their understanding of what *learning* mathematics meant and the learning strategies they currently used. These emphasised recall. I explained that my approach to teaching would be based on increasing their awareness of their learning and helping them take more control of their learning, and that I planned to do this through having them reflect on their learning, work together collaboratively, and seek to understand what they are doing and why they are doing that. The class related strategies to their list of difficulties. This provided a way of discussing what could contribute to effective learning, including what they could do themselves to improve their own learning.

Setting the scene for the changed instructional approach by suggesting it as a response to student concerns and difficulties, has proved most important. It contributed to the students' awareness of their learning and provided them with a way to have input into, and take some responsibility for, how learning would be approached in the class.

The students' role has become that of active partners in the instructional process, not that of subjects who have teaching *done* to them. I had used the same sort of approach to teaching the previous year, but without this preparation and involvement of the students. While the top third of that class responded very well, many others felt that I was being unreasonable, and was not doing my job as the teacher. They wanted me to just tell them what they had to know so they could learn it, and considered understanding to be an "optional extra".

Strategies being used with the class

The teaching/learning approach is based on students actively seeking deep understanding and taking control of their own learning, with an emphasis on their "making sense" of concepts, learning collaboratively, and being aware of their learning.

New concepts are introduced using an inquiry-oriented approach. Meaning is developed cooperatively as the students engage in mathematical activity and dialogue with the teacher concerning their interpretations and solutions, and reflect on the activity to ensure that the processes are explicit (Alro & Skovsmose, 1996). The problem or situation provides a context for the new learning. The students discuss, question, and conjecture possible solutions by extending their previous knowledge. Groups of students present their solutions to be critiqued by the rest of the class. This approach emphasises the importance of understanding what they are doing, and why they are using a particular method. The students are asked to focus on the processes they use to solve problems. They find that they generally have skills and knowledge to be able to solve the problems, and so realise that they do not need to depend on the teacher to provide set procedures or algorithms for every concept, but can find a method for themselves.

A collaborative approach allows the active involvement of the students in their learning. By working together, the students can explain their understandings and help each other recall previous learning. Meaning is made overt through the group discussion, and the students can appreciate the use of different solution paths. The small group situation provides a safe environment for suggesting or questioning a process. The teacher is no longer the sole source of knowledge in the classroom, and the students realise that they learn well themselves through explaining to their peers. These realisations imply change in the traditional roles of teacher and students, with the students adopting a more autonomous role.

Control over learning can be increased as students become aware of how they learn mathematics (Borasi & Rose, 1989). This is being done using reflective journals, and class discussion of teaching and learning strategies. Successful mathematics problem solvers have been shown to be more aware of what they know and able to plan how to use it; to more accurately monitor their strategies, actions and intermediate results; and to be better able to revise their thinking if necessary (Schoenfeld, 1985). The students are asked to reflect on their learning in general, and to focus specifically on the planning, monitoring, and evaluating of their problem solutions. They monitor their understanding and goals, both in preparation for assessment to consider what they need to do, and after assessment to become aware of what worked.

The effects of the changed approach

This discussion about the effects of this approach to teaching and learning focuses on the extent to which students see themselves as autonomous (in control of their learning); their concept of themselves as learners, and the effects on their results; the value given to deep understanding; and the classroom environment. It is supported by student comments made in interviews or journal writing, and the outcomes of a questionnaire concerned with autonomous behaviours, based on Higgs (1988) and Candy (1991). In the questionnaire, the students gave their perceptions of their degree of autonomy compared with the previous year on a 5 (highest) to 1 (lowest) scale.

The students see themselves as being more autonomous, as indicated by their responses to this item from the questionnaire:

> Your concept of yourself as an independent and self-directing learner, able to accept the responsibility for your own learning.

The mean for their autonomy this year was 3.86 compared with 2.79 for last year ($p = 0.0025$).

The approach to instruction has been that the students do the learning and the teacher facilitates this. Most have indicated that they are working harder in class this year, and are thinking more about what they are doing. One student stated:

> *I find because you're putting all the teaching on us, that we've got to understand it. We're accepting more responsibility, more than we did last year, because last year I kept wishing the teacher would do more. You just kept relying on the teacher, but now you realise it's how much you do yourself.*

I am able to act more as a resource person, doing a lot less talking and a lot more listening. Instead of needing to rush to attend to students needing help on nearly every question, I am able to spend five or ten minutes sitting with a group listening to their approaches, often without contributing at all, because they are working out what to do themselves. There has been no indication that any student considers that I am not teaching them, as occurred during a pilot study last year. Instead these students have described the approach as "*wanting us to understand, but not helping us too much*". Most students have stated that they feel more mature and responsible, and prefer it when:

> *The teacher doesn't do the work for us, makes us do it ourselves. The teacher is making more demands on us, which is good. You are teaching us to teach ourselves.*

The students are accepting the responsibility for their learning: they blame themselves if they have not done sufficient work. After a disappointing class result on an assessment task, I asked the students what they felt should have been done differently. They stated that it was their responsibility to manage their own learning. This is supported by a comment from a student's journal:

> *I did very poorly on my test. I think it's good that you make us responsible for our own work. I did not manage my time very well, however I think I have learnt, the hard way, but that was good. I don't think I would have learnt any other way.*

The students are more aware of what they are doing in class; and they are finding ways of solving problems without any learned procedures. Most are responding well to the freedom associated with greater autonomy:

> *Being responsible for my learning was great - I could go at my own pace - increasing it as I understood more. I found that monitoring my own work meant that I could work more on understanding rather than just doing the set work. That made it easier to get all the work done that I needed to. I like to work at my own pace and develop ways of doing things that I am comfortable with.*

The results of the student responses to the following items from the Autonomous Behaviours questionnaire indicate that the students perceive changes in their behaviours as learners:

Item	**Mean for current year**	**Mean for previous year**	***p***
Your concept of yourself as an effective learner, able to work confidently and to your own satisfaction.	3.71	2.36	0.0001
Your ability to plan ahead, setting goals and pursuing them with determination.	3.50	2.71	0.0099
Your ability to select and use effective learning strategies; asking for justification for rules, procedures, principles and assumptions; developing sufficient understanding of a concept so as to be able to explain it in words and use it in different situations.	3.50	2.36	0.0006
Your ability to work cooperatively with others, yet enjoy learning individually; to be willing and able to learn with others and share ideas; to know how and when to ask for help and direction; and to relate to teachers as facilitators of your learning.	4.57	3.00	0.0014

Your awareness of your learning, being able to identify your needs when you encounter a problem to be solved or skills to be acquired; your understanding of your own learning style, and of your strengths and weaknesses; and your ability to diagnose, prescribe and evaluate your own progress.	4.29	2.79	0.0001

There has been an overall improvement in results this year compared with the students' Year 11 results. Students have stated that they are better able to cope with problem solving:

> *Last year, if I'd had the problem I had today, I probably wouldn't have had any idea how to do it, because you learnt things but didn't really understand them. Today I found I could do something with it.*

At the beginning of this study, the students indicated that they often memorised rules, although did not know why and so forgot them immediately after assessment; were performing procedures without understanding why; and often equated memorisation with understanding. Students' beliefs about what learning mathematics involves now focus on the importance of understanding what they are doing. One student stated:

> *Understanding in maths is so important. That's what it's all about, isn't it? Isn't that the whole basis for learning? One can't learn without understanding.*

They have stated that understanding what they are doing is very important to them, and they want to know more than just how to do something: they want to know why they are doing it. I have been trying to emphasise this through encouraging their discussion and reviewing of solutions. One student commented about having other students provide solutions for the class:

> *It encourages other students to think about and ask questions about what is going on. I know I certainly concentrate more because I know that there'd be a larger possibility of the student making a mistake.*

Through working collaboratively in small groups, students have become aware that they can work with others towards understanding, can ask other students for help, and that their own understanding is enhanced through explaining concepts to others. One student said:

> *If someone in my group needs help, then I like to help them, because that helps me because I understand more what I'm doing. Like sometimes I know how to do it, but I don't understand why I'm doing it. But when I'm teaching them, people ask questions, like: Why are you doing that? How are you doing that? That makes me think about why I'm doing it.*

They have indicated an enjoyment of mathematics classes:

> *I enjoy maths so much now that I'm succeeding, and also the feeling in the classroom, almost like a bunch of friends getting together to have fun, and that fun turns out to be maths.*

There is also a sense that they are working together to improve the learning of all:

> *I think, the atmosphere is definitely more, you know, more a caring attitude. Everyone wants to learn for their own benefit as well as for everyone else's.*

The students also feel safe and supported in the classroom. I have found it important to provide scaffolding and gradually encourage students to take more responsibility for their learning, so that they do not become concerned that they are not being taught properly, and believe they cannot succeed with this instructional approach. Students have also indicated that they find classes worthwhile.

> *I really look forward to having Maths B lessons, because I know that if I go there, I'm going to learn something. And I know that I'm going to understand it, not just learn it. It's not just that I know how to do it but don't know why I'm doing it.*

Constraints

In implementing these changes, I have found time to be a major constraint. Having students explore questions, arrive at group solutions, and explain solution processes to other students, are all much more time-consuming than having the teacher provide the information. I have not spent as much time having the students reflect as I had wished, as asking students to reflect on their learning during or at the end of the lesson often seems an intrusion into the learning going on.

It can also be difficult finding problems suitable for an inquiry-oriented approach. These need to be questions that will extend students, but which are suitable for their current knowledge, so that, with some time and thought, they can succeed.

Conclusion

However, this particular class of students has responded enthusiastically to this approach to teaching and learning, with its emphasis on their deep understanding and responsibility for their learning. It has allowed the development of a partnership between the students and me, with their doing the learning and my facilitating it.

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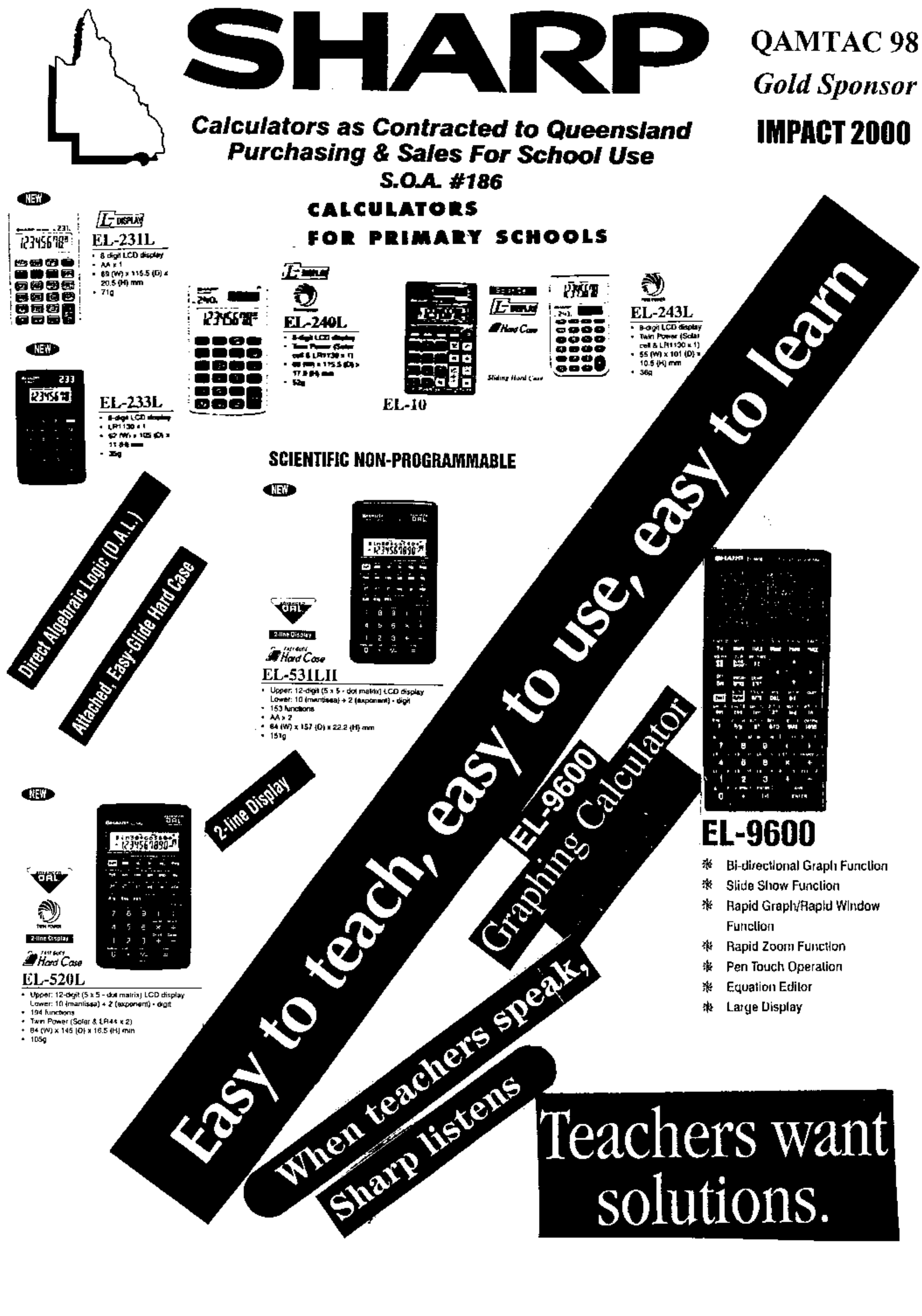
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Teachers want solutions.

Displaying Statistical Data

Steven Nisbet, Griffith University

Introduction

The purpose of this project was to investigate student teachers' methods of visually representing statistical data. *Representing Data* is the third construct of a learning framework for statistical thinking proposed by Jones, Thornton, Langrall, Perry & Putt (1998) which is currently being validated with respect to children in primary school. This construct incorporates constructing representations that exhibit different organisations of the data. As with the other constructs in the framework, four levels of thinking have been proposed for the construct. The four level statements are descriptions of students' displays in terms of two aspects of representing data, (i) validity of a display when asked to complete a partial graph, and (ii) degree of reorganisation of data when asked to produce a display. In this project, student teachers were given the task of designing a poster to represent some qualitative data relating to children's method of transport to school, and hence the data obtained relates chiefly to the second aspect of data representation in the framework.

According to the framework, at Level 1, the student produces an idiosyncratic display that does not represent the data set. At Level 2, the student produces a display that represents the data but does not attempt to reorganise the data. At Level 3, the student produces a display that represents the data but shows some attempt to reorganise the data. At Level 4, the student produces multiple valid displays, some of which reorganise the data. In this project, displays produced by the students were categorised according to visual features and implicit mathematical techniques, and then the categories obtained were assigned to one of the four levels.

Method

The participants in this study were 114 student teachers in 2nd year of BEd Primary or BSpecEd. They were given an A5 sheet of paper with the following printed instructions:

> *Some children were talking about how they travelled to school. This is what they said.*
>
> *Alice comes by bus. Brendan comes by car. Cathy rides a bike. Dennis comes by bus. Elouise walks to school. Francis comes by bus. Gail comes by car. Herby walks. Ilsa comes by car. Jack comes by bus.*
>
> *In the space below, design a little poster which shows this information in a clear way.*

Prior to doing this task, the student teachers had completed a short unit (four hours) on teaching the 'Chance and Data' strand and had covered skills of drawing various types of graphs – line plots, bar graphs, box plots, and line graphs. The responses were analysed first in terms of the display features and mathematical processes employed and assigned to one of the four levels within the statistical thinking framework.

Results

Analysis of the displays revealed a wide variety of methods employed by the student teachers. A total of 29 categories of methods were found, ranging from displays showing no grouping to various types of tables, pictographs, matrices and bar graphs. The 29 categories were then classified into 11 classes of displays in an attempt to relate the categories with their common elements. The categories and classes are listed below.

A: Ungrouped Class
1. Ungrouped; naive drawing
2. Ungrouped; same order (alphabetical)

B: Four Groups
3. 4 groups; names listed
4. 4 groups; names on tree diagram
5. 4 groups; 4 drawings to label groups
6. 4 groups; 5 drawings (bus, car, bike + 2 walkers)
7. 4 groups; 7 drawings (bus, 3 cars, 1 bike, 2 walkers)

C: Four Groups With Drawings
8. 4 groups (rows); 1 drawing/person
9. 4 groups (columns); 1 drawing/person
10. 4 groups; stick figures for each

D: Four Groups With Numerical Summary
11. 4 groups; drawing for each category; names on tree diagram; numerical summary

E: Tables
12. Table; 4 rows; names listed
13. Table; 4 columns; names listed
14. Table; 4 columns; drawings for headings; names listed

F: Pictographs
15. Table/pictograph; horizontal
16. Table/pictograph; vertical
17. Pictograph; vertical; axes labelled and scaled

G: Matrices
18. Matrix (names by mode)
19. Matrix (mode by names)

H: Tables With Numerical Summary
20. Table with tallies
21. Table with names and scale of no. of students
22. Table; 2 columns (mode of transport, number of students)

I: Line Plots
23. Line plot

J: Bar Graphs
24. Bar graph

K: Two Methods
25. Two methods: Tallies and groups lists
26. Two methods: Tallies and matrix
27. Two methods: Table and group drawings
28. Two methods: Table and bar graph
29. Two methods: Table and matrix

The display classes were then assigned to one of four levels in the Statistical Thinking Framework. As the task of designing a poster required students to organize and reduce the given data, as well as produce a visual display, it was appropriate to assign levels for *Organzing and reducing data* as well as *Representing data.* (See Table 1.)

Table 1: Levels assigned in the Statistical Thinking Framework for each class of display.

Class of display	Level: Organizing and reducing data	Level: Representing data	No. of students
A:Ungrouped	1 (A1) 3 (A2)	1 (A1) 2 (A2)	1 2
B: Four groups	3	3	31
C: Four groups with drawings	3	3	10
D Four groups with numerical summary	3	3	2
E: Tables	3	3	19
F: Pictographs	3	3	8
G: Matrices	3	3	18
H: Tables with numerical summary	3	3	3
I: Line plots	3	3	2
J: Bar graphs	3	3	6
K: Two methods	4	4	11

The results show that the majority of the student teachers were at Level 3 according to the criteria for both constructs *Organizing and Reducing Data* and *Representing Data*. A small number were at Level 4, and only one or two were at Level 1 or 2. Although assigned to Level 3 (Quantitative) according to the criteria, some categories were not necessarily quantitative. This result suggested that Level 3 could be subdivided to distinguish between the various features of the many classes and categories of displays included in the level. The following subdivision of Level 3 for *Representing Data* is proposed to acknowledge features such as the use of two dimensions to show groupings, the inclusion of numerical summaries, and the knowledge of formal metric methods. (Table 2.)

Table 2: Proposed subdivision of Level 3 of *Representing Data*

Level	Label	Classes included	Description
3a	Simple verbal	B: Four groups C: Four groups with drawings	These displays show groupings but have a simple visual organisation or no particular visual organisation.
3b	Verbal: 2 dimensional	E: Tables G: Matrices	These displays use two dimensions to show groupings.
3c	Numerical	D: Four groups with numerical summary H: Tables with numerical summary	These displays show groupings plus numerical summaries of the data.
3d	Metric	F: Pictographs I: Line plots J: Bar graphs	These displays use measurable quantities such as length or area to convey a message, and allow visual comparisons to be made.

The labelling and ordering of the sub-divisions has been done intuitively and according to what seems to be a reasonably logical progression. However, there is no evidence as yet to confirm that the progression is developmental. It should also be noted that all levels apply only to the responses, and do not indicate fixed levels for the student teachers and their ability levels. Their responses were probably influenced just as much by the context of a primary mathematics method subject and the associated valuing of creative thinking as by their knowledge of statistics and graphing techniques.

Conclusion

The outstanding feature of the results of this study was the wide variety of data displays produced by the student teachers. The large majority of the responses were assigned to Level 3 in the Statistical Thinking Learning Framework (Jones et al, 1998), and a proposed subdivision of that level identifies significant differences between the 11 classes and 29 categories of displays and acknowledges features such the use of two dimensions to show groupings, the inclusion of numerical summaries, and the knowledge of formal metric methods.

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Q. Why did the trigonometry teacher think her students looked pale?
A. Cos there was no sin of a tan.

Understanding r^2

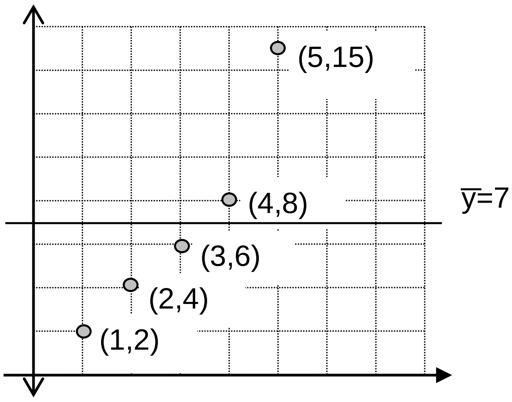
Rex Boggs, Glenmore State High School

Thanks to Al Coons for sharing his excellent explanation of r^2 (of which this article is an adaptation) and to Gretchen Davis from Santa Monica HS for this most useful dataset.

Consider the following set of paired data

(1,2), (2,4), (3,6), (4,8), (5,15)

plotted on the scatterplot below.



The mean of the y-values is

$$\bar{y} = \frac{2+4+6+8+15}{5}$$
$$= 7$$

If we know nothing else about this set of data, we can at least say that the mean of the dependent variable is 7. Hence a very simple model for this dataset is

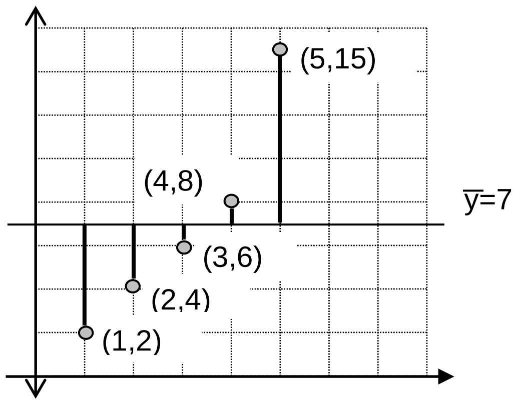
$$\bar{y} = 7$$

A better model will include the variability of the data about the mean.

point	(1,2)	(2,4)	(3,6)	(4,8)	(5,15)
$y_i - \bar{y}$	-5	-3	-1	1	8

However the sum of these differences is 0. As with our formula for standard deviation, we can remove the negative values by squaring these differences.

point	(1,2)	(2,4)	(3,6)	(4,8)	(5,15)
$(y_i - \bar{y})^2$	25	9	1	1	64

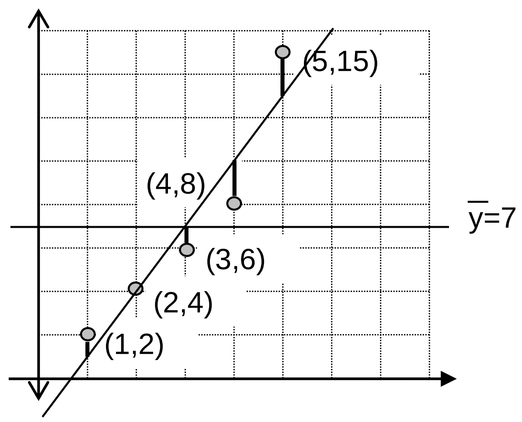


The total of the squared differences, called SST, equals 100.

In this improved model, $\bar{y} = 7$ and SST = 100.

A much better model for the data than the line $\bar{y} = 7$ is the line with equation $y = 3x - 2$, as it "fits" the data much better. We can determine the variability of the data about this line from the graph:

point	(1,2)	(2,4)	(3,6)	(4,8)	(5,15)
y_i - y	1	0	-1	-2	2
$(y_i$ - $y)^2$	1	0	1	4	4



A line that models a set of data is called a **regression line**. The values in the row headed $y_i - y$ are called the residuals. For each data value, the **residual** is the difference between the actual data value and the value predicted by the regression line (called the **fitted value**).

Residual = Data – Fit

The row headed $(y_i - y)^2$ contains the squared residuals. The sum of the squared residuals,

SSR = 1+0+1+4+4 = 10. So
SSR = 10

Can we find an equation with a smaller SSR?

It so happens that for this dataset, the line $y = 3x - 2$ is the line that gives the smallest value for SSR. To prove this requires using either calculus, or a lot of very clever algebra. The regression line with the smallest value for SSR is called the **least squares regression line**, because it is the **regression line** that makes the sum of the **squares** of the residuals the **least**.

How well does this regression line 'describe' the data?

One measure of this is the ratio of SSR to SST.

$$\frac{\text{SSR}}{\text{SST}} = \frac{10}{100} = 0.1$$

In a sense, 0.1, or 10%, of the "variation" about the mean *isn't* explained by the least squares regression line; hence 0.9, or 90%, of the variation *is* explained by the least squares regression line.

The statistic that indicates what proportion of the variation about the mean is explained by the regression line is called *r*-squared and is written as r^2. It is also called the **coefficient of determination**.

For this dataset, $r^2 = 0.9$. The coefficient of determination is what statisticians are referring to when they say, for instance, '90% of the variation in the data is explained by the regression equation.'

The formula for r^2 is:

$$r^2 = 1 - \frac{SSR}{SST} \ .$$

This is also the formula for another statistic, R^2, which is a measure of the variability that is explained by the least squares regression line in the multiple linear regression model

$$y = a_0x_0 + a_1x_1 + \ldots + a_nx_n \ .$$

In multiple regression, the dependent variable (called the response variable in statistics) is dependent upon multiple independent variables (called explanatory variables). The multiple regression model is used by graphing calculators for quadratic, cubic and quartic regression. For these models, the coefficient of determination reported is R^2 and not r^2 (even though it is really the same thing).

2 + 2 = 5 for extremely large values of 2

PYTHAGORAS REVISITED
Lessons from the Past

Paul Dooley, Fairholme College, Toowoomba. e-mail pmdooley@mathgym.com.au

Over the next four issues of *Teaching Mathematics* the author will attempt to show how historical, cultural, social and philosophical contexts can be used in the teaching of some pretty familiar, and sometimes dry mathematical concepts to secondary students. The author has found that adolescents show a keen interest in this approach , as it gives them some understanding of their place in space in time. The articles which appear in this journal are shortened extracts from a collection of essays and activities on the authors web site http://www.mathgym.com.au. To read a more expansive treatment and to obtain a copy of any of the Activity sheets - Click on the "history" button at the site and then follow the link to the activity you want. You can then print out the page to get a black-line master for reproduction.

Setting the Scene

Before we get on to the mathematics of the Pythagoreans, I like to "set the scene" with my students and do a bit of proportional thinking with this activity:

I get students to place some milestones in Western history (from 20 000 000 000 BC to 2000 AD) into a compressed time interval of 24 hours. This practices proportion and helps students appreciate the relatively short time Homo sapiens has been in existence. A reproducible master for this can be found on the web site.

Some background

Following this activity, I like to use this time-line to discuss with the students our present understanding of the major milestones which have led to Western Civilisation. We talk about (there is greater detail on this on the web site) the Stone and Bronze Ages and the type of nomadic existence we think humans had in those times, and the type of mathematics they found useful. We talk about the effect on world population of the passing of the Last Ice Age, the receding grasslands, and the gravitation towards, and eventual conflict over, the "Fertile Crescent(s)", leading to city life with its division of labour. (The Old Testament can be seen in this light – eg. Joshua and the Tribe of Israel at Jericho). I introduce some information on Babylonian and Egyptian society and in particular their mathematics, leading up to the remarkable sixth century BC.

I say remarkable because it was in the sixth century BC, humans seem to have asked the definitive questions "Why?" and "How?". This didn't happen only in Greece with the Ionian philosophers and Pythagoras but also in India (Buddha) and China (Confucius). Arthur Koestler [1] says of this century*, "A March breeze seemed to blow across this planet from China to Samos, stirring man[kind] into awareness, like the breath in Adam's nostrils"*. Koestler goes on to assert that *"Every philosopher of the period seems to have had his own theory regarding the nature of the universe around him"* and he likened the sixth century to an orchestra; *"expectantly tuning up, each player absorbed in his own instrument only, deaf to the caterwaulings of the others"*. He then describes his vision of the importance of Pythagoras, and it is beautifully put:

"Then there is a dramatic silence, the conductor enters the stage, raps three times with his baton, and harmony emerges from the chaos. The maestro is Pythagoras of Samos, whose influence on ideas, and thereby on the destiny of the human race was probably greater than that of any single man before or after him."

This vision of Pythagoras so appealingly painted by Koestler is the one which most of us identify with, and yet there is no direct evidence to support it. Much of what we know about Pythagoras has had to be inferred from texts written many centuries after his death and much of what has been passed off as Pythagorean is pure fancy.

John Burnet [2] gives a good description of the historical references to Pythagoras and the

Pythagoreans. In it he asserts that there is only one (sketchy) reference to Pythagoras which is contemporary to his generation, some credible references from the next generation, and some credible (but reserved) discussions in the next century by Plato, Aristotle, and Timaeus. In the 3rd century BC the essays on Pythagoras by Porphyry, Iamblichus, and Diogenes Laertius are viewed by Burnet as a *"mass of incredible fiction"* dwelling on the mystical cultish nature of the Order. Interestingly, in the earlier references there appears to be a tension between a respect for the philosophy of the Pythagoreans and caution not to appear sympathetic to the role of the Pythagorean Order in Ionian politics and religion. As in all historical study, one has to know the "political correctness" of the times to fully appreciate the opinions expressed.

Pythagoras (580?-501? BC) – his life and times

Burnet claims that we may be reasonably certain that Pythagoras passed his early manhood on the island of Samos, and was the son of Mnesarchus in the reign of Polycrates (532 BC). He is sceptical about the extensive travels attributed to Pythagoras by late writers, even his much quoted visit to Egypt. He cites credible sources that say Pythagoras left Samos (in order to escape from the tyranny of Polycrates), that he came to Italy in 529 BC and that he remained at Croton for twenty years. He retired to Metapontum when the Crotoniates rose in revolt against the authority of the Pythagorean Order.

Pythagorean Order

Burnet sees the Pythagorean Order as essentially "*a religious fraternity, and not, as has been maintained, a political league*." He asserts that the main purpose of the Order was the cultivation of holiness and that it resembled an Orphic society.

For a time the Pythagorean Order held supreme power in the Achaean cities, but the surge of democracy brought many purges against it resulting in the death or exile of members of the Order. At some time after this, the Pythagorean Order was able to return to Italy, and acquired great influence over the next two centuries.

The teachings of Pythagoras and the Pythagoreans

Burnet states: *"Pythagoras apparently preferred oral instruction to the dissemination of his opinions by writing, and it was not till Alexandrian times that anyone ventured to forge books in his name. The writings ascribed to the first Pythagoreans were also forgeries of the same period. The early history of Pythagoreanism is, therefore, wholly conjectural; but we may still make an attempt to understand, in a very general way, what the position of Pythagoras in the history of Greek thought must have been. "*

Teaching a way of life

We get some insight into the lives of the Pythagoreans when we realise that they taught the transmigration of souls (the belief that on death we return as another) and that they had many taboos. See for example the following edicts:

1. To abstain from beans.
2. Not to break bread.
3. Not to stir the fire with iron.
4. Not to eat from a whole loaf.
5. Not to eat the heart.
6. Not to walk on highways.
7. Do not look in a mirror beside a light.
8. When rising from the bedclothes, to roll them together and smooth out the impress of the body.

The Pythagoreans taught that the purpose of life was to purify the soul and body. They expanded on their Orphic beginnings to include the "purification" through science and knowing. So to reach purification one had to discover the "harmonies" of the cosmos – and scientific (mathematical) enquiry was the vehicle with which to find them.

Arguably the greatest scientific achievement of the Order was the discovery (attributed to "The Master" as Pythagoras was called) of the mathematical order in the musical scale and the harmonies so produced. It is not difficult to appreciate how the Pythagoreans would extrapolate from this success to the belief that, in the quest for the secrets of the cosmos "all is number".

Mathematical teaching

Tradition has it that most of the first two books of Euclid's Elements can be attributed to the Pythagoreans. If this is the case, then much of what is still taught to adolescents around the world today as geometry, number, space and algebra is essentially Pythagorean. Since the initial discovery of these concepts, text book writers have developed and abstracted them to the point where it is difficult for us to get a clear view of how the Pythagoreans actually understood their cosmos. The way they visualised number is different from the way we think of number. To the Pythagoreans number was a living thing - it was real, a substance of the landscape. It was there all around them, but hidden; the way to enlightenment was in the discovery of its secrets.

Pythagoras' Theorem – some background

Firstly, we need to appreciate that Pythagoras did not ***discover*** the relationship between the lengths of the sides of a right-angled triangle. This relationship had been known in Babylon and Egypt for centuries (if not millennia) before. Tradition has it though, that Pythagoras did find the first rigorous geometric proof. In the essay on the web site I describe an extraordinary clay tablet containing evidence of Babylonian knowledge of the relationship.

Numerical relationship

I like to start my lessons on Pythagoras' Theorem not with geometry, but with number patterns as this is recorded by Proclus (410-485 AD) as cited in Heath [3] to be the method used by Pythagoras:

"Certain methods for the discovery of triangles of this kind are handed down, one of which they refer to Plato, the other to Pythagoras. [The latter] starts from odd numbers. For it makes the odd number the smaller of the sides about the right angle; then it makes the square of it, subtracts unity, and makes half the difference the greater of the sides about the right angle; lastly it adds unity to this and so forms the remaining side the hypotenuse. For example, taking 3, squaring it, and subtracting unity from the 9, the method takes half of the 8, namely 4; then, adding unity to it again, it makes 5, and a right-angled triangle has been found…"

This method does not yield all Pythagorean triples (Pythagorean triples are the integers which can form a right triangle e.g. 3, 4, 5). It should be seen that the problem of finding Pythagorean triples **is to find two square numbers such that their sum is also a square.**

One possible way that Pythagoras may have arrived at the relationship is through the use of the *gnomon*. The *gnomon* is the name given to the oblique staff which is used to cast a shadow in the sundial. It is claimed that Anaximander, a contemporary to Pythagoras, introduced the *gnomon* to Greece from Babylon. As we will see soon, the *gnomon* became synonymous with the numbers formed by the difference between successive squares – the odds. We know that Pythagoras investigated the figurate numbers and their related patterns. He would possibly have represented the consecutive square numbers 1, 4, 9, 16, 25 as stones grouped as in the figure (figure 2A) below:

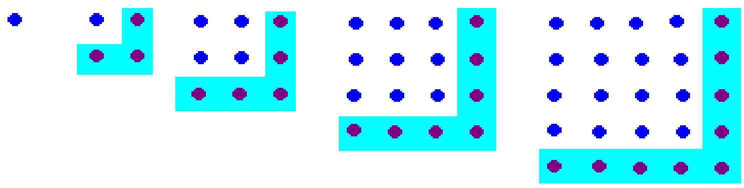


figure 2A - consecutive squares showing gnomon

With a bit of inspection, one can see that to make 4 from 1, it only takes the addition of the "L" shaped (*gnomon*) figure of three stones. To make 9 from 4 the *gnomon* is made up of the next odd number, 5. If you continue investigating this pattern you will eventually arrive at a shape where the *gnomon* is also square i.e. 16 + 9 = 25; the first Pythagorean triple.

A related, but more abstract and therefore less likely, way that the relationship may have been seen is by writing down three rows of numbers (a) the counting numbers, (b) their squares and (c) the successive odd numbers constituting the difference between the successive squares (figure 2B). By picking out the numbers in the third row which are squares, Pythagoras would have arrived at the same point as above.

Counting Numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Squares	1	4	9	16	25	36	49	64	81	100	121	144	169	196
gnomon		3	5	7	9	11	13	15	17	19	21	23	25	27

figure 2B

It seems unlikely that Pythagoras would use this device because Burnet argues there is no evidence to suggest that the concept of number had progressed much more than as a collection of pebbles at this time. It is only over the next few centuries that the Pythagoreans developed an abstracted number sense which would allow a representation as above.

It is worth noting that neither method would expose the incommensurables (the irrational numbers) and neither is useful as a general proof of the theorem.

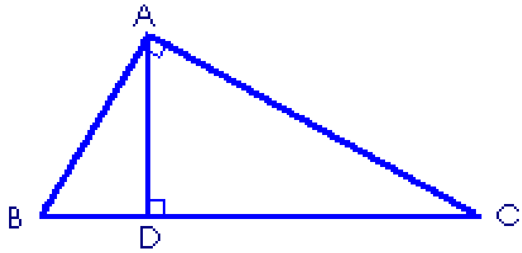
A reproducible black-line master that can be used to further investigate this numerical relationship can be found on the web site.

Geometric proof

Whether the relationship was taught to him, or whatever method Pythagoras used to find triples, it is generally accepted that he was the first to give a general proof for the relationship. Heath claims that the proof given in Proposition 47, Book I of Euclid's Elements is generally attributed to Euclid and makes use of Propositions not known to Pythagoras. He goes on to describe the following proof using similar triangles and proportion as the one most probably used by Pythagoras:

"One method is to prove, from the similarity of triangles ABC, DBA, that the rectangle CB,BD is equal to the square on BA, and, from the similarity of the triangles ABC, DAC, that the rectangle BC,CD is equal to the square on CA; whence the result follows from addition"

Notice how the product of CB and BD is described as "the rectangle CB,BD". This is how the early Pythagoreans understood multiplication – as a physical area (as apart from an abstract product). In modern courses this same explanation would be:



Triangles ABC, and DBA are similar. (AAA)

$$\therefore \frac{BC}{AB} = \frac{AB}{BD}$$

$$\therefore AB^2 = BC.BD ....1$$

Similarly triangles ABC and DAC are similar

$$\therefore \frac{BC}{AC} = \frac{AC}{DC}$$

$$\therefore AC^2 = BC.DC ....2$$

Combining 1 and 2 we get

$$\begin{aligned} AB^2 + AC^2 &= BC.BD + BC.DC \\ &= BC(BD + DC) \\ &= BC^2 \end{aligned}$$

This is the familiar theorem. There are many different proofs for this theorem of Pythagoras'. On my web site I have a neat little interactive animation which students can use to demonstrate the theorem by one of the many dissection methods.

In the next Edition of *Teaching Mathematics* I will try to describe how the Pythagoreans saw and analysed number.

References:

[1] Koestler, Arthur, *The Sleepwakers - A History of Man's Changing Vision of the Universe.* London: Hutchinson & Co. 1959.

[2] Burnet, John, Early Greek Philosophy, 3rd Ed., London: Adam and Charles Black, 1920.

[3] Heath, Sir Thomas L., *Euclid - The Thirteen Books of The Elements Second Edition Vol 1*, N.Y. Dover Publications (orig. 1908).

Evolution of the Maths Problem (a slightly cynical view)

1959: A logger spent £94.10.11½d cutting down a tree. He sold 42 feet 8½ inches of timber at £3.13.6d per foot. What was his profit?

1969: A logger spent \$400 cutting down a tree, then sold 7m^3 of wood at $\$90/\text{m}^3$. What was his profit?

1979: A logger exchanged a set T of timber for a set M of money. The cardinality of the set M is 100 and each element is worth \$1. The subset C of M, with cardinality 70, is his costs. The profit P is defined by $P = M \setminus C$. Draw 100 dots to represent the set M, then find the cardinality of C.

1989: A logger sold 7m^3 of timber for \$1600. His production cost was \$960. Your assignment: underline the number seven.

1999: A logger cut down a beautiful rainforest tree that had been standing for over 300 years. She then sold it for \$180. What do you think of this way of making a living? Topic for class discussion: 'How would the possums feel?'

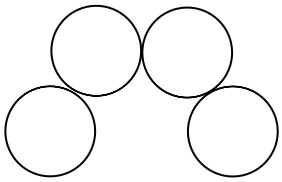
EUCLIDEAN COINS

In Euclidean geometry, students produce exact geometric arrangements of lines using a compass and straight-edge. Working out how to produce certain arrangements can be a worthwhile thinking exercise.

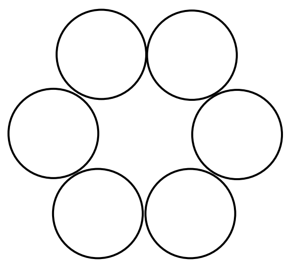
The same kind of thinking can be involved in producing exact arrangements of coins on a table. The coins can be placed on the table and slid from one position to another. But they cannot be lifted. Nor can any measuring or marking be done. Nor can any extra coins can be used. Just as in Euclidean geometry, a good approximation by eye is not what is required.

Here are a couple of examples, the first one fairly easy, the second a bit more challenging.

Arrange four 20c coins as shown, such that, if a fifth 20c coin were produced, it could be placed in the centre of the semi-circle and touch all of the other four.



Arrange six 20c coins such that, if a seventh 20c coin were produced, it could be placed in the centre of the circle and touch all of the other six.



These examples were taken from *The Big Book of Puzzles and Games* by Gyles Brandreth, Treasure Press. Solutions in the next issue.

STUDENT PROBLEM PAGE

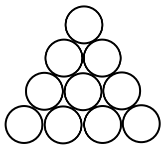
Cheryl Stojanovic Forest Lake College
Garnet Greenbury Publications Committee

Question 1

A store increased the prices of boxed chocolates by 20%. In the clearance sale after Easter, all the prices were reduced by 25%. How do the sale prices compare with the original prices?

Question 2

Boxed chocolate Easter eggs are arranged in a triangular pattern, as shown. The diagram shows 10 eggs arranged with 4 eggs in each of the outside rows. If 78 chocolate eggs were arranged in a similar pattern, how many would there be in the outside rows?

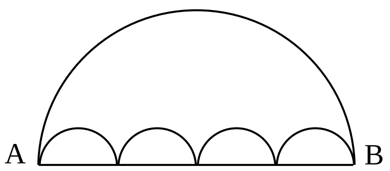


Question 3

At a family reunion, there are 8 teenagers of the same age and five other teenagers of the same age, but different from the first 8. The total of all their ages is 184 years. What age are the older teenagers?

Question 4

Mother Rabbit travels from A to B with one large semi-circular hop. Baby Rabbit travels from A to B using four smaller semi-circular hops as shown in the diagram. Which of the two curved paths is the shortest?



Question 5

Leyah has 100 chocolates. Starting from today, she plans to eat one quarter of the number that remain from the previous day. She will eat chocolates whole and will eat a maximum each day, but never more than a quarter of the remainder. How many chocolates will she eat on day 10 if she follows this plan?

Submitting solutions

Students are invited to submit solutions. Include your name, the problem number, your school and year level (clearly printed). Send them to Garnet Greenbury, Unit 14 Greenleaves Village, Upper Mt Gravatt, 4122. Closing date: 1st May 1999.

Solutions to the problems, Vol. 23 No. 4

1. 23 and 24
2. 33.3%
3. 600 cm²
4. 18 correct
5. 7.5 cm

Prizes

Prizes for solutions to the student problems in *Teaching Mathematics* Vol. 23 No.4:

Katey Robinson of The Gap SHS is the winner of the Penguin book prize.

Christelle Hunt of The Gap SHS is the winner of the annual subscription to *Tenrag*.

Solutions were received from:

Thulimbah State School
The Gap State High School
Bundaberg State High School
Centenary Heights State High School
St Mary's College
Canterbury College
Urangan State High School
Siena Catholic College
William Ross School
Rockhampton Grammar School
University of the Third Age
Stanthorpe State High School