TEACHING MATHEMATICS

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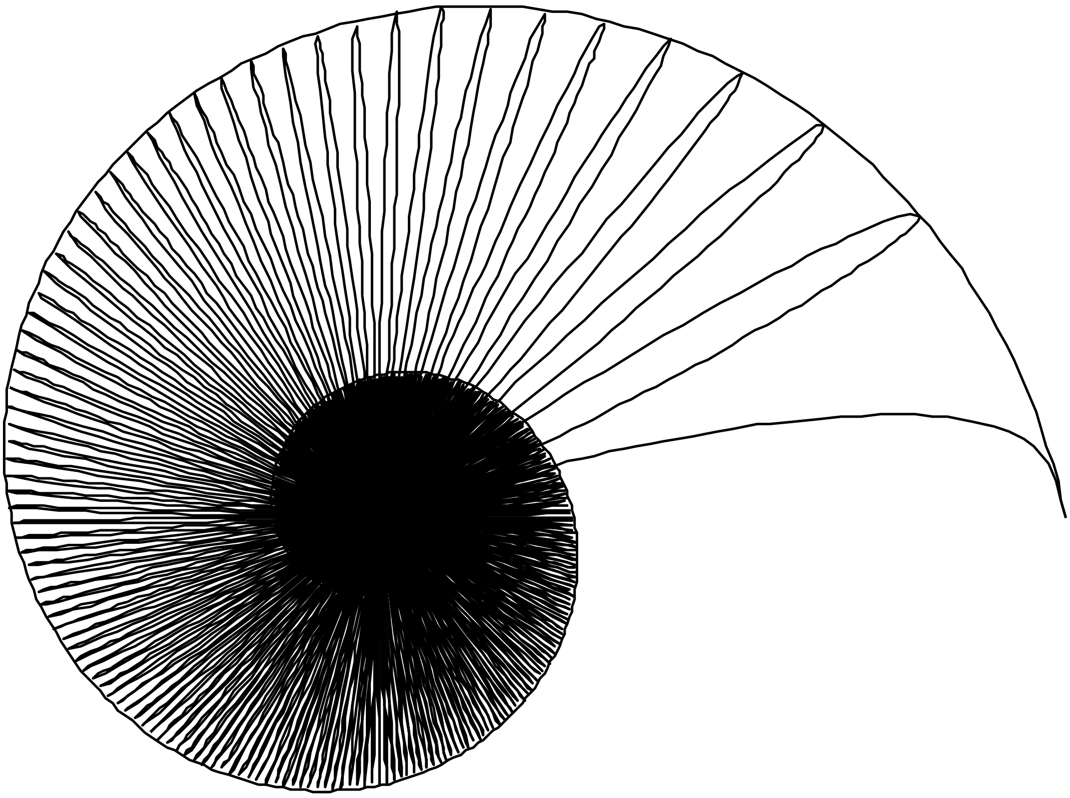

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Teaching Mathematics is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, August and November) and is provided as a right to all members of the association.

The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- and to speak and act on matters related to mathematical education.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the editor, David Ilsley. The preferred way is by e-mail to ilsley@cyberbiz.net.au Materials may also be sent on floppy disk. Contact details are as follows:

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Word for IBM (up to Word 97) is the preferred format. All receipts will be acknowledged – if you haven't heard within a week, ring David to check.

Copy dates are: mid-February; mid-May; mid-July; mid-October.

If you have any questions, contact David Ilsley.

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FROM THE PRESIDENT

John McKinlay
Carmel College

As this will be the last column I write as President of QAMT, I take this opportunity to look back over the past two years and reflect on the work that your association has accomplished. Obviously, limitations on space prevent me from acknowledging all that has occurred (my President's Report at the September AGM will be more substantive) but I can easily describe some of the highlights.

It is clear that QAMT has increased its role as a service provider for teachers of mathematics especially given the demise of the Teacher Support Centres. Thanks largely to Gary O'Brien and his team on the Professional Development Committee, QAMT now runs a very full PD program. We offer workshops to Novice teachers, to secondary teachers wanting to learn more about technology and mathematics, to primary school teachers of mathematics and we ran a forum on the review of the Maths ABC syllabuses, to name a few.

The publications portfolio of QAMT has grown considerably too, with more in the pipeline. David Ilsley has revamped the journal, we published the AToMIC Project and work will soon be completed on the Sub-AToMIC project and an Applications Booklet for Senior Mathematics will soon be under way. Sue Reid has given the newsletter a facelift and now that it is posted separately from the journal, we can communicate more frequently with our members.

It has been one of my ambitions this term to strengthen our ties with the various Local Branches, and I am pleased to say that there is great rapport between QAMT and the various branches. Certainly the opportunity to offer Graphics calculator workshops together has been well received by all. That the Rockhampton Local Branch took up the opportunity to stage the QAMTAC is a testament to their continued interest in working together for the betterment of mathematics education.

As in any large organisation, QAMT continues to function only due to the great work done by a great number of people – from June our office assistant, without whom we could not function as we do, to all those on Committee who regularly attend our monthly meetings and either take the minutes (thanks Merrilyn) or attend to the office (thanks John Butler) or keep us on the straight and narrow (thanks Marjorie).

Thanks to you our members without whom QAMT would not exist – and that our membership numbers have been steadily increasing will be one of the most satisfying memories of my presidency. All the best to the future Executive - we are in for more interesting times in mathematics education in the years ahead!

FROM THE EDITOR

David Ilsley
Queensland School Curriculum Council

I would like to take this opportunity, on behalf of the executive and indeed all our members, to thank John for his hard work and the direction he has given as President over the past two years – and indeed in other executive roles over a considerably longer period.

All the executive and committee positions come up for re-appointment at the AGM at the annual conference in September. I would like to encourage members to consider nominating for one or more of these positions. As the activity of the association increases, we need more and more people to be involved so that individual workloads remain manageable. A nomination form is included with this journal. You can nominate even if you cannot make it to the conference.

Which brings me on to the conference. This is the first time we have held it outside of Southeast Queensland. We hope to get many delegates from country areas, but we are also hoping that many people from Southeast Queensland will be able to attend. Things are looking encouraging at the moment. If you are thinking of going up from Brisbane or nearby, you might consider the Tilt Train – a stress-free seven hour journey in comfort for $134 return.

Hope to see you there!

What's happening?

in mathematics education

Mathematics Education in the UK - The National Numeracy Strategy

Cal Irons QUT

The beginning of a new millennium seems to be the start of many significant curriculum initiatives in mathematics education. In England, the implementation of a new mathematics framework for the primary school begins September, 1999. The change is significant in the content to be covered, the sequence of the development, and the approach to be followed. While observers might criticise the project, there are many proponents for the change and precedents for its success.

The National Numeracy Strategy (NNS) began more than 4 years ago with intensive study and data gathering in countries that were achieving excellent results in mathematics. The study analysed the scope and sequence as well as the classroom teaching practice in mathematics in these countries. Following this analysis, the National Numeracy Project was established in 1996 and implemented in 13 education authorities in separate parts of England. The results in these authorities were closely monitored and the curriculum that forms the numeracy strategy slowly developed. Over the next three years (1999 to 2002), the content of the framework will be implemented in schools throughout England, Scotland and Wales. The material in the strategy does not represent a new curriculum as such. However, the mandatory national curriculum is expected to adopt most if not all of the initiatives in the strategy.

The Numeracy Strategy is significant in many respects. The following points summarise the major changes or directions of the strategy.

1 The materials contain more detail than any previous UK mathematics curriculum. The sketchy outcomes that formed part of the national curriculum have been replaced with more detailed statements as well as many more exemplars at each of the grade levels.

2 A greater emphasis is given to mental calculation. This might be described as a focus on thinking in the context of calculating. The report and the materials provide many examples to help teachers include more strategies and opportunities for mental calculation from year 1 (the second year at school). Paper and pencil calculation is included as one of the calculation strategies but the formal algorithms are not to be introduced until the last term of the fourth year at school.

3 Teachers must provide a daily mathematics lesson. This uninterrupted time is divided into 3 parts. The first part of approximately 10 minutes helps children develop their mental calculation skills. The main part of the lesson is a whole class discussion and supporting activities with a specific focus. The final portion of the lesson is a plenary discussion to 'pull together' the content that was developed.

4 The implementation of the curriculum is thorough and very well organised. Over 300 consultants have been employed to implement the strategy. Extensive amounts of money have been provided for professional development and the preparation of comprehensive course materials.

The materials and the dedication of the people involved in implementing the numeracy strategy impress this reviewer. The ideas and publications that are flowing from the energy of the UK project will provide us in Queensland with a wealth of ideas as we pursue our curriculum review.

From the Journals

Merrilyn Goos
The University of Queensland

The focus for this edition is on students' beliefs about mathematics, how these beliefs affect their subject and career choices, and the ways in which beliefs are shaped by students' classroom experiences.

Australian Senior Mathematics Journal, Volume 13, Number 1 (1999)

Peter Brinkworth and John Truran report on a survey of almost 400 Year 12 students in South Australian schools which aimed to identify their reasons for choosing, or not choosing, to study mathematics, and relationships between students' beliefs about mathematics and their career aspirations.

The status of mathematics as a prerequisite for further tertiary study was a powerful factor in influencing students' decisions to enrol in a Year 12 mathematics subject. Perceived ability and interest in the subject were also important. Nevertheless, most students associated mathematics classes with routine, traditional activities such as copying notes and memorising rules and procedures, even though these activities were not generally seen by students as important for their learning.

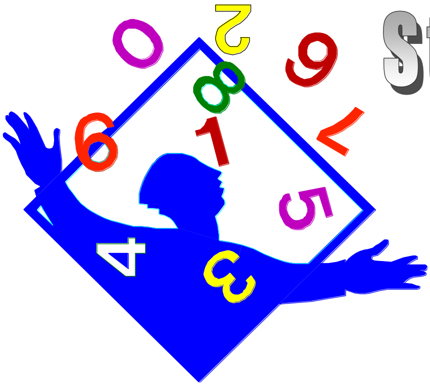
Students' beliefs about mathematics and the work of mathematicians revealed a limited understanding of the relevance of mathematics to a wide range of careers, such as legal or environmental studies, and suggested that students saw professional mathematicians as isolated individuals who are out of touch with issues affecting the wider community.

Queensland Journal of Educational Research, Volume 14, Number 1 (1998)

Similar issues were addressed by Jackie Walkington in research on tertiary pathways taken by girls studying mathematics, science and engineering in Queensland. A longitudinal study of girls in Years 11 and 12 is attempting to identify the major barriers they face in completing their secondary mathematics and science studies. Like Brinkworth and Truran, Walkington also found that perceptions of ability, interest in the topics studied, and information on future career choices affected girls' selection of mathematics and science subjects for Year 11.

Interviews with the students have additionally compared their actual learning experiences in Year 11 with prior expectations, and revealed factors perceived as influencing their success. The teacher was unanimously seen as the most crucial influence on how students experienced the subject, while the nature of classroom activity could also either support or hinder success. In particular, the girls favoured interactive, practical activities over passive note taking. Clearly, teachers play a vital role in shaping students' perceptions of mathematics, and it is recognised that teachers' own beliefs about mathematics are important in guiding their planning and instruction.

However, the relationship between teacher beliefs and classroom practice is not straightforward, as illustrated by two recent studies by Mal Shield (Queensland) and Bob Perry, Peter Howard and Danielle Tracey (NSW). The latter authors report on a survey of the espoused beliefs of 233 secondary mathematics teachers **(Mathematics Education Research Journal, Volume 11, Number 1, 1999)**. These teachers expressed strong support for practices consistent with the National Statement, such as problem solving and collaborative learning, yet about half maintained that the role of the teacher is to transmit knowledge and verify that students have "received" that knowledge. Similar contradictions were observed by Shield, who conducted an intensive case study of one Year 8 teacher and his classes **(Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia, Adelaide, 4-7 July 1999)**. This study revealed how the constraints imposed by syllabuses, assessment requirements, and the daily pressures of teaching can lead teachers to make curriculum decisions inconsistent with their beliefs.



Strategies for Calculating Exact Answers Mentally

Geoffrey R. Morgan
Undurba State School

The resurgence of interest in mental computation, and the recognition of the continued social usefulness of being able to perform a wide range of mental calculations, have contributed to a need to redefine the relative importance of the various methods of calculation in school mathematics. A focus on mental computation fits with a constructivist theory of learning in mathematics, and the belief that paper-and-pencil skills should receive decreased attention – particularly the traditional written algorithms. The focus needs to be shifted from answers per se to the mental strategies employed. By so doing a deeper understanding of the number system is facilitated (Reys, 1984).

The legitimacy of the development of mental skills as a key outcome of school mathematics needs to be recognised. But for this to happen, teachers need to gain familiarity with the range of predominantly self-initiated strategies employed by students as they calculate mentally. This is reminiscent of Colburn's (1830) advocacy: "If ... teachers would have the patience to listen to their scholars and examine their operations, they would frequently discover very good ways that had never occurred to them before."

Numeracy and Number Sense

A goal of school mathematics is to allow students to develop the mathematical knowledge and skills essential to their becoming numerate adults. "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (AAMT, 1997). One aspect of numeracy is the ability to choose and use appropriate computational procedures – mental, written or technological – many of which may be informal and idiosyncratic. This choice is based on salient features of the problem context and the individual's unique pattern of mathematical knowledge. Hence, to be numerate implies personalised responses in the use of mathematics to solve problems.

These responses are dependent upon a well developed sense of number. This is reflected by, for example, an individual's depth of understanding of the meaning of numbers, an ability to comprehend numbers in a range of equivalent forms, and to know the relative effects of operating on numbers (NCTM, 1989). Individuals with number sense tend to analyse the whole problem first, rather than try immediately to apply a standard algorithm. They look for relationships among the numbers, the operations, and the contexts involved. The computational procedure chosen or invented takes advantage of these observed relationships.

Research suggests that an emphasis on calculating mentally significantly contributes to the development of number sense through the fostering of ingenious ways in which to manipulate numbers. Hence a focus on mental computation, besides its social utility, constitutes a means for promoting thinking, conjecturing, and generalising based on an understanding of numbers and their relationships. Most importantly, it reflects an emphasis on self-initiated, rather than imposed, calculation methods.

Mental Strategies

Essential to a focus on mental computation is the recognition that mental strategies are characterised by flexibility, variability, and transience; in contrast to standard paper-and-pencil methods. Additionally, the range of contexts for which particular strategies are appropriate is limited. Most importantly, however, understanding is required to arrive at a solution (Plunkett, 1979).

Further, the manipulation of quantities rather than symbols is facilitated (Carraher, Carraher, & Schliemann, 1985). This contrasts with the use of the standard written algorithms, the efficient use of which requires the symbols (digits) to be dealt with separately without reference to their meaning.

Significant differences exist in the nature of the mental strategies used by students who are proficient with mental computation and those who are not so proficient. Nonetheless, many efficient mental strategies are able to be used by the majority of students, with the more able acquiring them sooner than those who find mental calculation difficult (McIntosh, 1998). Efficient strategies are generally those that involve a *working from the left* approach, and which often produce a single result at each step, thus reducing the load on short-term memory – e.g. calculating 39 + 25 as: 30 + 20, then 50 + 9, and finally 59 + 5 to get 64 (see *incorporation* in Table 3).

Proficiency with mental calculation is not so much related to having a large short-term memory capacity as to having a richly interconnected store of number facts and strategies in long-term memory. Individual differences in mental calculation are conjectured to depend upon differences in the choice of computational strategy. This choice is dependent upon an analysis of the elements of a problem situation and their perceived relationships to the store of number facts held in long-term memory.

The research that has so far been undertaken reveals the diversity of the idiosyncratic nature of mental strategies. But, as yet, no framework or nomenclature for their classification has been agreed upon. The various mental strategies will here be classified as: (a) counting strategies, (b) strategies based upon instrumental understanding, and (c) heuristic strategies based upon relational understanding (Morgan, 1999). Those in the second category (Table 2) are strategies often used by individuals during their transition from using counting strategies (Table 1) to heuristic ones (Table 3).

Whereas the strategies based on instrumental understanding, in Skemp's (1976) terms, involve the application of rules without reason, heuristic strategies rely on knowledge that allows the individual to know what method will work and why. Such strategies are referred to as heuristics to emphasise the flexible manner in which the numbers are decomposed and recomposed in order to transform a given calculation into one which is easier for the individual to undertake mentally (Murray & Olivier, 1989). The use of heuristic strategies not only assists in the development of number sense, but may also be considered an indicator of its presence (Sowder, 1992).

Tables 1, 2 and 3 constitute a synthesis (Morgan, 1999) of some of the mental strategies identified by a number of researchers. In the majority of studies subjects worked with one or more of the four operations, usually with whole numbers. Samples varied with respect to age, grade, ability, and cultural background. Although it is the use of heuristic strategies that is a desired outcome of school mathematics, counting strategies are often used by the less proficient, with these strategies becoming increasingly inefficient as the complexity of the numbers and the operation increases. However, not all of the heuristic strategies are likely to be used by all students, particularly those involving the distributive principle, for example.

Concluding Points

These mental strategies are presented with a view to enabling insights into the range of self-developed strategies used by students. It should be noted that the examples related to each strategy are for illustrative purposes only, and that it should *not* be interpreted that any particular example should only be calculated using the strategy it illustrates. Any example may be calculated using a number of mental strategies, each appropriate to the numbers involved and the individual doing the calculation. For example, 23 + 9 could be calculated as: 23 + 4 + 5 (*add parts of the second number*), or as 23 + 10 – 1 (*compensation*).

Further, it is *not* intended that the heuristic strategies should be taught as standard algorithms. To do so would remove the freedom of students to choose or invent a strategy which best suits their particular pattern of numerical knowledge and their understanding of particular computational situations. Nonetheless, it is essential that a focus is placed on activities that develop number sense and that opportunities be given to students to discuss the strategies they use. Moreover, it is legitimate for teachers, in this context, to introduce particular strategies as alternative approaches to solving problems.

Table 1
Counting Strategies

Elementary counting

Counting-on in ones

min of the addends

23 + 9: 24, 25, 26...32; 32

min of the units

23 + 9: 29 + 3; 30, 31, 32; 32

Counting-back in ones

24 – 6: 23, 22, 21, 20, 19, 18

Counting in larger units

Counting-on/back in twos/fives/tens

80 + 60: 90, 100...140; 140
38 – 14: 28, 26, 24; 24
28 – 15: 23, 18, 13; 13

Counting-back to a second number in twos/fives/tens

140 – 60: 130, 120, 110...60; 80

Repeated addition

15 × 50: 50 + 50 + 50 + 50 + 50 = 250; 2 × 250 = 500; 500 + 50 + 50 + 50 + 50 + 50 = 750

Repeated subtraction

150 ÷ 30: 150 – 30 – 30 – 30 – 30 – 30 = 0; 5

Note: Adapted from: McIntosh (1990, 1991) and Resnick & Omanson (1987)

Table 2
Strategies Based Upon Instrumental Understanding

Used place value instrumentally

Removed zero

90 – 70: 9 – 7 = 2; add a zero; 20

Used written algorithm mentally

39 + 25: 9 + 5 = 14; carry the 1; 1 + 3 = 4 + 2 = 6; 64

For multiplication:

- no partial product retrieved (use of written algorithm mentally)
- one partial product retrieved

48 × 25: 48 × 5 is 8 × 5 = 40, carry 4, 24, 240; 48 × 2 = 96, 960; 240 + 960 = 1200

* two partial products retrieved

250 × 12: 250 × 2 = 500; 250 × 1 = 250, 2500; 500 + 2500 = 3000

* stacking

999 × 8: 9 × 8 = 72, 72(0), 72(00)

Note: Adapted from: Hope & Sherrill (1987) and McIntosh (1990, 1991)

Table 3
Heuristic Strategies Based Upon Relational Understanding

Add or subtract parts of the first or second number

34 + 48: 30 + 48 = 78, 78 + 4 = 82
46 + 38: 46 + 30 = 76, 76 + 8 = 84
33 – 16: 33 – 10 = 23, 23 – 6 = 17

Use fives, tens and/or hundreds

Add-up

317 – 198: 198 + 2 = 200, 200 + 100 = 300, 300 + 17 = 317; 119
51– 34: 34 + 10 = 44, 44 + 7 = 51, 17

Decomposition

200 – 35: 200 = 100 + 100, 35 = 30 + 5, 100 - 30 = 70, 70 - 5 = 65, 165
252 – 57: 252 – 52 = 200, 200 – 5 = 195

Compensation

28 + 29: 30 + 30 = 60, 60 – 2 – 1 = 57
25 + 89: 89 + 11 = 100, 25 – 11 = 14, 100 + 14 = 114
86 – 38: 88 – 40 = 48

Work from the left

Organisation

58 + 34: 50 + 30 = 80, 8 + 4 = 12, 80 + 12 = 92
36 – 23: 30 – 20 = 10, 6 – 3 = 3, 10 + 3 = 13

Incorporation

39 + 25: 30 + 20 = 50, 50 + 9 = 59, 59 + 5 = 64
51 – 34: 50 – 30 = 20, 20 – 4 = 16, 16 + 1 = 17
43 – 26: 40 – 20 = 20, 20 + 3 = 23, 23 – 6 = 17

Work from the right

mental analogue of standard written algorithm

58 + 34: $4 + 8 = 12$, $5 + 3 = 8(0)$, $80 + 12 = 92$

74 – 28: $14 - 8 = 6$, $60 - 20 = 40$, $6 + 40 = 46$

place-grouping

439 – 327: $39 - 27 = 12$, $4(00) - 3(00) = 1(00)$, 112

Use known facts

29 – 14: $2 \times 14 + 1 = 29$, $1 \times 14 + 1 = 15$

Use factors

general factoring

60 × 15: $60 \times 3 \times 5 = 300 \times 3 = 900$

half-and-double

60 × 15: $30 \times 30 = 900$

aliquot parts

25 × 48: $48 \times (100 \div 4) = (48 \div 4) \times 100 = 12 \times 100 = 1200$

exponential factoring

32 × 32: $(2^5)^2 = 2^{10} = 1024$

iterative factoring

27 × 32: 27×2^5; 27, 54, 108, 216, 432, 864

Use distributive principle

additive distribution

64 ÷ 4: $(60 \div 4) + (4 \div 4) = 15 + 1 = 16$

21 × 13: $(20 \times 13) + (1 \times 13) = 260 + 13 = 273$

subtractive distribution

8 × 999: $8 \times (1000 - 1) = (8 \times 1000) - (8 \times 1) = 8000 - 8 = 7992$

fractional distribution

15 × 48: $(10 + 5) \times 48 = 10 \times 48 = 480$, ½ of 480 = 240, 480 + 240 = 720

quadratic distribution

49 × 51: $50^2 - 1 = 2500 - 1 = 2499$

Note. Adapted from: Carraher et al. (1987); Hope (1987); McIntosh (1990, 1991)

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APPLICATIONS IN GEOMETRY for Mathematics A and B

Mal Shield, QUT Kelvin Grove

Both the Mathematics A and Mathematics B syllabuses emphasise the application of mathematical ideas to real-life situations. The various topic areas involving geometry (Elements of Applied Geometry, Linking Two and Three Dimensions, etc) provide many possibilities for developing real-life applications. In this paper I will outline a few resources and ideas that I have found useful in senior geometry.

Plans for subdivisions

The Mathematics A syllabus mentions the use of plans of a sub-division as a learning experience. Such plans are drawn up by civil engineers who design everything from the roads to the provision of amenities such as water, drainage and power. The actual plans for all of this consist of quite a number of separate drawings which these days are all prepared on computers using CAD software. The whole set of plans can be easily printed out for use. I have used a set of these plans provided by a friend who works in this area. If you do not know a civil engineer yourself, perhaps a parent of one of your students does this type of work. There should be no problem using this material in the classroom but permission needs to be obtained from the engineer as plans are usually copyright.

The first page of a set of plans for a sub-division is usually the general layout as shown below.

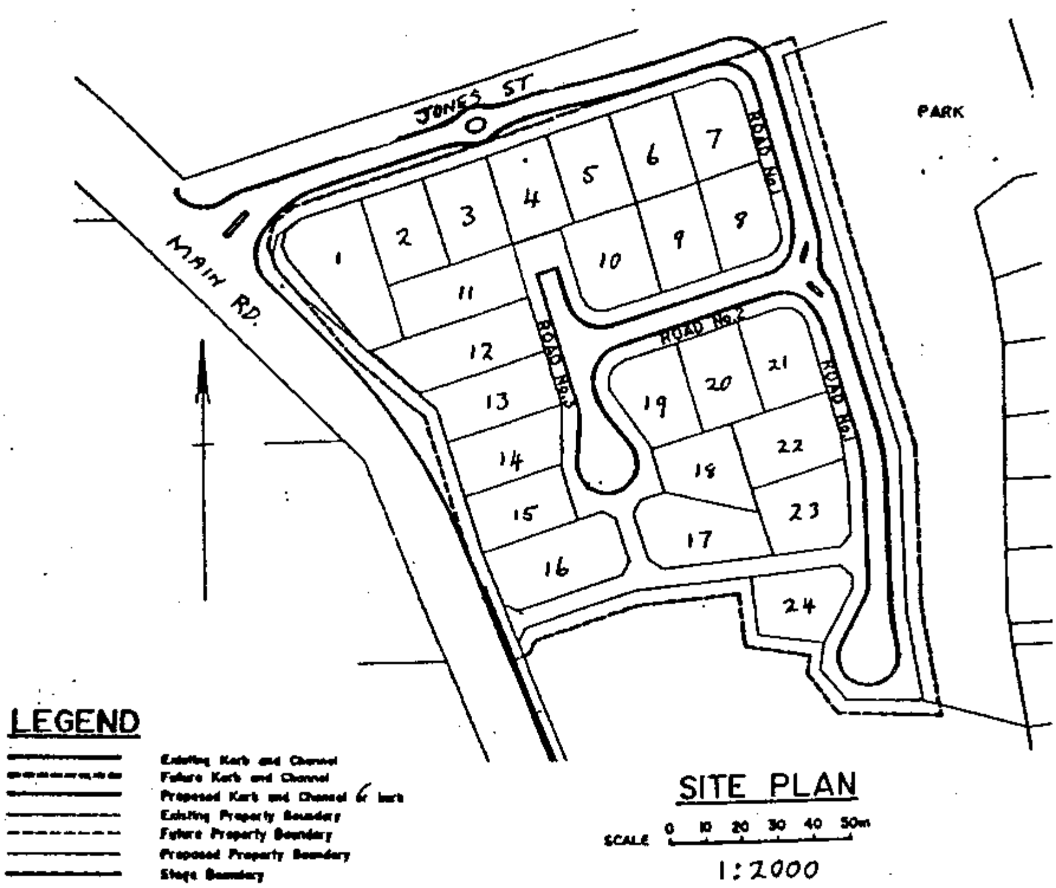

Figure 1

This particular plan shows quite a number of lots with unusual shapes. The engineer's job includes calculating the area (in square metres) of each lot. It can become a problem-solving exercise for students to calculate the area of a block like lot 17. One problem that exists in going to view a particular lot on a new estate is to actually identify the block you are looking at. It is an interesting exercise to ask students to provide detailed directions for locating, for example, lot 14. How would you direct a prospective buyer to lot 14 from the main road?

The syllabuses mention estimating quantities and costs using areas and volumes. The plans for a sub-division include large numbers of cross-sections showing how the existing profile of the land is to be changed during the development. Two such cross-sections including a road are shown below.

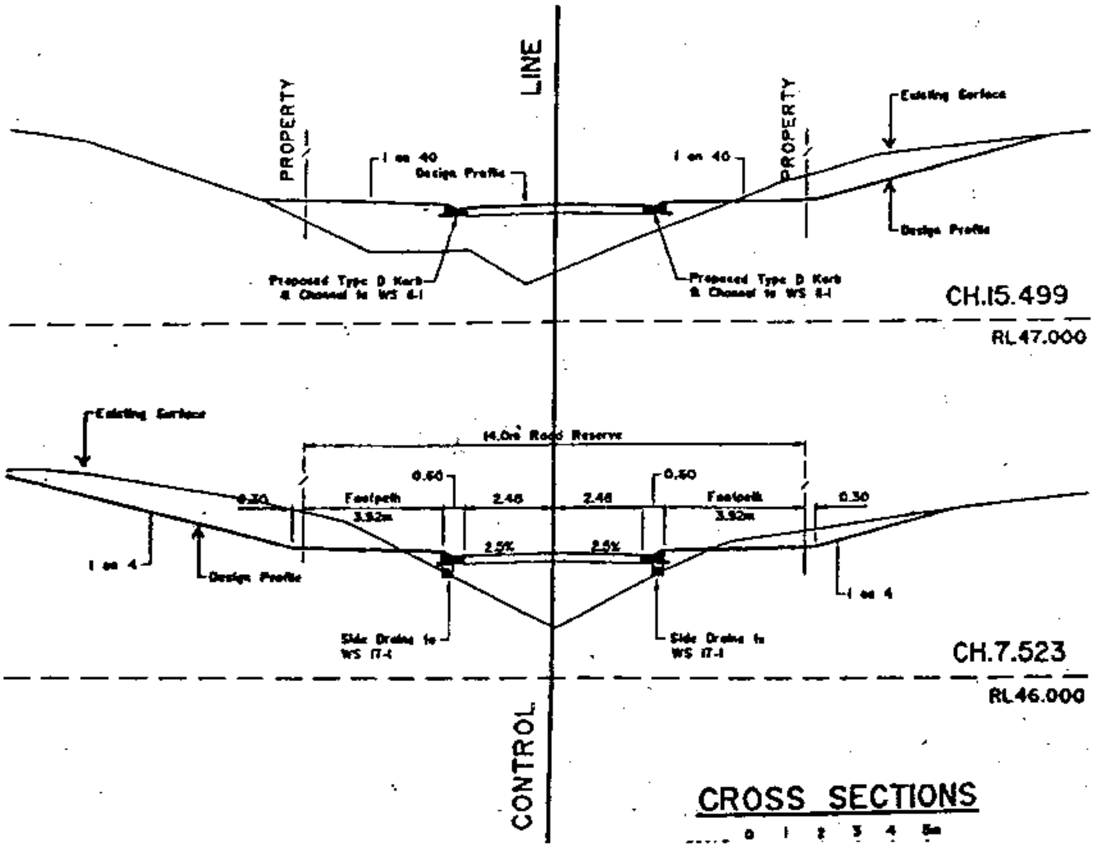

Figure 2

As you can see the existing profile needs to be changed so that the bed for the road is raised and levelled and the land either side has a 1 in 4 gradient. The two cross-sections shown are from two positions along the road. To estimate the amount of fill required to make up the road bed between these two positions, the engineer calculates the areas of cross-section of the parts to be filled at each end and averages those. The average area is then multiplied by the distance between the two cross-sections to estimate the amount of fill required. I am sure you can see other maths problems in these diagrams, as you will in the many other drawings that make up the plans for a sub-division.

Garden projects

Both the Maths A and Maths B syllabuses mention learning activities involving watering systems for gardens and some schools already include some sort of garden design project in their assessment programs. The company Hardie Pope, which makes a large range of components for garden watering systems, has two publications *Easy Guide: Installing drip & spray systems* and *The Hardie Pope Easy Guide to When I Should Water*. These are free publications and I obtained mine from the Kmart garden section. These two publications provide a wide range of ideas for enriching the mathematics in a garden project.

They include diagrams showing the coverage of various types of sprayers, flow rates, water requirements for various types of soils, and much other information. The possibilites in these free booklets for mathematical applications are many.

Aspect ratio

Aspect ratio is a simple concept that has a surprisingly large number of applications. Its study in mathematics can be included in a section on similar shapes and ratio. It is defined as follows.

*The **aspect ratio** of a rectangular shape is the ratio of the longer side length to the shorter side length.*

Aspect ratio is usually written as a single number, for example, a ratio of 3 : 2 is the same as 1.5 : 1 and is written as an aspect ratio of 1.5.

School mathematics has usually included one very famous aspect ratio, the *Golden Section*. Other applications include television screen sizes, paper sizes, photograph sizes, and aircraft wings. I will discuss some below and other information can be found in the book *Mathematics at Work* (Lowe, 1988, p. 289).

Television screens all have an aspect ratio of approximately 1.3 (that is, length : breadth = 1.3 : 1). Students can check their sets at home. The question that students should consider is “Why?”. Television screen sizes are specified by the diagonal measurement, which provides an interesting problem in calculating the dimensions using the aspect ratio.

One of the parameters determining the thickness of glass used in a window is the aspect ratio. For a given area of glass, the smaller the aspect ratio, the thicker (and therefore stronger) the glass must be. Again, think about why a square sheet of glass needs to be stronger than a long, narrow one of the same area. The Standards Association of Australia publishes a guide book for glass installation. In this book there are some very interesting graphs which could be used in Maths B. The required thickness of the glass is a function of the design wind pressure (the maximum to be allowed for), area of the glass panel, and the aspect ratio of the panel. The graph has a series of straight lines for different aspect ratios using wind pressure and area on log-scale axes. The relationships are quite complex and take considerable investigation to explain.

In the design of aircraft, the aspect ratio of the wing is one of the most important parameters governing performance. Of course the plan form of a wing is usually not rectangular, so the aspect ratio of a wing is defined as the ratio of span to average chord. (Note that chord is measured in the direction of travel and not perpendicular to the edge of the wing.)

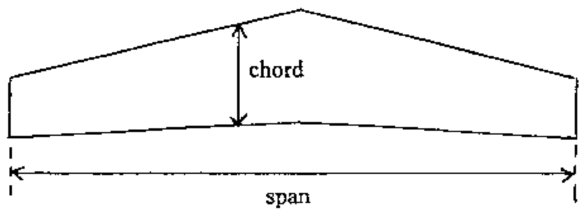

Figure 3

In general, high aspect ratio wings (long and narrow) are best for long range cruising, the extreme examples being sail planes (gliders) and the American U2R reconnaissance aircraft. Low aspect ratio wings are best for manoeuvrability, such as for jet fighters. Two examples of plan shapes of aircraft are shown below.

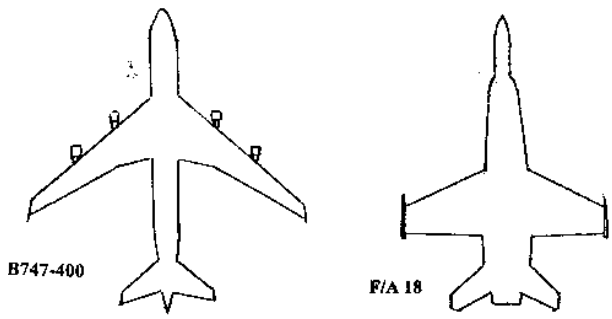

Figure 4

Students could investigate the aspect ratios used on different planes such as the extreme range Rutan aircraft, or answer questions such as: How does the aspect ratio of an F111 change as the wing position changes from minimum sweep to maximum sweep and why is this advantageous? Sweep angle of wings could also be investigated.

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What is a Billion? (or Trillion …)

***Jack Oliver* (*Northern Territory University, Retired*)**

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In my presentation 'The Naming of Numbers' given at the AAMT 17th Biennial Conference in January, I mentioned that the controversy between the 'English' billion (10^{12}) and 'American' billion (10^{9}) seems to have been resolved in favour of the 'American' version. In discussion time, my audience felt that the two different definitions are alive and well and my further reading seems to confirm this.

American references either refer to the 10^{9} meaning only or add the English 10^{12} meaning as an afterthought. English references seem to use the 10^{12} meaning consistently. As an example American references refer to the age of the Earth as 4.6 billion years while English references use 4 600 million years. Australian references use either. In a recent ABC-TV news broadcast, within the same news item I heard that a company had a 6 month profit of $1 800 million and, in the next sentence, a profit of $1.8 billion.

In Australia we have 2 competing '-illion' notations. Let us have a look at each one of them:

American '-illion' Notation

Name	Number	Power
Unit	1	10^{0}
Thousand	1 000	10^{3}
Million	1 000 000	10^{6}
Billion	1 000 000 000	10^{9}
Trillion	1 000 000 000 000	10^{12}
…	…	…
Centillion	…	10^{303}
N-illion	…	10^{3N+3}

Each row shows a number which is larger than the number in the preceding row by a factor of 1000. As an example of how the first and second columns are related, look at the 'Trillion' row. 'Tri' indicates 3 and in the second column we have 3 lots of '000' and 1 extra '000'. The rule doesn't apply to the first 2 rows.

English '-illion' Notation

Name	Number	Power
Unit	1	10^{0}
Thousand	1 000	10^{3}
Million	1 000 000	10^{6}
Billion	1 000 000 000 000	10^{12}
Trillion	1 000 000 000 000 000 000	10^{18}
…	…	…
Centillion	…	10^{600}
N-illion	…	10^{6N}

Ignoring the first 2 rows, each row shows a number which is larger than the number in the preceding row by a factor of 1 000 000. As an example of how the first and second columns are related, look at the 'Trillion' row. 'Tri' indicates 3 and in the second column we have 3 lots of '000 000'. As in the 'American' notation, the rule doesn't really work for the first 2 rows.

Comparing the two notations, the 'American' scores in as much as the factor between steps is 1000 rather than 1 000 000 which fits in better with SI where the factor between adjacent multiples and sub-multiples is also 1000. On the other hand, the 'Rule' 10^{6N} is simpler than the 'Rule' 10^{3N+3}.

We could of course avoid this sort of thing altogether if we just used exponential notation. We could refer to a profit of $1 800 000 000 as '1.8 by 10 to the 9th dollars' and write it '$\$1.8 \times 10^{9}$'. This should be unambiguous and would tell us immediately just how many trailing zeros we were dealing with. Could we sell it to the populace? What do you think?

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Maths in the Mall

Jan McDowell QAMT

Following the success of the pilot project in Brisbane last year, QAMT invited schools across the state to take mathematics to the people.

A state-wide mail-out to all government and non-government schools was conducted in late March. Expressions of interest were received from Dutton Park State School, Kallangur State School, Our Lady of the Rosary School, Kenmore, Robina State School, St Mary's College, Ipswich, Springfield State School and the Toowoomba and District Branch of QAMT.

On Tuesday 4 May the Year 7 students from Our Lady of the Rosary School in Kenmore brought a set of problems to the Kenmore Shopping Village. These included a wide range of surveys and length and volume measurements. The photographs show students of Our Lady of the Rosary School working on these with members of the public.

On Wednesday students from the Year 10 Challenge/Science class at St Mary's College, Ipswich visited the Ipswich Shopping Mall. They prepared posters, activities and handouts for an interactive display around the theme "Your Finances – Everyday Decisions Can Save You Money". They were well prepared with relevant examples and made good use of their lap-tops, using Internet sites to help them talk to the public.

Also during the week, students from Springfield State School visited their local shopping centre and students from Robina State School attended a shopping centre on the Gold Coast.

All the students who participated benefited from the experience of interacting with members of the public and were able to raise the profile of mathematics within the general community. Feedback received from the teachers who were directly involved was positive and they were happy to be part of the National Science Week activities.

Page 15 Problem

Solution to 'Moonwalk'

Once the sun goes down it will not rise again for 14 days. However, the earth would be easily bright enough to allow them to walk after the sun sets. The earth will maintain roughly the same position in the sky throughout the journey and will remain more than half full for several days.

The essential supplies are:
- Moon map on computer disk (200g); battery-powered computer (6kg); battery for computer (3kg)
- Sufficient food, water and oxygen for the duration of the journey.

To reach the base by alternately walking and resting would take 58 hours. For this they would need 3 oxygen tanks each plus food, water and the computer equipment. This is more than their weight limit. Less intrepid travellers might just sit down and die, but Marcus and Anastasia make it alive! This is what they do.

Marcus sets off with 3 oxygen tanks, Anastasia with two, the computer gear, 5 litres of water and 3 packs of sponge cake. They walk for 10 hours, rest for 8 hours, then return, leaving behind two full oxygen tanks and one pack of sponge cake. In order to make it back on 3 tanks of oxygen, Marcus switches tanks after 10 hours, leaving the original one half full for Anastasia when her first one runs out.

They rest another 8 hours then set off again with two full tanks each. Marcus takes the computer gear and 3 packs of sponge cake. Anastasia takes 14 litres of water. They walk for 10 hours to where they dumped the supplies, rest for 8 hours, then head off again, each with two full oxygen tanks. Marcus takes the computer gear and the remaining 3 packs of sponge cake. Anastasia takes the remaining 10 litres of water. They walk for 10 hours, rest 8, walk 10, rest 8, then walk the remaining 10 km to the base. This second part of the journey takes 40 hours. They have just enough oxygen, and adequate water and sponge cake. The total journey time is 76 hours, inside the 80 hours allowed by their space suits.

Sun up

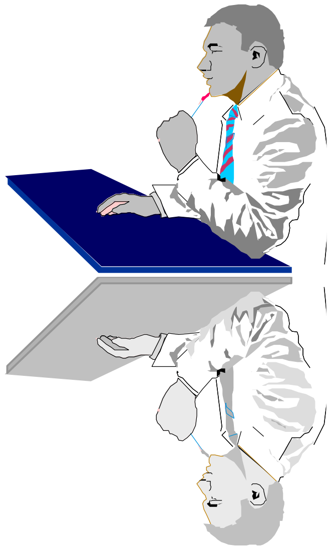
On the June solstice, the sun is 23.5° north of the earth's equatorial plane. If Brisbane is at latitude 27.5°S, calculate how long between sunrise and sunset on the June solstice.

Level of mathematical knowledge required: Year 10 or 11. But some tricky 3-dimensional thinking is involved.

REFLECTION ON LEARNING STRATEGIES FOR USE IN SENIOR MATHEMATICS

PART 1

Lyn Nothdurft
St Patrick's College, Gympie



This is the Part 1 of a two-part paper based on a presentation at the Queensland Association of Mathematics Teachers One Day Conference on 1 May 1999 in Brisbane.
Part 2 will be published in the next issue of *Teaching Mathematics.*

Introduction

A focus in senior schooling is the development of independent autonomous learners, who contribute to decisions about how they learn and who accept responsibility for their own learning.

Autonomous students are knowledgeable about themselves and their goals; they have an awareness of their strengths and weaknesses in learning; and they understand their learning needs. These students are knowledgeable about learning strategies: able to use a range of different strategies, able to exercise metacognitive awareness and control; able to evaluate the effectiveness of strategies; and able to reflect on the learning process. Also, they are knowledgeable about content: able to relate new knowledge to previous understandings to create further new knowledge, to apply knowledge, to analyse and practise what is known, and to acknowledge what is not known.

Learning occurs as learners reflect on their experiences to connect and make sense of them in the context of knowledge they already possess. So reflection contributes to knowledge in all these areas.

Hiebert et al. (1997) described reflection as follows:

> Reflection occurs when you consciously think about your experiences. It means turning ideas over in your head, thinking about things from different points of view, stepping back to look at things again, consciously thinking about what you are doing and why you are doing it. (p. 5)

Southwell (1999) has related personal reflection to the mathematical transformation of reflection. When a shape is reflected, its position and orientation may be changed, but it retains all its other original characteristics. When a person reflects, again the essential characteristics are retained, while the experience is looked at from another position. Thus, the process of reflecting on one's experiences involves thinking carefully about them in an attempt to make sense of them. This enables the establishment of new relationships and the checking of old ones. Hiebert et al. (1997) stated that reflection is almost certain to increase understanding.

The term 'metacognition' is also used to describe this conscious reflection on oneself and one's actions, as it is:

> What one knows or believes about oneself as a learner and how one controls and adjusts one's behaviour. … Metacognition is a form of looking over one's shoulder – observing yourself as you work and thinking about what you are thinking. (Reys, Suydam, Lindquist & Smith, 1998, pp. 26-27)

Metacognition involves organising thinking by using problem solving, decision making, and evidence seeking strategies. It also involves reflecting on thinking in progress, pondering strategies, and revising them accordingly. There is conscious control of learning and change to learning behaviours: activities are planned (task demands are analysed, outcomes predicted, strategies scheduled and forms of trial and error used); activities are monitored (strategies are monitored, tested, revised, re-scheduled, and judgment is made as to when the problem is solved or insoluble); and outcomes are checked.

Advantages of reflection

The encouragement of student reflection on learning is a response to concerns about learning in mathematics including students' passive dependence on the teacher, their use of surface learning strategies, their inappropriate application of procedures, their inadequate monitoring of their progress, their lack of transfer of knowledge, and their beliefs that mathematics is not to be expected to make sense so that the best approach to learning is memorisation. In contrast to this, autonomous learners are reflective and self-aware.

Reflection enables students to develop deep understanding of concepts. Students who reflect on their learning find new ways of doing things, clarify issues, develop their skills, and better synthesize, validate and integrate their knowledge (Southwell, 1999). This is illustrated by Southwell's description of the role of reflection using Polya's (1957) problem-solving sequence. To understand what the problem is about, the student initially reflects on the language of the problem statement, the concepts involved, and the goal to be achieved. The student then evaluates possible strategies and selects an appropriate one. Carrying out the strategy requires previous mastery of that strategy. Finally, the student reviews the problem and its solution, reflecting on the strategy itself, on the solution, and on possible extensions or generalisations evolved from the solution process.

The reflective student thus makes connections between new and old knowledge, and thoughtfully applies the knowledge. The following student's comment described a reflective approach to learning:

> *I have thought a lot more about learning and about how I am doing it. I use class time better. Last year, I would just sit and listen, but this year, I am trying to think about how it works and what to do with it.*

Reflection enables students to become aware of their beliefs and expectations and of how they learn, and so influences their motivation and their use of learning strategies. In particular, students can critically analyse their assumptions about learning, and become more aware of the effectiveness of the learning approaches they are using (Borasi & Rose, 1989). The following reflection indicates a student's awareness of her attitude and acceptance of the responsibility for her understanding:

> *I should be studying earlier, look at each day's work. Asking questions to either you or group members helps a lot! Should try to apply concepts to real life to help understanding. Should try to concentrate more on working in class, instead of thinking I can't understand and giving up. Should see it as the best time to get help and gain understanding before I forget it and have to re-learn it just before the exam. Should use own rules and procedures. More writing down how I do it works.*

Situations where reflection may occur

Reflection may occur: in response to structured **journal writing**; through writing about processes being used in **collaborative problem solving**; and through thoughtful **class discussions**.

Journal Writing

Journal writing enables students to examine thoughts and actions to determine what they really mean, what has been understood, and how this is connected to previous understandings. Students can:

- express and reflect on their feelings about learning mathematics;
- increase their knowledge of content as they develop better and more personal understanding from writing about content;
- improve their learning and problem-solving skills by articulating and reflecting on their process of doing mathematics; and
- make explicit and consequently re-evaluate their beliefs on the nature of mathematics and thus move towards a better understanding of the discipline (Borasi & Rose, 1989).

Such writing can also improve the instructional situation, as teachers can do the following in response to students' reflections:

- provide more appropriate evaluation and remediation of individual students from their increased knowledge about them;

- make immediate changes and improvements in response to students' feedback;
- make long-term improvement in response to new insights gained about students' learning; and
- respond directly to students' questions, problems and suggestions (Borasi & Rose, 1989).

As well as reflecting in their journals, students can be encouraged to write in their exercise books next to their solutions about strategies they are using, why they are using these, and questions they have about their use.

Collaborative Problem Solving

A second type of reflection occurs as students work collaboratively. In general, reflection will be enhanced through:

- using a problem-solving approach and providing more open-ended investigative activities so that students need to reflect on what they are doing;
- asking questions of a probing or challenging nature so that the students need to think about what knowledge and strategies will be needed; and
- using cooperative learning activities to take advantage of the stimulus of peers (Southwell, 1999).

A collaborative learning environment enables students to become more aware of their strengths and weaknesses in learning and of their own learning needs. Also, as they explore different strategies, it enables them to evaluate the effectiveness of these strategies. Students can become more aware of their responsibility for their own learning, and realise that they understand concepts better by explaining them to other students (Hiebert et al., 1997). This is explained in the words of a student:

> *If someone in my group needs help, then I like to help them, because that helps me because I understand more what I'm doing. Like sometimes I know how to do it, but I don't understand why I'm doing it. But when I'm teaching them, people ask questions, like why are you doing that? How are you doing that? That makes me think about why I'm doing it. There is a purpose to what I am doing.*

Research has supported the effectiveness of asking students to provide explanations to their peers in small groups (Hendry, 1996). Deep processing and real understanding occurs as students elaborate, question, and examine their ideas and procedures, that is, reflect on their learning.

Class discussions

A third type of reflection occurs through class discussions and evaluation of the learning that has been occurring. Students can suggest approaches that they find effective, and explain why other teaching and learning strategies have not been effective for them. This may be followed up by individual journal writing. The focus is on what the learning community, that is the students and I, can do to enhance learning.

Specific strategies to encourage student reflection

Initial reflection on learning

This is used to set the scene for the year. It can harness the students' initial enthusiasm and determination to work hard and do well, and allow discussion of the planned teaching/learning approaches for the year. It contributes to a collaborative effort to enhance learning.

The class discusses the following questions:

- What do you find difficult about learning maths?
- How do you go about learning maths?
- What does "learning" maths mean?

The first question is reflected on by individual students. Small groups then discuss their ideas. Finally, groups contribute their responses in a whole-class forum. The responses are written on the board, and the class structures them into themes. The same process then occurs for the next question. The second question allows the discussion of strategies which students have used in the past and a chance to evaluate their effectiveness. The third question about the meaning of learning provides an opportunity to reflect on the importance of understanding and of the students' responsibilities in the learning process.

As a follow up, students are asked to reflect in their journals on themselves as learners of mathematics, writing about one page titled "Maths and me". This provides an opportunity for students to reflect on the class discussion and to relate those ideas to their personal experiences.

Later in the year, a copy of their list of difficulties and strategies is reviewed by the students. They reflect on their current difficulties and the strategies they are using. They can see if they have developed as learners of mathematics, and consider what they are doing and whether they should be doing anything differently.

Reflection on individual lessons

Students reflect on their understanding of mathematical concepts as well as on their behavioural strategies. They are asked to write in their journals for five minutes before the end of the lesson to review the purpose of the lesson, to assess their understanding of the concepts, and to plan future actions. They respond to questions of the type:

- What was the lesson about?
- What did I learn today?
- Where does this fit in with what I already know?
- What don't I understand?
- What steps will I take to overcome difficulties?

Similarly, they can be asked to analyse their approaches to and success with their homework:

- Were there any problems? What can be done about that?
- Which questions were similar to other questions?
- How easy or difficult were the questions (and why)?
- How did you go about solving the questions?
- Are the solutions correct? And how do you know?
- Could the questions have been done any other ways?

Students can also review their involvement in the lesson and reflect on how well they have learnt, and whether there were factors which enhanced or hindered their learning. They could be asked to reflect on this in their journals, with questions of the type:

- How well do you use time, both in class and at home?
- How are you organising your homework and study? How much time are you putting in? How often?
- How effective is your help-seeking?
- How effective was your learning this lesson?
- Should anything have been different?

Further strategies for encouraging student reflection will be presented in Part 2 of this article in the next issue of Teaching Mathematics *to be published in November 1999.*

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Geometry: **What the acorn said when it grew up.**

Hexagon: **What the Italian said when the doctor removed the curse.**

REVIEWS

UNILEARN Introductory Mathematics

Queensland Open Learning Network

This is a mathematics program designed for open learning situations. It would be suitable for students with a primary school mathematics background and it takes them to about a Year 10 Extension level. The study materials consist of five books totalling about 1000 pages.

Number receives a good coverage. Algebra is treated very extensively, the algebra sections occupying about half of the study notes and including introductions to exponential functions, logarithms and matrices as well as the traditional material. Some use is made of the graphics calculator in the development of algebraic concepts. Statistics also receives good coverage and standard deviation and correlation are included.

Less than 50 pages are devoted to space and measurement, however, with no mention of some important ideas like volume, mass or time. And no probability is included.

The study materials are designed to take students through the course without the input of a teacher or other written materials. They contain detailed explanations of ideas along with plenty of examples and student exercises. The answers to the exercises in the backs of the books are backed up with worked solutions, so that, if students are stuck, they can see how the problem can be tackled.

The sequencing of ideas is logical; the language used would be accessible to the majority of students; and the pages have an uncrowded, 'friendly' appearance. There are not many graphics, but those present are clear and to the point. The approach is fairly traditional and algorithmic, but this is probably necessary for students who have to do most of the learning by themselves. Progress Tests are included at the end of each topic.

Although the materials are designed for open learning, they might well be useful in school situations where students need to develop some knowledge without direct teacher input. This could be where a student misses work through absence, where a student wishes to do further study on a topic s/he has not done well enough on in assessment, or where a student wishes to change from a slower to a faster class and needs to catch up on some missed units.

The algebra section might provide a bridging course to Mathematics B for students who have not completed sufficient Extension material.

The cost of the study materials is $25 per book. Students may enrol in the course for $495. For this they get all the study materials, an HP38G graphics calculator, tutorial support by phone, fax or e-mail as they need it, marking of and feedback on the Progress Tests and access to further materials at Open Learning Centres. A formal supervised examination is available on completion of the course. A Statement of Attainment is provided upon successful completion of the examination; a Statement of Completion is provided to students who complete the Progress Tests, but who elect not to sit the examination.

Other courses in this series are *Senior Mathematics* and *Statistics*. Reviews of the materials for these will appear in the next two issues of *Teaching Mathematics*.

David Ilsley
Queensland School Curriculum Council

Teaching Number Facts Using a Number Sense Approach - Addition and Subtraction

Teaching Number Facts Using a Number Sense Approach - Multiplication and Division

Author: James Burnett

Consulting Author: Dr Calvin Irons

Publisher: Prime Education

RRP: $24 each

These two books support the teaching of number facts in the primary school.

The clusters of thinking strategies, around which each book is structured, parallel those detailed in the sourcebooks which support the Queensland Years 1 to 10 Mathematics Syllabus. These clusters are, for addition: *Count-on 1*, *Count-on 2*, *Count-on 0*, *Doubles*, *Double-Add-1*, *Double-Add-2*, *Use Ten*; and for subtraction: *Count-back*, *Count-on*, *Use Doubles*, *Use Ten*. The clusters of strategies for multiplication focus on: The *Tens* facts, *Fives*, *Ones*, *Zeros*, *Twos*, *Fours*, *Eights*, *Nines*, *Sixes*, and the *Threes* facts. A similar sequence is used for division, viz *Fives*, *Twos*, *Fours*, *Eights*, *Nines*, *Sixes*, and the *Threes* facts.

The links between addition and subtraction and multiplication and division is emphasised. By focusing on the development of number sense, recognition is given to the supportive role that the thinking strategies related to each of these clusters plays in the development of an ability to mentally compute beyond the basic facts.

Activities related specifically to number fact development together with a series of associated number sense activities are presented on facing pages in each book. The latter encourage the use of activities that engage children in short five to ten minute mathematical discussions. Each is classified as one of three types - *Initiation*, *Companion* or *Extension*. These types parallel the three phases – *Introduction*, *Reinforcement*, *Practice* – that children pass through as they gain mastery of the number facts for each operation.

The number sense activities are characterised by their variety, with clear and concise directions provided for each. Clear directions are also provided to guide the introduction of each number fact strategy. These include reference to a range of reproducible blackline masters included in each book for constructing mathematical games relevant to introducing, reinforcing and practising particular thinking strategies.

These two books are highly recommended, particularly for the way in which the development of number sense is specifically linked to the learning of the basic facts. As noted in each book's introduction, the thinking that results from the approach presented provides "an excellent foundation for all computation and [is] particularly useful for mental computation", the desired method of first resort for all calculations.

Geoff Morgan
Undurba State School

Queensland Mathematics Workbook 3

Hellyn Goodman and Neville Goodman

McGraw Hill 1998

RRP $11.95

128 pages

This workbook is designed for students of Year 10. It would fit well with the Future Maths 10 textbook topics but it can clearly be used with any text or any Year 10 Mathematics course.

It is structured under five major headings corresponding to the 5 strands of The National Statement – Number, Measurement, Space, Chance & Data and Algebra. Each of the five main sections is further subdivided into between 10 and 20 'homework-sized' chunks. The standard of questions ranges from Level 5 to Level 7 in the main.

There is actually a sixth section to this workbook. This is the Investigation section that contains 20 investigations ranging from games of Hex to investigations into finding a way of making a cone with the largest volume. There is plenty of lined paper provided for students to do their working on.

A mechanism for student profiling has been included at the beginning of the workbook. There is space to record the date on which the subsection was completed and the Level achieved can be recorded. There is also space to record the student's achievements on the investigations.

Being a workbook, there is plenty of space in which students can write their answers to all of the questions. I think this is a major strength that will make the book useful to many teachers and students. Teachers can set homework quickly and easily from it and it is easy to check if it has been done.

The workbook could be used for assessment by being collected and marked, with results recorded. There is a separate booklet of answers for the teacher that is available free if a school purchases a class set of workbooks. If it is used as a teacher resource book only, then pages could easily be photocopied for class use when teachers are absent etc.

I think that this workbook has been designed well for its purpose. The pages are well set out with a good-sized font used. It costs around $12 which is very reasonable and affordable. Take a good look next time the book rep comes!

Lydia Commins
Ipswich Girls' Grammar School

Special Feature – Review of the Senior Mathematics Syllabuses

The Board of Senior Secondary School Studies syllabuses in Mathematics A, B and C are currently under review. In the following two articles Stephen Norton and Neville Grace have presented some thoughts to stimulate discussion in relation to this review.

Effects of the Senior Mathematics Syllabuses

upon Teachers' Goals and Practices

Stephen Norton
QUT

The intention of this paper is to discuss some of the unforseen effects of the Syllabuses in Mathematics A, B and C upon teachers' goals and behaviours. The insights come from my observations of the classes of ten experienced leading senior mathematics teachers and subsequent discussions with them. The classes were without exception well ordered and the teachers' knowledge of the subject matter excellent. It is argued in this paper that their teaching strategies reflected their attempts to satisfy their understandings of the syllabuses. In a number of instances this caused them some tension in that sometimes their teaching strategies did not reflect their beliefs and best practices. The findings have implications for the rewriting of the senior mathematics syllabuses.

The effect of the syllabuses upon teachers' goals

All three senior syllabuses have a similar underpinning philosophy and similar criteria for assessment. Mathematics B is examined in greatest detail because in the schools studied, about half the students took that subject (Board of Senior Secondary School Studies, BSSSS, 1992).

The assessment guidelines of the Senior Syllabus in Mathematics B stipulate that students need to perform in three criteria: communication, mathematical techniques, and mathematical applications (p. 4-5). Mathematical techniques includes the use of procedures in familiar situations. Mathematical applications involves applying mathematics in unfamiliar situations, both life-related and purely mathematical.

In order to be awarded a Very High Achievement level, students need to demonstrate extensive knowledge in both techniques and applications. For a High Achievement level, students need to demonstrate substantial knowledge of techniques, generally provide solutions in simple unfamiliar situations, and occasionally provide solutions in complex situations. Thus good students aiming for the top two achievement levels need to solve problems and model mathematically as well as to apply procedures in familiar situations.

When teaching able students, teachers in this study attempted to help the students meet these criteria by attempting to teach conceptually. That is, they reasoned that in order to be able to solve problems in unfamiliar contexts the students would have to understand the underlying concepts.

However, in order to gain a Sound Achievement, the students have only to occasionally provide solutions in simple, unfamiliar situations. In effect this meant that problem solving and mathematical modelling were not necessary. These students had only to demonstrate a working knowledge of techniques.

Thus, the assessment criteria of the syllabus do not encourage teachers to attempt to teach their middle and lower ability students how to solve problems and to model. It could be argued that the way those teachers interpret the syllabuses tended to justify their holding of different goals for different ability students. Alternatively, it could be argued that the senior syllabuses fostered the holding of different goals for able and less able students.

Certainly there is strong evidence that teachers held "conceptual" goals for able students and "calculational" goals for less able students (Thompson, Phillip, Thompson & Boyd, 1994). Teachers express their calculational goals for less able students by talking about "getting the students a pass," "getting the students to jump through those hoops (pass the techniques section of the exam)" and often were candid in admitting that the students did not understand the underlying concepts.

The effect of the syllabuses upon practice

The teachers in this study demonstrated three types of teaching practice. These practices are described and then teachers' reasons for using them are related to the syllabuses. Not all the teachers exhibited all the behaviours when operating in the three behaviour modes but the extended list gives a rich description of practices that many teachers will recognise.

The first behaviour pattern I have termed *"show and tell."* That is a teacher shows students how to do mathematical procedures and then tells them why it is important to do them. Usually the "why" is to pass the examination. The following behaviours illustrate *show and tell* pedagogy.

1. Short wait time. If students could not answer a question quickly the teacher would answer it. In this way the lesson flowed smoothly but students were not forced to think.
2. Letting students who did not know answers "off the hook."
3. Questioning used to maintain discipline rather than promote reflection.
4. Reducing the cognitive load by breaking the task into simpler and simpler steps to the extent that, often, most of the cognition was taken out of the task.
5. Rarely asking students to explain and justify answers.
6. Teaching predominantly from the front of the room. (This was important in maintaining discipline.)
7. Limiting student discussion until they were doing the practice of procedures from a text.
8. Presenting mathematics as a set of rules.
9. Rare use of diagrammatic models.
10. Non-use of investigative texts like Mary Barnes (1993) and *Access to Algebra* (Lowe, Willis, Kissane & Grace, 1994).
11. Very limited use of computers and graphing calculators.

In all the lessons observed these behaviours resulted in orderly student behaviour. However, it seemed that in these classes conceptualisation was not the goal, but, rather, getting students to perform mathematical techniques was the prime objective. Several teachers noted that they used *show and tell* strategies in part as a response to the large amount of content material to be covered. That is these strategies were viewed as time efficient.

The second sets of behaviours are termed "*explain*" behaviours. Generally the teachers who exhibited these behaviours frequently were very experienced and had a very good knowledge of the subject material. They had to have been knowledgable in order to construct the logical explanations observed. The following behaviours were typical:

1. Careful explanation of the logic underlying mathematics rules. Prior knowledge was linked to new concepts.
2. Usually short wait time.
3. Questions sometimes used to maintain discipline.
4. Reducing the cognitive load by breaking the task into simpler and simpler steps.
5. Teaching mostly from the front of the room.
6. Often the teachers did not rely on textbook explanations but constructed their own. Usually these explanations were very logical and framed in contexts that were relevant to students.
7. Some use of diagrammatic models.
8. The process of modelling and application might come after careful explanation if there was time (usually there was not).
9. Use of group work not to investigate phenomena but to enable students to peer assist in doing practice activities or to help each other interpret the teachers' explanations.

10. Some use of graphing calculators or computers, mostly to “crunch numbers”.
11. Rare use of investigative texts such as Barnes (1993).

The third group of behaviours has been termed “*learner-focused*.” This was characterised by:

1. Long wait time.
2. Students asked to explain and justify answers frequently.
3. Questions not used to maintain discipline.
4. Often most of the lesson taken up by students’ focusing upon investigative tasks such as those in Barnes (1993) and Lowe *et al*. (1994).
5. Students required to report on their progress to the class and justify their findings.
6. While students worked on tasks, the teacher acted more as a facilitator and gave structural hints rather than explanations. However, if the structural leading failed explanations followed.
7. Teachers spoke about the importance of “doing mathematics.”
8. Drill type questions set for homework and usually did not take up class time.
9. More frequent use of graphing calculators and computers.
10. Noisy classrooms with students moving between groups.

Three teachers exhibited all three behaviour patterns. Two teachers used mostly *show and tell* and five of the ten teachers used mostly *explain* or *show and tell* in the lessons observed. What was interesting was that all teachers used *show and tell* almost exclusively with less able students. That is, they used strategies that were consistent with their calculational goals for students who did not need conceptual understanding to get a sound on assessment instruments.

It could be argued that the assessment format of the current syllabuses supports or at least condones the use of teaching strategies that many educationalists claim are unlikely to foster conceptual understanding of mathematics concepts (Cobb, Wood, Yackel & McNeal, 1992; Thompson, et al., 1994). Less able students in particular get little cognitive development from their mathematics study and, as two teachers noted, soon forget which rules to apply. The teachers in this study were aware of this problem and were uneasy about their role in teaching less able students.

The five teachers who mostly used *explain* strategies with their better students when teaching new work (all teachers used *show and tell* during the revision phase) believed that these strategies were more time effective and efficient in transmitting understanding than getting students to investigate mathematical concepts. This may or may not have been the case. What is important is that eight of the experienced teachers in this study, which included AST's (Advanced Skill Teachers) and Heads of Department, mostly used teaching strategies that were not consistent with the rationales of the syllabuses.

Thus, it is apparent that some six years after the senior mathematics syllabuses have been implemented, most teachers have not adopted or have minimally adopted the investigative approaches it recommends. Further, as we have seen above, the wording of the assessment criteria has either fostered the maintenance of such strategies or at least failed to encouraged teachers to adopt the learner-focused and investigative approaches it recommends.

Concluding remarks and recommendations

The data collected by observing senior teachers’ practice and discussing their goals has indicated that it is likely that the goals and intentions of the existing syllabuses with respect to recommended teaching strategies have not been implemented by many teachers. We can speculate as to why this might have occurred. One possibility is that the rationale and the recommended learning activities have not been well understood by teachers. Another is that the recommended strategies have been understood, but have been rejected. There was some evidence that this was the case with some of the teachers and these teachers cited the amount of content to be covered and assessment criteria as important considerations.

Further it is apparent that teachers have been able to maintain strategies that focus on the achievement of some technical competency but with little conceptual understanding (particularly in the use of *show and tell* strategies) because, at the moderation panel level, this has been allowed to happen and possibly even fostered.

Either way there seems little point in continuing with syllabuses that are not achieving their objectives. Clearly, some change is needed. The following are a few recommendations:

1. The syllabus authors need to reflect upon the validity of their current rationale. Do they really want to make recommendations in relation to teaching or do they want to simply stipulate what needs to be taught and leave the manner of the teaching to teachers? (a minimalist approach)
2. If Queensland wishes to maintain senior syllabuses, which emphasise investigative learning, then the rationale and recommended learning activities need to more clearly articulate this. That is, the teaching strategies that the authors recommend need to be spelt out (an interventionist approach).
3. Whatever path the syllabuses take, that is, minimalist or interventionist, the assessment and moderation process needs to be consistent with the rationale and supportive of the rationale and recommended learning activities.

The intention of this article has been to foster debate. I think that this is a debate that we need to have. I hope that this article provides a framework upon which to begin the debate.

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A Store of Application Questions
for Mathematics A, B and C

A Call for Contributions

Coming up with good application problems for Mathematics A, B and C assessment can be difficult. But most teachers have, in their time come up with a few quite inspired questions. Around the state, there must be hundreds of such questions. Finding out about them and getting hold of them are the problem, though.

QAMT is planning to put together a store of application questions for Mathematics A, B and C. It is hoped that the store will contain a few of hundred questions organised by subject and topic. The store will be published in print form and on the web.

This is a call for contributions. If you have or know of one or two questions which have worked well with your students, consider sending them in. Contributions will be acknowledged unless the contributor wishes otherwise.

Contributions can mailed to Brad Barker, c/- QAMT (address inside the back cover) or e-mailed to Brad at bandc@fan.net.au.

Ten myths plus one

The many myths about mathematics education in the senior years and some modest proposals for change

Neville Grace
Member of QAMT and past long-term member of the executive

Background

A well attended forum was held on 5 May, hosted by QAMT and the BSSSS, to discuss the field of mathematics education at Years 11 and 12. Special attention was given to approaches taken in other states and other countries and especially to the advances in computer and calculator technology that have occurred in recent years.

It was a lively forum, with an excellent opening input from Jeff Baxter of South Australia and carefully considered responses from a panel of educators with experience in secondary and tertiary education. Both QAMT and the BSSSS are to be congratulated for providing this opportunity for yet another exchange of ideas in the lead up to revision of the Mathematics A, B and C syllabuses.

I understand that the BSSSS has plans only for a *revision* of the existing syllabuses. However, as a non-teaching observer, I was somewhat concerned that the profession is spending much time on narrowly focused issues to do with how the content might be juggled a little within the existing three-syllabus framework. I was left with a strong feeling that we had passed this way before – and a fear that in another decade or so we, or at least the ones still teaching or still alive, would be sitting in a similar forum going over the same ground yet again.

This time of syllabus revision provides a stimulus for us to re-examine some of the prevailing ideas of mathematics education as they relate to Years 11 and 12. It may be that some readers see these ideas as part of sacred scripture – but in this paper they are seen as myths in need of critical review.

A discussion of myths is offered for consideration. Then a few suggestions are made as to how the profession of mathematics education might act to help curriculum development and delivery structures cope with the changes that have occurred in technology and, equally importantly, in the school-client climate. The suggestions relate both to the current minor revision of existing syllabuses and, looking to the longer term, to ways in which curriculum-making structures might be expanded.

Disclaimer

It is recognised that there are significant needs for fairly rudimentary courses that allow students to revisit primary or lower secondary mathematics. It is important that efforts be put into providing age-appropriate pathways to allow students to achieve these ends. Existing pathways include the BSSSS subject area specifications in *Trade and Business Mathematics* and *Numeracy and Literacy - Strand B: Consumer Mathematics*. This paper takes the view that these courses, however appropriate for particular groups of students, have little to do with mainstream mathematics at Years 11 and 12.

Myth 1: Queensland is a world leader in curriculum development

Whoa! There is little doubt that three decades ago Queensland made great strides in moving to school-based assessment. Fine tuning has taken place at times and the BSSSS does a great job in managing complex machinery to oversee processes for accrediting school work programs and moderating school-based assessment and reporting.

But 'the times they are a-changin' as the saying goes. Clients are seeking different pathways to future study and employment. The *Queensland University of Technology* is one that actively advertises the possibility of *alternative* ways of entry to tertiary study. Information technology makes possible curriculum delivery methods that were not dreamed of ten years ago. As Jeff Baxter told the recent forum, it is expected that soon up to 50% of candidates at external senior examinations in South Australia could be arriving via pathways that do not involve regular attendance in traditional classrooms.

We can sit and bask in past glory. We might find, however, that unless new curriculum responses are developed we will one day be sitting in a rusty vessel, the SS *Curriculum Cutting Edge*, wondering why we are on a sand bank and hoping desperately for the tide to return.

Myth 2: External examinations are all bad

Really! ... Then how is it that most of the world is not convinced?

Perhaps the time has come to recognise that assessment has to meet a wide variety of purposes, some educationally acceptable and others to do with competition and ranking that are necessary – but purposes with which we, as educators, would rather not be too closely associated.

We should at least ask the question as to whether the time has come to acknowledge that there are heavy costs in a fully school-based assessment regime and that there are some limited things that external examinations can do quite successfully. Can we contemplate a mix?

Myth 3: We need more time for mathematics in schools

No! No! With great respect to those who argue for more time, the effect will be comparable to that of King Canute with the tides. Let us be honest in this argument. If a student chooses to study say, French, History and Mathematics can we honestly say that one should have more time devoted to it than another. We run a great risk of being seen as sour members of a previously privileged enclave and as narrow thinkers who simply do not know the difficulties, or value, in learning languages or history.

Students who study both Mathematics B and C elect to allocate one-third of their school time to mathematics. We should recognise that many choose, for very sound educational and personal reasons, not to become so narrowly focused.

We need to be passionate about our learning area – but not fanatical. Fanatics, whether mathematical or ecclesiastical, are a pox on society. We need to devote our energies to ensuring quality time and quality modes of delivery rather than in fighting colleagues for time.

Myth 4: We need three syllabuses, no more and no fewer

Well ... maybe? But if by ‘three syllabuses’ we mean three rigid pathways, then it is likely that clients need more than just three syllabuses. On the other hand, we could loosen up our thinking about syllabuses by concentrating more on clients and needs rather than on assessment and issues to do with comparability. A variety of pathways to competence in areas of need should be identified and supported.

Myth 5: Syllabuses stay in place for many years

Sadly true ... at least in the past! The BSSSS operates within legislative requirements and has to carry a heavy load in providing levels of accountability and comparability that are acceptable to the community. There will be a continuing need to undergo complex processes that will operate against pressures for rapid response and frequent change.

The situation will change over time in response to changing demands on education. In the meantime, perhaps the mathematics education profession can work with curriculum authorities to expand the pathways available to students and help build a climate in which procedures for curriculum authorisation and certification of achievement can expand also.

Myth 6: Syllabuses are for teachers

Why? ... Why not for clients? The syllabus-for-teachers approach has been just too prevalent in the past. Syllabuses have been larded with all manner of very sound advice for teachers – about teaching approaches, learning theories, suggested learning experiences and resources. The intentions of writers were quite honourable, but have resulted in syllabuses that were readable only by somebody with the background and interests of a teacher.

In the past, the interests of clients have not been well met by syllabuses. The student, or parent, has a right to ask: 'As a result of engaging in this study, what is the student required to know and to do with that knowledge?' – and also a right to expect that a syllabus will provide the answers clearly in accessible language. There is a desperate need to pare syllabuses back to be clear statements of expected outcomes and to let teachers grapple with the very difficult problems of how to assist students to achieve what has been stated. Support for teachers can be provided through a variety of media – not by confusing what a syllabus should be.

Myth 7: Students need to sit in classrooms for *x* hours per week

Many will not! Jeff Baxter spoke at the forum about the expectation that some clients will be studying at different sites, or using information technology, and might attend a school only for tutorial sessions. We need to think about the future, or even current, relevance of a syllabus statement that *the minimum time-tabled school time, including assessment, for this subject is 55 hours per semester*.

Myth 8: Technology should not drive syllabus development

Why not? ... This is akin to saying that *Telstra* should not be driven by technology!

During the last syllabus production event, writers could live with the view that content generally stayed in place and that technology had potential only as a tool to support the learning of what has traditionally been important in mathematics education. This position is no longer tenable. The technology is now widely available and can allow users to do, with a few buttons, all the content that we used to teach. The question as to WHAT we should now teach cannot be avoided – and the responses to that question should be driven very much by technological considerations.

Myth 9: There is too much content in current syllabuses

Perhaps? Have we examined the reasons carefully? Are we assuming that students have eight terms when they really have seven, given the chaos of fourth term in Year 12.

Have we asked whether some of the problems arise from lack of clarity in current syllabuses? Do we have shared understanding about the intentions of existing syllabuses or do teachers feel pressured to pack in most of the content that has been there for years just in case it might be intended?

Myth 10: We can teach higher-order thinking and problem solving

Ah ... If only this were true! We certainly know a great deal about how to provide students with a rich learning environment that includes: relevant applications; interesting challenges; different solutions; open-ended problems; a variety of resources; and opportunities for collaborative learning.

What little evidence there is suggests that senior students, even those who are more mathematically able, do not develop the ability to transfer their learning from one type of problem to another. John Belward gave a heart-felt plea at the forum for students to come into undergraduate courses with less rigidity in their thinking. Mathematics teachers can respond to this plea by providing rich learning environments. Let us not kid ourselves, however, that we know how to *teach* flexibility and problem solving – much less should we be sucked into thinking we know how to judge whether we have been successful.

Perhaps it is time for an increased focus on what we *can do well* as mathematics teachers.

A myth for the road: All courses must be two-year courses

Why? Why is it essential that the good ship SS *Curriculum* be fitted only with two-year guided missiles?

Some tentative proposals

The identification of commonly held beliefs is an easy task. The harder task is to propose some alternative courses of action that will expand pathways for our student clients. The following suggestions are offered to stimulate thought.

Proposal 1: A needs-based alternative to Mathematics C

There is a group of students who would rather not undertake Mathematics C because of workload and competitive pressures. Some of these will find, when they begin tertiary study in Engineering and other tertiary courses with a heavy mathematics orientation, that they are short of certain mathematical content and have to engage in bridging studies.

Can we arrange a certificate that students could gain through study in their own time? The process might have the following features:

- A competency-based certificate could be offered by QAMT in conjunction with a small group of secondary schools, or with a group of tertiary institutions. With the right promotion and attention to quality it might be possible to make such a certificate a highly valued one. Jeff Baxter, at the forum, spoke of the esteem accorded to the *International Baccalaureate* by an increasing number of schools in other Australian states.
- An examination offered twice a year would allow students to sit more than once if need be. Candidates could be required to pay a fee – perhaps $200.
- Schools might organise support in the form of a sponsored study circle or a fortnightly tutorial. Schools, state or non-state, might charge a fee or might offer this as a service to clients.
- QAMT could commission a self-paced course of study based totally on use of computers and the Internet. The course might offer perhaps 40 hours of study and have many links to on-line resources. It might draw on some of the current US materials that Jeff Baxter cited in his talk at the forum.
- The certificate could indicate that the standard had been achieved, and be attached to a clear statement of knowledge and skills covered and the criteria for achievement. A category of excellence might be added to satisfy our needs to rank people.
- The course content would be decided solely by consideration of entry to a small set of tertiary courses and of needs not met by Mathematics B.

The existing Mathematics C could be retained generally in its present form and allowed to find its place through market forces.

Proposal 2: Expanding pathways in Mathematics B

Mathematics B is the jewel in the crown of secondary mathematics. It is likely that there will be a growing demand for ways to access this mainstream course in mathematics. The current revision of Mathematics B will surely give attention to how the content can be more clearly defined and perhaps pared back a little to fit the real time available in schools.

The proposal is to retain mathematics B, but to ensure that it is written in a way that will allow schools to offer more pathways, perhaps three as follows:

- A course substantially as it stands.
- A computer based version that really takes into account the student who will have very significant access to computer and graphics calculator technology.
- An enhanced external examination-based version on the assumption that students will increasingly be encouraged and allowed to enter via this route.

Issues to do with comparability and fitting of students in a single rank order will have to be accommodated – but these issues exist already with the internally and externally assessed versions.

Proposal 3: Reinvigorate Mathematics A

Who are the students undertaking Mathematics A? What are their destinations in terms of further study and what are their needs?

The proposal is to allow needs to be met through a one-year course in mathematics. An exploration of needs might show that a course should be based on money management, data analysis and inferential statistics. Much of the content in the current Mathematics A syllabus looks to be of marginal importance to most clients – and some even looks like busy work to make up a two-year course of study.

Issues to do with contribution to tertiary entrance ranking will have to be addressed. It seems likely that clients of other subject areas would also opt for one-year courses if they became available.

Conclusion

As Jeff Baxter reported about the situation in South Australia, clients vote with their feet and leave mathematics if given half a chance.

I commend both QAMT and the BSSSS for the initiative in stimulating discussion about mathematics in Years 11 and 12. One, perhaps unintended, outcome has been this article in an attempt to reinvigorate thinking about an important sector of mathematics education.

It matters little whether or not readers agree with the ideas expressed. It would, however, be a matter for great concern if we just fiddle a little with the content of three syllabuses and argue some more about whether an incomplete response to a CIII question should contribute to achievement on this criteria or that.

If this happens, then mathematics teachers will be sitting in a very rusty SS *Curriculum Cutting Edge* in ten years time – polishing up the nice nameplate, wondering where the river mouth has shifted to and entertaining each other with tales of the good-old-days.

The challenge is over to you!

Basketball Maths

Adrian Hekel and Simon Gills
The University of Queensland

This is the third of three articles resulting from seminars on technology-based mathematics teaching given by Dip. Ed. students at the University of Queensland under the guidance of Merrilyn Goos. The first article and some background information were published in Volume 24 No. 1 of *Teaching Mathematics*, pages 16-18. The second article was published in Volume 24 No. 2, pages 11-13.

Year Level: Year 11 Mathematics B.

Topic: Introduction to Functions (perhaps linked with Applied Statistical Analysis, although regression is not a formal syllabus topic).

Source of Activity: Contemporary Precalculus Through Applications (Department of Mathematics and Computer Science, North Carolina School of Science and Mathematics, 1993). See task sheet.

Activity – Basketball Shooting Percentages

The table below gives data on basketball shooting percentages at various distances from the basket.

Distance (feet)	Shooting percentage
3	62
6	52
9	40
12	32
15	28
18	24
21	21
24	20
27	18
30	17
40	13

Source: Hays, James G., The Biomechanics of Sports Techniques, 2nd Edition, Englewood Cliffs, NJ: Prentice Hall, 1978.

1. Find a function that models the relationship between shooting pecentage and distance.
2. What shooting accuracy would you anticipate from a half-court (45 feet)?
3. Over the past several years the distance for a three-point basket has varied. Compare the shooting accuracy from 19, 21 and 24 feet.
4. Which strategy would be likely to score more points for a team in the long run: taking 100 three-point shots from 25 feet or taking 50 two-point shots from 8 feet?
5. If k three-point shots are taken from 22 feet, how many two-point shots from 20 feet would you expect to need in order to score the same number of points?

Value

The purpose of this activity is threefold. First, it provides students with a meaningful, relevant and fun learning activity on the topic of *Functions*. In particular, it provides a link between the idea of graphing the relationship between two data sets (with a certain domain-range) with the concept of representing such functions by equations. The activity would thus reinforce many concepts from analytical geometry, as the students can see the application of polynomials, hyperbolas and exponential functions as a means of modelling a very practical situation. Also, concepts such as domain, range, dependent and independent variables, table-graph representations, and especially the ideas of interpolating and extrapolating, could be drawn out from the students.

Secondly, the activity draws on some ideas from *Applied Statistical Analysis*, and thus could be used to discuss regression (as an extension C3 topic), probability (with the concept of "expected number") and sampling. For instance, the derivation of the shooting percentages could be left as a research project, and the statistics for NBL and NBA games could be compared and contrasted.

Finally, this activity enables the student to enhance and further their knowledge of the uses of technology within mathematics, especially in the context of a real-life situation. Graphics calculators give the students a fast, easy way to solve the problem, so more time can be devoted to the conceptual aspects and they can develop a good intuitive "feel" for modelling techniques.

Possible Solutions

(Solutions are illustrated with the aid of the Casio CFX-9850G plus graphics calculator; however, the activity is suitable for other calculators which can produce scatter plots and perform the required regression analyses.)

1) The data are first entered in a table via the STATS menu, and then a scatterplot is obtained by pressing GRPH - GRPH1 (a scatterplot is the default setting). Figure 1 shows the data table and scatterplot displays:

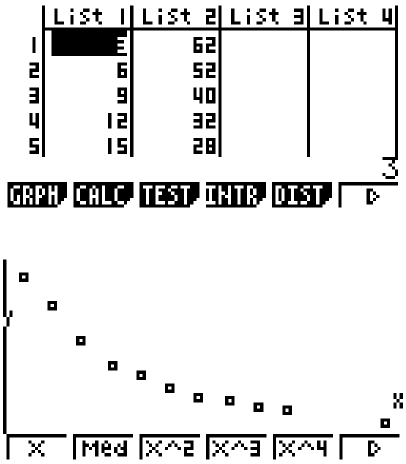


Figure 1

Intuitively, one would guess that the most appropriate fit is given by an exponential regression, since this function cuts the *y*-axis at a finite value, and tapers off to zero when the distance from the hoop becomes very large. A graphics calculator makes this model very easy to explore. The regression options are displayed below the *x*-axis, and by selecting EXP, the equation of best fit is displayed (Figure 2):

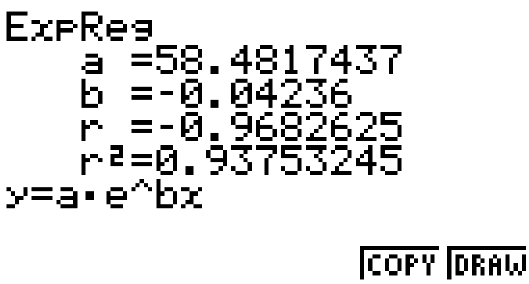


Figure 2

The graph of best fit then can be superimposed on the scatterplot (Figure 3):

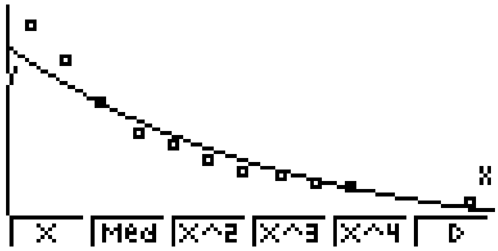


Figure 3

From the graph and the r² value, it can be seen that the exponential model lacks some accuracy, especially for distances close to the hoop. (This could be analysed further by looking at the residuals at each data point.) However, the model is the only one that makes physical sense, and attention can be drawn its limitations.

2) This question involves interpolating from the regression graph. An easy way to do this is to use the TABLE function from the main menu, with which we can specify the interval and range of *x*-values. Scrolling down (with interval = 1), we find that a distance of 45 feet corresponds to a shooting percentage of 8.7 % (as per Figure 4).

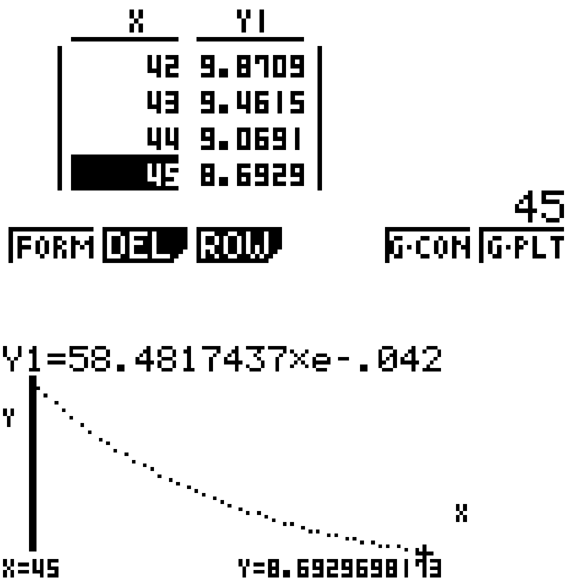


Figure 4

The Graph-Table option can then be selected to display the graph and table values simultaneously (as per Figure 5). Questions 3 - 5 can be solved in a similar fashion to Question 2, although different models might be more appropriate for certain shooting distances. They also involve some discussion of game tactics, given the different point-value of shots outside the three-point line.

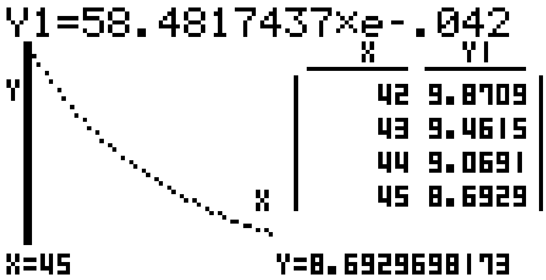
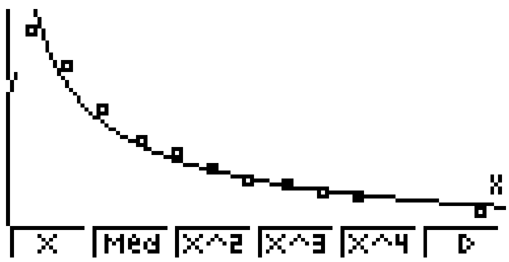


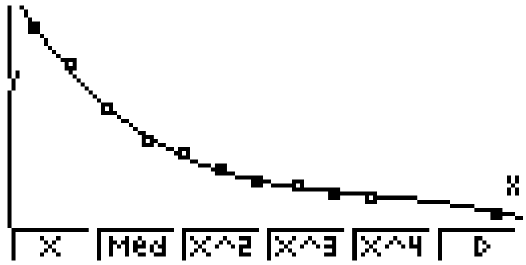
Figure 5

Exploring Other Models

When other regressions are tried, it is found that quartic and power regressions give very high r^2 values: 0.9975 and 0.9749 respectively (Figures 6 and 7). This raises some interesting discussion points such as the constraint that percentage must be less than 100 for zero shooting distance (which rules out the power model as it asymptotes to infinity). This model, on the other hand, gives a valid and accurate result for large *x*-values, while the quartic model (or any other polynomial) breaks down for large distances. But then again, a basketball court has finite length (90 feet)!



Power Model
Figure 6



Quartic Model
Figure 7

Conclusion

This activity raises a host of issues which demand discussion, particularly concerning the accuracy and validity of the various models. It is suggested that Question 1 is discussed thoroughly (with different groups perhaps trialing different models and debating their appropriateness), before Questions 2-5 are looked at using the best models. A homework or assessment task could then be appropriate, with the data still stored on the calculator and easily accessible. Extension activities might include: shooting some hoops and recording/ analysing/ comparing the students' own data, recording data from a basketball video, or discussing some basketball tactics from a statistics-viewpoint.

References

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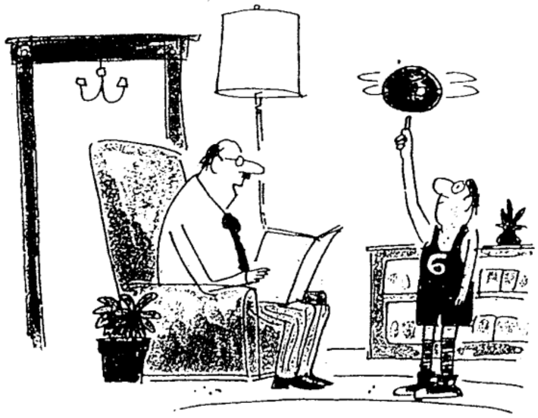
Department of Mathematics and Computer Science, North Carolina School of Science and Mathematics (1993). *Contemporary Precalculus Through Applications*. Dedham, Mass.: Janson Publications.



"Remember, Son, winning isn't everything.
You need good sponsorship deals too."

PYTHAGORAS REVISITED
Lessons from the Past

Paul Dooley, Fairholme College, Toowoomba. e-mail pmdooley@mathgym.com.au

Over the last two issues of *Teaching Mathematics* the author has attempted to show how historical/cultural/social/philosophical contexts can be used in the teaching of some pretty familiar, and sometimes dry mathematical concepts to secondary students. The author has found that adolescents show a keen interest in this approach , as it gives them some understanding of their place in space in time. The articles which appear in this Journal are shortened extracts from a collection of Essays and Activities on the authors web site http://www.mathgym.com.au To read a more expansive treatment and to obtain a copy of any of the Activity sheets - Click on the "history" button at the site and then follow the link to the Activity you want. You can then print out the page to get a black-line master for reproduction.

Introduction

In the previous essay I concluded with the "logical scandal" of the **incommensurables**. The realisation that all numbers were not Natural caused a major problem for the Pythagorean Order, not only because their faith was based on it, but also because their **theory of proportions,** used at the time to *solve* equations, relied solely on Natural number. This 'loss in confidence' with number caused Greek Mathematics to move away from numbers and to use measures in their place i.e. lengths, areas, volumes. Because geometrical properties could be physically constructed (with straight-edge and compass) and their accuracy 'seen', geometry was considered to be verifiable and rigorous. After the experience with the incommensurables, Greek mathematicians considered working with number as unreliable, resulting in the development of algebra as a geometrical construct over the next few centuries. This confidence in the 'tangibility' and rigour of geometry lasted through to the 17th century. Sir Isaac Newton, considered to be influential in the acceptance of algebraic procedures, would not publish his monumental *The Mathematical Principles of Natural Philosophy* until he had established the geometric proofs to replace his algebra and calculus. Kline [1] states:

In **The Principles** he [Newton] used geometric methods and he gave theorems on limits in geometric form. He admitted much later that he used analysis to find the theorems in The Principles, but he formulated the proofs geometrically to make the arguments as secure as those of the ancients.

Before I start, it is important to restate what I have said before; that it is impossible to know for certain which of the following is the result of Pythagoras and which is the work of later Pythagoreans or even later Mathematicians. All of the following ideas come from the early Books of *The Elements* [2]. It is generally, but not universally, agreed that the ideas in these Books are essentially Pythagorean in origin.

It is very tempting to give this essay a rigorous treatment as developed in *The Elements*, but I will leave interested readers to pursue such a treatment in the references below. The purpose of this essay is to give the reader an appreciation of the development of Greek algebra in as much as it was geometric in nature.

A. Geometric Basis for Arithmetic Operations

It is important that we make a distinction between the methods of commerce and those of philosophers. To perform everyday calculations in the marketplace and elsewhere, the Greeks used both the abacus, and certain algorithms performed with a stylus on a wax tablet. The methods described below were those of the intellectuals who sought to discover the meaning of existence and were therefore searching for truth, and were drawn to the purity of Mathematics.

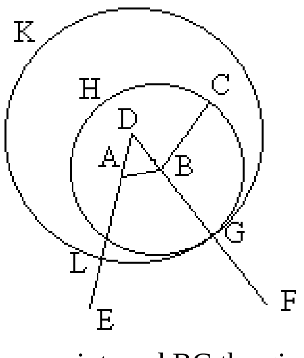
As explained above the early Greek mathematicians resorted to geometry to rigorously prove (and demonstrate) their ideas. It is unclear what their initial tools were but by the time of Euclid [2] all constructions were to be done by **straightedge and compass**, so it is useful to digress for a moment to describe these tools. A **straightedge** is essentially an idealised ruler, with the difference that it is unlimited in length and has no scale markings. It is not a measuring tool but a construction tool, allowing the user to produce a straight line of any length. The **compass** of *The Elements* is similar to the one used today with one very important distinction - the classical compass **collapses after use**. By this I mean that you can open the compass to draw an arc but once that arc is drawn the compass collapses so that the **size of the arc is not preserved** between applications. This feature of the classical compass renders some common constructions (easily taught in school using the "preserving" compass) somewhat non-trivial.

Addition and Subtraction

The early Definitions, Postulates, Common Notions and Propositions of Book I account for these constructions. Without repeating these I will show how addition and subtraction were constructed. Firstly Proposition 1, Book I of *The Elements* shows how to construct an equilateral triangle. Armed with this, **Proposition 2** shows how

To place at a given point (as an extremity) a straight line equal to a given straight line

This construction is described in Heath [2] as below:



Let A be the given point, and BC the given straight line. Thus it is required to place at point A (as an extremity) a straight line equal to the straight line BC.

From the point A to the point B let the straight line AB be joined; and on it let the equilateral triangle DAB be constructed.

Let the straight lines AE, BF be produced in a straight line with DA, DB; with centre B and distance BC let the circle CGH be described; and again with centre D and distance DG let the circle GKL be described.

Then, since the point B is the centre of the circle CGH, BC is equal to BG.

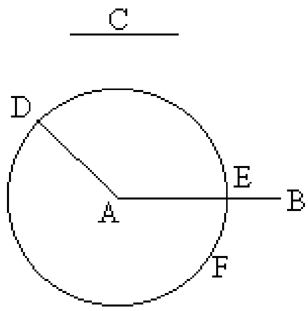
Again, since the point D is the centre of the circle GKL, DL is equal to DG. And in these DA is equal to DB; therefore the remainder AL is equal to the remainder BG.

But BC was also proved equal to BG; therefore each of the straight lines AL, BC is equal to BG. And things which are equal to the same thing are also equal to one other; therefore AL is also equal to BC.

Therefore at the given point A the straight line AL is placed equal to the given straight line BC. (Being) what it was required to do.

We can now consider **Proposition 3**. Though specifically used for subtraction of lengths, it could also be used to add two lengths. This Proposition states

Given two unequal straight lines, to cut off from the greater a straight line equal to the less.



Let AB, C be the two given unequal straight lines and let AB be the greater of them.

Thus it is required to cut off from AB the greater a straight line equal to C the less.

At the point A let AD be placed equal to the straight line C (Proposition I.2 above); and with centre A and distance AD let the circle DEF be described.

Now, since the point A is the centre of the circle DEF, AE is equal to AD. But C is also equal to AD. Therefore each of the straight lines AE, C is equal to AD; so that AE is also equal to C.

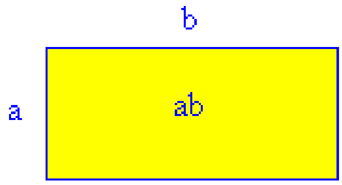
Therefore, given the two straight lines AB, C, from AB the greater AE has been cut off equal to C the less.

(Being) what it was required to do.

To add C to BA one has simply to extend BA and consider the sum to be where the circle DEF intersects with BA extended.

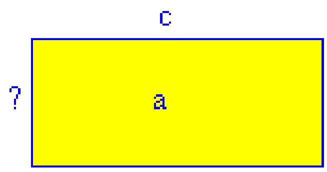
Multiplication

We have already seen in an earlier essay that the Pythagoreans considered multiplication as the area of a rectangle. When they said **'...the rectangle contained by two straight lines of length a and b...'** they meant the area of the rectangle of sides ***a*** and ***b*** or the value of the **product of *a* and *b*** as shown below.



Division

To divide ***a*** by ***c*** the Pythagoreans made a construction that was essentially applying to the straight line of length ***c***, a rectangle of area ***a***. The width of the constructed rectangle would be the desired quotient.



This is the so-called '***application of areas***'. This example is a specific case of **Proposition I. 44**. Of this Proposition Heath claims:

This proposition will always remain one of the most impressive in all geometry... The marvellous ingenuity of the solution is indeed worthy of the 'godlike men of old' as Proclus calls the discoverers of the method of 'application of areas'; and there would seem to be no reason to doubt that the particular solution, like the whole theory, was Pythagorean ...

The Pythagoreans also used **geometric proportion** to perform division. This is discussed in Section C below. The addition or subtraction of products is, in geometric algebra, the addition and subtraction of areas of rectangles and squares, the answer being transformed into a single rectangle by the *application of areas* to any line of any length. To find a square root the Pythagoreans would use a construction that converted a rectangle into a square of equivalent area.

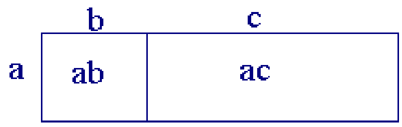
B. Algebraic Identities

The first ten propositions of Book II of *The Elements* are essentially algebraic identities written in geometric form. Most junior high school students would recognise the first four which I reproduce here:

Proposition 1. If there be two straight lines, and one of them be cut into any number of segments whatever, the rectangle contained by the two straight lines is equal to the rectangles contained by the uncut straight line and each of the segments.

In algebraic form this is written as: **a (b + c + d +...) = ab + ac + ad +....** This proposition is what today we call the **Distributive Law** and in its simplest form is written as: **a (b + c) = ab + ac** and its converse **ab + ac = a (b + c)**.

The Pythagoreans demonstrated this Law using the area of a rectangle of width **a** and length **b + c**, as in the diagram shown.

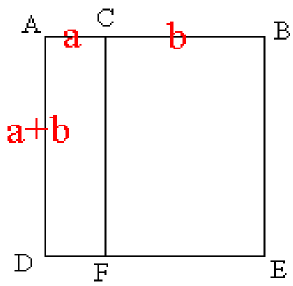


The large rectangle is divided up into two smaller rectangles of size **a** by **b** and **a** by **c** respectively. The area of the large rectangle is **a** times **(b + c)**. But the area of the larger rectangle is the sum of the areas of the two smaller ones or **ab** plus **ac**. So **ab + ac = a (b + c)**.

Proposition 2. If a straight line be cut at random, the rectangle contained by the whole and both of the segments is equal to the square on the whole.

In algebraic form this is written as:

$$(a + b)\,a + (a + b)\,b = (a + b)^2.$$

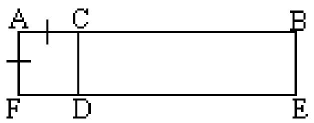


This is easily seen with a construction as shown. The line AB is cut at random at C. A rectangle is constructed on AC and AB so that the length of AD, CF and BE are equal to AB. This makes the square ABED. If we let AC be ***a*** and CB be ***b*** then AB and AD equal **(a + b)** in length. So the area of the square is $(\mathbf{a} + \mathbf{b})^2$ and the area of the rectangles is **(a + b) a** and **(a + b) b**. It remains to notice that the square is formed as the sum of the two rectangles.

Proposition 3. If a straight line be cut at random, the rectangle contained by the whole and one of the segments is equal to the rectangle contained by the segments and the square on the aforesaid segment.

This is essentially the Distributive Law in that $(\mathbf{a} + \mathbf{b})\,\mathbf{a} = \mathbf{a}^2 + \mathbf{ab}$.

I leave it to the reader to prove it from the construction.

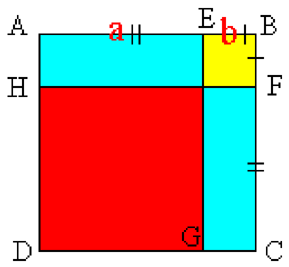


Proposition 4. If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments.

This is the binomial expansion

$$(\mathbf{a} + \mathbf{b})^2 = \mathbf{a}^2 + 2\mathbf{ab} + \mathbf{b}^2.$$

I leave it to the reader to prove it from the construction.



The other six Propositions are similar in nature. Interested readers can investigate these in Heath [2].

C. Solving Equations

Eves [3] states:

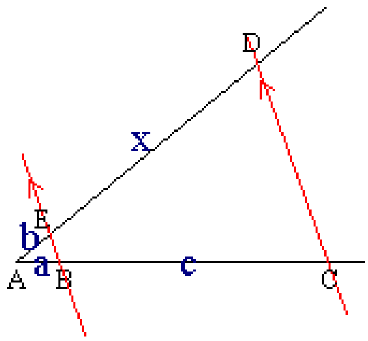
In their geometric algebra, the Greeks employed two principal methods for solving certain simple equations – the method of proportions and the method of application of areas. There is evidence that both of these methods originated with the Pythagoreans.

The Pythagoreans identified many different ‘styles’ of quadratics, and applied their geometrical propositions to each different style to arrive at a unique solution for each. This essay will show only a few of their techniques; the interested reader should look in Books II and VI of *The Elements* for a more comprehensive treatment.

Solving Equations by Proportion

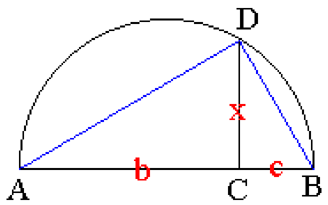
As mentioned previously, the early Pythagoreans defined ratio in terms of the Natural numbers. Their techniques for solving what we would call algebraic equations used proportions based on this definition. The discovery of the incommensurables caused this definition for ratio to be discarded. It wasn't until Eudoxus' outstanding work 150 years later that an effective definition for ratio was found and a rigorous theory of proportions was established. To overcome this problem in the meantime, the Pythagoreans resorted to geometrical proportion using the properties of similarity.

To solve the linear equation **ax = bc**, they used two properties which later appear as Propositions 8 and 9 in Book VI of *The Elements*. This involved making the construction below, the length **x** being the solution. The technique is based on the relationship **a:b = c:x** which results from the parallel lines **EB** and **DC**.



For example, if they wished to solve $\mathbf{4x} = \mathbf{2} \times \mathbf{32}$, they would construct a straight line ABC 36 units long with B 4 units from A. At an angle to AC, a second segment AED would be drawn with E placed at a distance of 2 units from A. EB would be joined and CD constructed parallel to BE. The resulting segment ED would represent the solution to the equation (16 units).

To solve the quadratic equation $\mathbf{x}^2 = \mathbf{ab}$, they used what was to become Proposition 13 in Book VI of *The Elements*. This involved making the construction below, the length **x** being the solution. The technique is based on the relationship **b:x = x:c** which results from the properties of the similar triangles ACD and DCB. This process was also used to find the square root or to "square the rectangle".



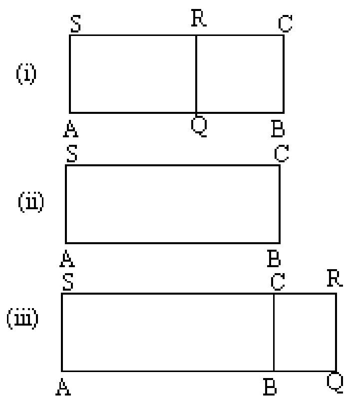
For example, to solve $\mathbf{x^2 = 8 \times 2}$, they would construct a semicircle with diameter AB 10 units in length and with AC 8 units. The length of the perpendicular from C to the circumference of the semicircle (CD) is the solution of the equation (4 units).

Solving Equations by Application of Areas

Heath quotes Proclus on the method of 'application of areas' as writing:

These things says Eudemus, are ancient and are discoveries of the Muse of the Pythagoreans, I mean the **application of areas**, their **exceeding** and their **falling short**.

To appreciate application of areas, consider the rectangles ABCS and AQRS where the side AQ lies along the ray AB.

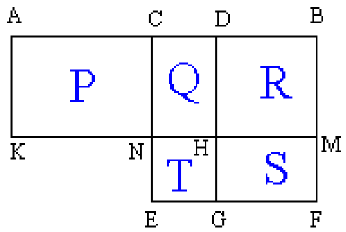


When AQ is less than AB (i), the Pythagoreans would say that the rectangle AQRS is **applied to segment AB, falling short by rectangle QBCR**; when AQ is equal to AB (ii), they would say that the rectangle AQRS is **applied to segment AB**; when AQ is greater than AB (iii), they would say that the rectangle AQRS is **applied to segment AB, exceeding by rectangle QBCR**

Proposition 5 of Book II of *The Elements* states:

If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the points of section is equal to the square on the half.

The diagram to demonstrate this is shown below.



The straight line is AB, and it is cut equally (i.e. into two halves) at C and unequally at D. The rectangle ADHK is constructed where DH = DB. The square on CB and auxiliary lines are also constructed. The proposition states that the area of the rectangle ADHK (P+Q) plus the area of the square NHGE (T) is equal to the area of the square CBFE (Q+R+S+T). I will leave it to the reader to establish this.

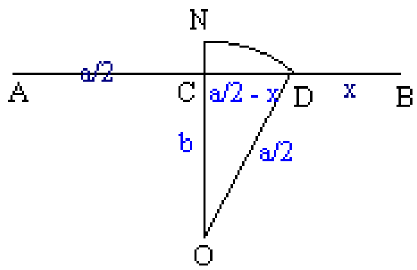
From the figure we can also establish that:

- the area of the rectangle ADHK (P+Q) must equal the area of the gnomon NCBFGH (Q+R+S) since area NHGE (T) is common; and
- that DBMH is square.

Now if we let the area of the said gnomon be $\mathbf{b^2}$ (say) and let AB = **a**, and DB = **x** then we can say that $\mathbf{ax - x^2 = b^2}$. This is the quadratic

$$\mathbf{x^2 - ax + b^2 = 0}\ .$$

According to Heath the solution of $\mathbf{ax - x^2 = b^2}$ was obtained with the construction "**To a given straight line (a) to apply a rectangle which shall be equal to a given square ($\mathbf{b^2}$) and shall fall short by a square figure**" as follows:



Draw AB equal in length to **a**. Bisect AB at C. Draw CO perpendicular to AB and equal in length to **b**. Produce OC to N so that ON is equal in length to CB (**a/2**). With O as centre and radius ON, describe a circle cutting CB in D. DB is found which equals **x**. This is because triangle CDO is right-angled with CO (= **b**), OD (= **a/2**), and CD (= **a/2–x**) being Pythagorean triples. Thus

$$(a/2)^2 = b^2 + (a/2 - x)^2$$
$$a^2/4 = b^2 + a^2/4 - ax + x^2$$
$$\text{i.e. } 0 = x^2 - ax + b^2$$

Though the justification for this construction may require some understanding, the construction itself is quite straightforward. Of course for a Real solution to this style of quadratic, b^2 must not be greater than $(a/2)^2$.

Conclusion

This essay has attempted to demonstrate how the early Pythagoreans were able to use their discoveries of proportion and geometry to solve linear and quadratic equations. In this way they were able to perform algebraic processes similar to those expected of junior high school students today; a degree of sophistication not often credited to them.

References

[1] Kline, Morris B. , Mathematics – The Loss of Certainty, N.Y. :Oxford University Press, 1980.

[2] Heath, Sir Thomas L., Euclid – The Thirteen Books of The Elements Second Edition Vol i, N.Y.: Dover Publications (orig 1908).

[3] Eves, Howard., An Introduction To The History of Mathematics 5th Ed., N.Y.: Saunders College Publishing, 1983.

LETTER TO THE EDITOR

Dear Sir,

As author of MathsMania Year 7, which was reviewed in your recent publication, I would like to thank both you and the reviewer, Jill Hedley, for taking the time and interest in the year seven edition of the series of student activity books. The series was originally written to support student learning through an organised, integrated and motivating approach by working through a variety of activities which reinforced learning experiences provided by the teacher. The format was also designed to provide teachers with a developmental program on which to base learning experiences to promote quality outcomes in mathematics education. A series of Teacher Resource Books for all year levels provides the teacher with assessment of each unit in the form of a mini Maths trail for direction with extension and remediation and, at a later date, inclusion in the student profile.

I agree wholeheartedly with the reviewer that, although the use and understanding of associated vocabulary, investigative approaches and problem solving, concrete materials, collaborative learning, discussion, mental computation and estimation before calculation have been emphasised throughout the series, strategies and teaching approaches for these have not been included. All of these issues I support, and have discussed at length through a series of presentations introducing the series throughout Southeast Queensland late last year, as well as through many conference presentations over the past few years. However, my intention with the series is to provide both teachers and students with a developmental program of activities addressing Queensland curriculum aspects, not a theory of teaching practice. This may be a future project, but with the costing limitations of a student activity book, this was not the place for my espousing. The references to the promotion of use and understanding of these issues are to inspire and direct thought for teacher instruction and to encourage their inclusion into teaching programs.

The books are currently being used as school programs, as support activities, as homework contracts, for teacher planning, in home schooling – for many uses, even collecting dust on reference shelves, but they are being used to support both teacher and student learning which is the reason they were written. I agree with the reviewer that, if used alone, any text or workbook could lead to a narrow approach, but in the hands of the talented teachers we have in Queensland, MathsMania provides a worthwhile starting point for mathematical thinking, working, planning and assessment in the pursuit of quality teaching and learning in mathematics education, but then maybe I'm biased.

Andrew Boswell

STUDENT PROBLEM PAGE

Cheryl Stojanovic Forest Lake College
Garnet Greenbury Publications Committee

Question 1

What is the smallest number that is divisible by all of the integers from 1 to 10, inclusive?

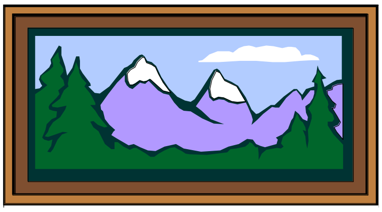
Question 2

A 10 metre length of tape is to be used to mark the boundary of a figure with four straight sides. What is the largest area that can be enclosed within the tape?

Question 3

45% of the inhabitants of QAMTOWN are female. The males of the town outnumber the females by 132. What is the population of QAMTOWN?

Question 4



An unframed rectangular painting measures 32 cm by 54 cm. How wide should the frame be made so that the width of the framed painting is two thirds of its length?

Question 5

A hotel required 1470 brass numerals to number the doors of all its guest rooms. The rooms were numbered consecutively from room number 1. How many guest rooms are in the hotel?

Submitting solutions

Students are invited to submit solutions. Include your name, the problem number, your school and year level (clearly printed). Send them to Garnet Greenbury, Unit 14 Greenleaves Village, Upper Mt Gravatt, 4122. Closing date: 1st October 1999.

Solutions to the problems, Vol. 24 No. 2

1. 6 minutes
2. 11 boys
3. 50 tiles
4. 7 chefs
5. No solution*

*Note on Question 5

The editor would like to apologise for a typographical error in the printing of Question 5, the result of which was that there was no solution to the problem. The seat mentioned in the fifth row should have been number 33. There would then have been 10 rows and 7 columns.

Cheryl is now sending me the questions by e-mail to ensure that this does not happen again.

Prizes

Prizes for solutions to the student problems in *Teaching Mathematics* Vol. 24 No.2:

Monique Fenech of Pimlico State High School is the winner of the Penguin book prize.

Shyamini Gunuratne of Brisbane State High School is the winner of the annual subscription to *Tenrag*.

Solutions were received from:

St Mary’s College, Cairns
Pimlico State High School
Kingaroy State High School
Brisbane State High School