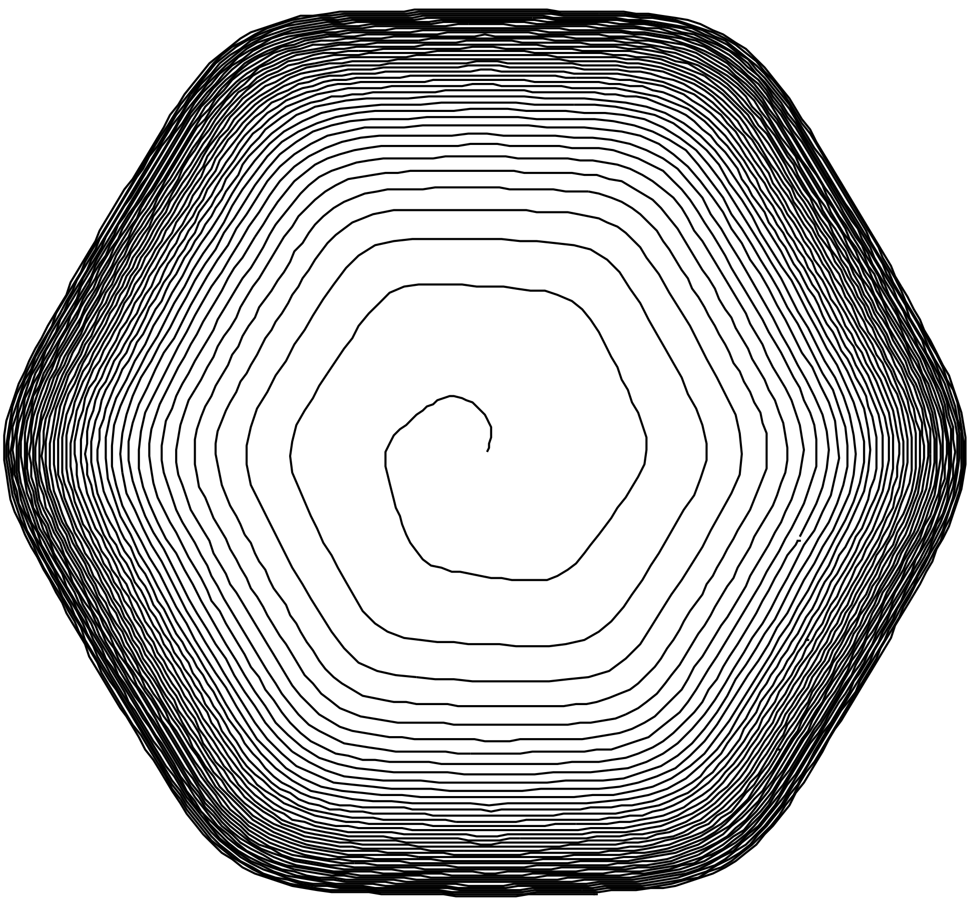
TEACHING MATHEMATICS

Volume 24 No. 4 November 1999

The Journal of the Queensland Association of Mathematics Teachers Inc



$r = [1 + 0.04 \cos(6\theta) + 0.006 \cos(12\theta)] \log(\theta) \quad \{1<\theta<250\}$

Teaching Mathematics is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, August and November) and is provided as a right to all members of the association.

The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- and to speak and act on matters related to mathematical education.

Advertising material published in and accompanying this journal does not imply endorsement by the association.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the editor, David Ilsley. The preferred way is by e-mail to ilsley@acenet.net.au Materials may also be sent on floppy disk. Contact details are as follows:

David Ilsley	Phone (home):	(07) 3807 1327
Queensland School Curriculum Council	Phone (work):	(07) 3228 1501
PO Box 317	Fax (work):	(07) 3228 1555
Brisbane Albert Street	e-mail (home):	ilsley@acenet.net.au
Qld 4002	e-mail (work):	david.ilsley@qscc.qld.edu.au

Word for IBM (up to Word 97) is the preferred format. All receipts will be acknowledged – if you haven't heard within a week, ring David to check.

Copy dates are: mid-February; mid-May; mid-July; mid-October.

If you have any questions, contact David Ilsley.

Publications sub-committee members are listed below. Feel free to contact any of these concerning other publication matters.

Books, software etc. for review should be sent to Lydia Commins; information to go in the newsletter should be sent to Sue Reid.

Newsletter copy dates are a month earlier than those for the journal.

QUEENSLAND ASSOCIATION OF MATHEMATICS TEACHERS INC

Executive:

President:	John Butler (BSSSS 3864 0289)
Vice Presidents:	Brad Barker (Carmel College 3206 0000)
	Merrilyn Goos (University of Queensland 3365 7949)
Secretary:	Gary O'Brien (Cannon Hill Anglican College 3896 0468)
Assist. Sec:	Ian Cronk (BSSSS 3864 0285)
Treasurer:	Lydia Commins (Ipswich Girls Grammar School 3281 4300)

Committee:

Kerry Bates, Rex Boggs, Marjorie Carss, Jan Cavanagh, Peter Cooper, Tom Cooper, Judy Hartnett, John McKinlay, Geoff Morgan, Sue Reid.

Executive Sub-Committees:

Policy:	John Butler (Convenor), Marjorie Carss, Tom Cooper, John McKinlay.
Publications:	Brad Barker (Convenor), Kerry Bates, Rex Boggs, Marjorie Carss, Peter Cooper, Lydia Commins, Garnet Greenbury, Judy Hartnett, David Ilsley, Sue Reid.
Professional Development:	Merrilyn Goos (Convenor), Brad Barker, Kerry Bates, John Butler, Marjorie Carss, Jan Cavanagh, Peter Cooper, Tom Cooper, Ian Cronk, Judy Hartnett, Geoff Morgan, Gary O'Brien.
Branch Liaison:	Tom Cooper, Ian Cronk.
Membership:	Lydia Commins.
Finance	Lydia Commins (Convenor), Marjorie Carss.

Sponsorship and Marketing: Geoff Morgan.

QAMT Office:

QAMT Inc
PO Box 328
Everton Park 4053

Phone and fax : (07) 3864 3920
e-mail: qamti@gil.com.au

Office hours: 9 – 4, Wednesday and Thursday, term weeks and some holiday weeks
Office Administrator: June Nowitzki

Membership and advertising enquiries: contact the QAMT office.

Advertising Rates	1 Issue	2 Issues	3 Issues	4 Issues
Quarter page	$40	$80	$105	$120
Half page	$60	$120	$165	$200
Full Page	$100	$200	$285	$360
Insert (single A4 or folded A3)	$200	$400	$585	$760
Other: Contact QAMT				

QAMT Website: http://qamt.cqu.edu.au

SMARD Website: http://smard.cqu.edu.au

QAMT Mailing List: to subscribe, e-mail majordomo@cq-pan.cqu.edu.au saying 'subscribe qamt'.

If undelivered, return to:

QUEENSLAND ASSOCIATION OF MATHEMATICS TEACHERS INC.
P.O. BOX 328   EVERTON PARK   QLD 4053

TEACHING MATHEMATICS

Print Post Approved
No: PP451223/00246

November 1999 Volume 24 No 4

TEACHING MATHEMATICS

CONTENTS

Regular Features

From the President	John Butler	2
From the Editor	David Ilsley	2
What's Happening?	Neville Grace	3
Page 15 Problem	Ed.	15
Reviews	various	20
QAMT Problem Solving Competition	Neil Williams	32
Student Problems	Cheryl Stojanovic, Garnet Greenbury	39
Index to Volume 24		40

Conference Proceedings

Annual Conference, Rockhampton, 1999	Ed.	6
The Literacy of Numeracy	Judy Hartnett	8
The Glass Slipper	Merrilyn Goos	12
Reflection on Learning – Strategies for use in Senior Mathematics – Part 2	Lyn Nothdurft	22
Linking 2 & 3 dimensions – a challenge for students and teachers	John Butler	27
MSWLogo	Paul Dooley	31

Special Features

R (-squar) ED – DANGER	Helen MacGillivray	16
Pythagoras Revisited – Lessons from the Past	Paul Dooley	34

FROM THE PRESIDENT

John Butler
Queensland Board of Senior Secondary School Studies

Once again we have been able to provide a very successful and exciting annual conference for our members. This year as you know, for the first time we ventured outside the south-eastern sector and held the conference in Rockhampton. On behalf of all QAMT members I thank the organising committee for their tireless efforts in planning and executing a smooth and efficient operation. Congratulations are extended to Kiddy Bolger as convenor and his team: Pat Moran, Rex Boggs, Leon Le Cudennic, Robert Bowser, Rhonda Farragher, Milton Fuller, Peter Cooper, Simon Warren and Russell Ogilvie. A big Thank You is also extended to Central Queensland University as host, the conference sponsors, and the students of the university who dutifully tended to the 'house-keeping' tasks under the direction of Pat Moran.

The outstanding success of this conference indicates to us that we need to seriously consider holding future conferences in other regional locations and hosted by the local sub-branches. Planning for the 2000 conference is already under way and it will be held on the Gold Coast. I urge each of the sub-branches to consider hosting the 2001 annual conference.

Following the AGM, the 1999-2000 committee has been formed and planning for next year's activities is well under way. I take the opportunity here to welcome Kerry Bates and Judy Hartnett to the committee, and sadly to farewell Jill Hedley and Lynn Squire thanking them sincerely for their contributions.

An initiative planned for 2000 is the introduction of an organised borrowing system for our library. From the beginning of 2000, members and member institutions will be able to access resources from the library. A six-month trial will be conducted whereby QAMT will meet the postage costs for sending resources to members or member institutions. Costs associated with the return of resources will be met by the borrower. Resources may be borrowed for a period of six weeks with some provision for extension. Requests for loans will be made by FAXFORM. A copy of the faxform with the conditions of loan (overleaf) is provided as an insert with this journal. Keep a watchful eye on the web-site for a catalogue of resources, along with policy details for borrowing. More information will be available in future journals and newsletters.

Finally, I wish to publicly thank the out-going president John McKinlay for his inspirational leadership over the past two years. As I indicated at the conference dinner, it will be a real challenge for me to follow in John's footsteps.

FROM THE EDITOR

David Ilsley
Queensland School Curriculum Council

Welcome to the last issue of *Teaching Mathematics* for the year (and indeed for the century). This issue includes some of the papers presented at the Annual Conference in Rockhampton. We have a new policy of offering to presenters to publish their conference papers on the QAMT web site. So you can find further interesting reading there.

The QAMT site has improved considerably over the past few months, thanks to the work of Peter Cooper. If you haven't visited it for a while, it could be worth a look. The address is http://qamt.cqu.edu.au – and don't forget the SMARD site at http://smard.cqu.edu.au.

We would like to include more papers in the journal (and on the web site) which are of interest to primary teachers – the journal has had a bit of a secondary bias at times and this issue, I'm afraid, is no exception. There were lots of people at the conference who presented material of primary interest. If you haven't done so, you might consider sending something in. If you wish to send it directly to the journal, my contact details are inside the front cover. Please note that my home e-mail address has changed since the last issue.

The March issue of *Teaching Mathematics* will contain further papers from the conference as well as other material. In the mean time, I hope you all have a good Christmas break and come back refreshed and ready to face the new year.

What's happening?

in mathematics education

In this issue, Neville Grace talks about the initiatives coming out of Education Queensland's 2010 Review and their relation to the production of KLA syllabuses by the Queensland School Curriculum Council

A Rich Pedagogical Soup

Extracts from QAMT Conference address (September 1999)

Neville Grace
Senior Education Officer (Mathematics)
Education Queensland

Introduction

There is currently a paradox that many quite able students do not demonstrate success in school learning. To put this another way, what is learned in school appears to be of little relevance or applicability to many students. QSCC syllabuses are one response in an attempt to state basic requirements more clearly. The 2010 review within Education Queensland has thrown up two responses to this paradox: school differentiation and *New Basics*.

I argue that the QSCC and Education Queensland initiatives need not be seen as incompatible.

First initiative – QSCC syllabuses

On the one hand we have the Queensland School Curriculum Council hard at work developing a new mathematics syllabus for Years 1 to 10. Plans are in train also to produce a large set of supporting materials for teachers. We can look forward to excellent output in several years time.
The new syllabus will be outcomes-based. Its intent will be to provide a much clearer statement of expected learning outcomes than we have had in the past. I am quite sure that the developers of all QSCC syllabuses are concerned to reduce the gap between "street smartness" and "school failure" that bedevils so many students.

Differentiation

The move within Education Queensland to encourage "differentiation" among schools is an extension of self-management in state schools. The idea is that each school will have the power, the right and indeed the responsibility to negotiate with its various communities to badge itself so as to respond more effectively to local needs and concerns. I believe it will be a healthy position for all of us if parents and students can choose a state school because of its special ethos and set of curriculum offerings rather than simply because it is the nearest state school and just like any other.

At the same time, in order to remain a system, instead of a disparate collection of differentiated schools, it seems reasonable that state schools retain a common focus – that they provide a guarantee as to what sets of essential learning outcomes are to be provided for all students who attend. A current initiative is to provide this curriculum guarantee through the identification of *New Basics*.

Preliminary thoughts about New Basics

1. One belief that underpins the current *New Basics* project is that, if our objective is to provide all students with a rich set of applicable knowledge and skills, attempts to organise learning in schools in boxes labelled KLAs will be counterproductive. Too often, such formalised bodies of knowledge tend to: become crowded with content that favours the learning area over the needs of the learner; allow their inherent subject logic to dominate the pedagogy; and be bound by expectations and credentials that cut many students out of success.

2. In the ill-fated experiment with SPS outcomes, failure could have been partly due to emphasis on reporting rather than on curriculum planning. However, that experience and reports from some other states suggest that dozens of outcomes tend to confuse rather than assist curriculum planners. The current initiative is attempting to identify *New Basics* as a few big task outcomes, in which categories of knowledge and skill are embedded, rather than build up from multiple outcomes.

3. The plan being developed within Education Queensland is for a small set of schools to pilot ideas for the next couple of years. It is expected that the other 1280 or so state schools will continue with curriculum development using QSCC syllabuses. They will not be required to do anything in immediate response to this *New Basics* project. However the project will be transparent. The draft outcomes are already up for view and discussion, on a website. We can be sure that they will alter over time as trial schools develop shared understanding and come to grips with implementation issues.

4. I am confident that there will be significant benefits for learners in having schools proceed with implementation of the QSCC-developed Science and HPE syllabuses – and others as they come on line. Benefits for students will flow from having whole school communities focused on attainment of comprehensive sets of defined learning outcomes.

5. There should also be at least three clear professional development benefits since teachers implementing QSCC syllabuses will be gaining experience with: an approach to outcomes-based education; school curriculum planning across significant spans of schooling; and strategies for generating shared understanding within schools and clusters about outcomes, standards and ways of judging performance.

6. Materials and ideas in the *New Basics* project are being circulated and change almost daily. Some clarity about overall direction has been established and will be discussed under four headings:
   a. Categories for the *New Basics*
   b. Junctures for reporting
   c. Task outcomes
   d. Reporting framework

Categories for the New Basics

I think these are not new and will be no surprise.

1. *Life pathways and social futures*
2. *Communications media*
3. *Active citizenship*
4. *Environments and technologies*

The suggested junctures

These might be a source of some surprise. Juncture 1 will be about the middle to end of Year 3; juncture 2 will be about the middle to end of Year 6; juncture 3 will be about the middle to end of Year 9.

The Year 9 juncture carries messages about Years 10 to 12 providing multiple pathways to future study and careers. A review team is currently looking at issues to do with pathways at Years 11 and 12. Richard Smith from Griffith University, Alan Cummings from QUT and Gabrielle Matters from The BSSSS are part of the review team.

I feel confident that current fascination with two-year courses of study and narrow entry pathways will eventually implode. There is a need for opportunity to build all manner of pathways through two-year, one-year and even semester courses – some perhaps with certificates and credentials based on competency tests. See my recent article in the QAMT journal *Teaching Mathematics* for some small suggestions. We need to make schooling through to Year 12 more attractive and rewarding for all.

Another part of the reasoning about junctures is that 6 or 7 or 8 levels of outcomes are too many – too many outcomes – too difficult to discriminate between them – too difficult to report on achievement – and too tied up with ideology about supposed developmental levels of students.

In the 1999 AAMT Virtual Conference, John Pegg provided a keynote address with a catchy title, *Outcomes based education in Australia: nice idea, pity it lacks a framework*. He argued in favour of a framework based on theories of child development. John says that without a developmental framework, *the only way for teachers to survive is to remember what is in what level by rote*. While I do not agree with all that is in John's paper, I recommend it to you as thought provoking.

Tasks outcomes

These are a new concept. They are certainly not meant to be "assignments" or "projects" or in any way trivial. They are intended to incorporate many outcomes, in the KLA sense, from SOSE and Science and HPE etc and of course Mathematics – but I doubt that they will incorporate a full set of core outcomes from any of the syllabuses.

Each task must:

1. Have substantial intellectual substance, rigour and value
2. Be interdisciplinary – across categories of *New Basics*
3. Be problem related, with relevance and power
4. Have validity with educators and community – acceptance as significant, powerful and important
5. Have depth and breadth to guide curriculum planning across a three year span of schooling

The issue of reporting

Thinking is still tentative here. I believe that, at the three junctures, reporting on the *New Basics* will be in terms of each task outcome i.e. about 6-9 outcomes at a juncture. Reporting at other times, and reporting on school options, would be up to the school.

Conclusion

The implementation of a *New Basics* curriculum will require a multidisciplinary approach to teaching – the *rich pedagogical soup* that I refer to in the title.

The website for the frameworks project is a good place to start:

www.qed.qld.gov.au/news/framework/htm

A Store of Application Questions
for Mathematics A, B and C

A Call for Contributions

Coming up with good application problems for Mathematics A, B and C assessment can be difficult. But most teachers have, in their time come up with a few quite inspired questions. Around the state, there must be hundreds of such questions. Finding out about them and getting hold of them are the problem though.

QAMT is planning to put together a store of application questions for Mathematics A, B and C. It is hoped that the store will contain a few of hundred questions organised by subject and topic. The store will be published in print form and on the web.

This is a call for contributions. If you have or know of one or two questions that have worked well with your students, consider sending them in (with their solutions!). Contributions will be acknowledged unless the contributor wishes otherwise. To avoid copyright infringements all contributions will need to be original.

Contributions can mailed to Brad Barker, c/- QAMT (P.O. Box 328, Everton Park Q 4053) or e-mailed to Brad at bandc@fan.net.au.

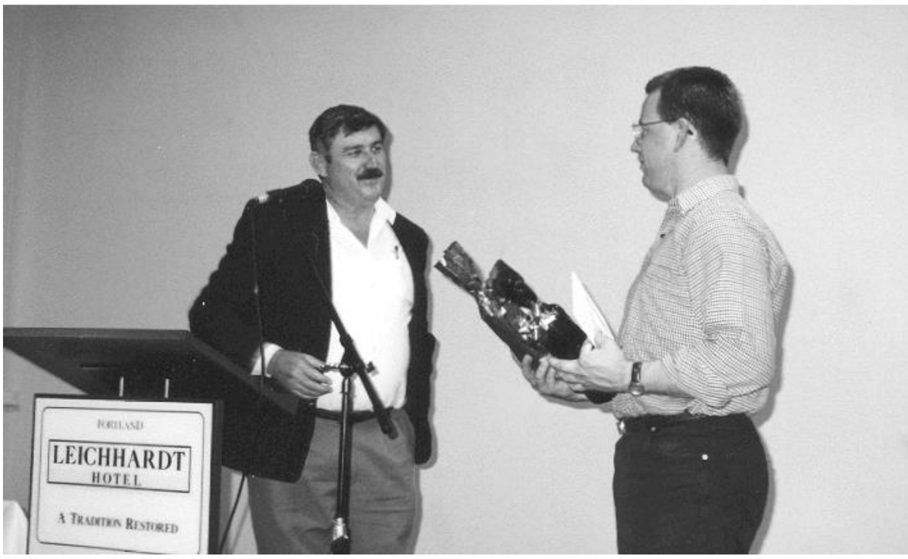
ANNUAL CONFERENCE

Despite being held for the first time outside the southeast corner, this year's annual conference in Rocky attracted a record number of participants and a record number of presenters. A very wide choice of presentations was offered at all sessions and the only complaint from participants was of not being able to attend more of them. Professional growth and a good time were had by all.

Left:

Four distinguished guests enjoying a drink before the conference dinner.

From right to left: Barry Kissane from Western Australia, Gillian Kidman from QUT, Susie Groves from Victoria and Kerri Hill from Townsville Grammar



Right:

Handing over the presidency – and the bottle.

John Butler, incoming president, makes a presentation to John McKinlay, outgoing president, in recognition of his service to the association.

ROCKHAMPTON 1999

Congratulations and thanks are due to the organising committee consisting of Kiddy Bolger, convenor,: Pat Moran, Rex Boggs, Leon Le Cudennic, Robert Bowser, Rhonda Farragher, Milton Fuller, Peter Cooper, Simon Warren and Russell Ogilvie. Thanks also to Central Queensland University for providing the venue, to the conference sponsors, and the students of the university who looked after the 'house-keeping' tasks.

Right:

Merrilyn Goos, Vice President presenting Pat Moran of Central Queensland University with a token of the association's appreciation for the many days of hard work she put into the organisation of the conference.

Left:

Anna Weathereld, Delvene Wildey and Amy Fatialofa, Batchelor of Education students from the University of Queensland, enjoying the conference dinner.

Anna was one of six students sponsored by Casio to attend and make a presentation at the conference

The Literacy of Numeracy

Judy Hartnett
Education Officer – Mathematics
Brisbane Catholic Education.

The development of mathematical concepts and the nature of mathematics itself is vitally connected with language. To understand and communicate mathematical ideas students need experience with reading, viewing, speaking, listening and writing mathematics. Are the skills required for these strands of English the same as for numeracy ? This paper will look at each of these strands and compare the way students process information in English and how the related skills match or do not match Mathematics.

Several Commonwealth initiatives in the past couple of years have lead to a focussed interest in literacy and schooling. Numeracy is often implicitly included in these documents.

Is numeracy mathematical literacy?

At a recent workshop mathematics teachers were asked to define literacy. Responses included:
Literacy is understanding and using language;
Literacy is communication;
Literacy is a social skill;
Literacy is making sense of information;
Literacy is comprehending and composing.
Literacy is an ability to convey meaning;
Literacy involves speaking, listening, reading, viewing and writing;

The participants were then asked to consider numeracy and to assess how many of the definitions for literacy still applied. We were able to see a connection with all these definitions of literacy for numeracy.

In his book *Innumeracy*, Paulos, (1988, p.3), says that mathematical illiteracy 'plagues far too many otherwise knowledgeable citizens.' He gives the example of a grammarian friend who was incensed at the confused use of 'continually' and 'continuously' yet was unperturbed when a 'T.V weathercaster announced that there was a 50 percent chance of rain on Saturday and a 50 percent chance of rain on Sunday and, therefore, a 100% chance of rain that weekend.' Paulos claims that it is imperative to be 'mathematically literate' in the modern world, especially with the use of statistics in the media and political propaganda circles. A mathematically literate person would know when the mathematics used makes sense. They would know if the type of mathematics used was a sensible choice in the first place.

I believe that mathematical thought can be accessed through natural language. I further believe that prose can assist people to explore the essential nature of mathematics and build mathematical literacy.

Literacy involves the integration of speaking, listening, viewing and critical thinking with reading and writing. Effective literacy is intrinsically purposeful, flexible and dynamic and continues to develop through an individual's lifetime. Successful students in mathematics need to develop a variety of literacy skills in order to develop and convey their knowledge, skills and understandings of mathematics.

The Queensland English syllabus is based on purposeful communication which serves particular social functions including building social relations. This purpose or function determines the mode and medium and all other aspects of the communication. The strands of this syllabus – Reading and Viewing; Speaking and Listening; and Writing will be used to investigate numeracy and mathematics for the rest of this paper.

Reading

The NSW Department of Education notes that in studying mathematics, students are expected to read and locate specific information, and understand concepts and procedures, as well as to interpret problems. When reading familiar texts we often leave out words, change their order or even substitute words. Language is normally full of redundant information. This allows us to understand by skim reading or to gain meaning from the use of key words and contextual clues. Consider the following English text.

The red headed boy with the skinny legs sprinted across the dusty pavement from the dubious protection of the banyan tree to the infinite safety of his own iron clad patio and his mother's protective arms.

This could easily be replaced with the following text which holds the same meaning.

The boy ran home.

Mathematics texts, however, are often lexically dense. This means that few words are used, all essential to the meaning. Also, as part of the literacy demands on mathematics, word order is very important. Consider the following two questions which contain exactly the same words.

Sixty is half of what number ? *Half of sixty is what number ?*

There are no redundant words here, all words are important and the conjunctions cannot be skimmed over as they have important meaning as well. Children need to use different skills to read mathematical text than they do for other text.

The NSW Dept of Education (1997) notes that the order in which information is presented in language is often at odds with the order in which it is processed in mathematics. This mismatch occurs even with very simple questions such as "Take 6 from 12". Weaker readers process information in the order in which it is encountered. Even students fluent in everyday spoken English may still have problems with "the number 5 is 2 less than what number?" The 5, 2 and less is that order suggest the answer is 3. The way the words are put together (the syntax) produces a different result. The mental restructuring that is necessary to recover the meaning may overload a student's processing and memory capabilities. Students often give up and simply guess what to do with the numbers.

Malcolm Swan (1990) used the following page from a primary Chinese mathematics textbook to illustrate that most word problems can be solved by using context cues.

1 輪船上有乘客共 2672 人，其中中國籍人仕有 2098 人，問該輪船上外籍乘客有多少人？

2 玩具 24 件，平均分給 8 人，每人可分得幾件？

3 李宅本季水費 105 元，恰是陳宅的 3 倍，陳宅本季水費若干元？

4 被加數是 2405，加數是 7504，和是多少？

5 每週上數學課 6 節，19 週共上數學課幾節？

Readers are able to work out the required operations for each example using similar context cues that children use when reading mathematics problems:

- If there are more than two numbers, add them;
- If two numbers are similar in magnitude, subtract the smaller from the larger;
- If one number is relatively large compared to a second number, divide;
- If the division answer has a remainder, cross out your work and multiply.

MacGregor (1990) noted that mathematical statements need to be analysed into their structural components and these components then need to be related to each other. Major stumbling blocks for students trying to read mathematical and other technical texts are:

- Not being aware that analytic reading is necessary;
- Lacking experience and skill in analytic reading;
- Unfamiliarity with the standard uses of prepositions.

Viewing

"Viewing is making meaning from information encountered in the visual or nonverbal mode. Written language is often accompanied by visual language including images such as photographs, diagrams, graphs, tables and symbols." (Queensland Department of Education English Syllabus, 1994). Mathematics includes the use of such graphic information and this can cause difficulties for some students.

"We are bombarded with data everyday, from train timetables to graphs showing fluctuations in interest rates. Many students and adults find such data at best confusing and at worst incomprehensible. While most students are capable of learning many of the technical skills associated with point plotting and so on, few are able to interpret the meaning of global features contained in a graph, even when it is set in a familiar context. Many students interpret graphs as if they are pictures of given situations, rather than abstract representations." Malcolm Swan (1990)

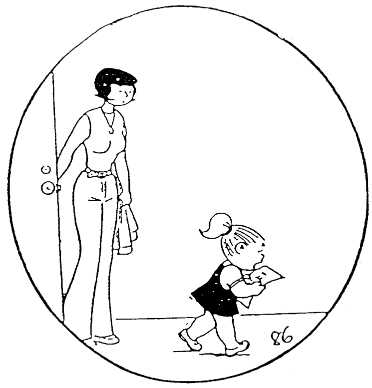
Possible causes and implications for this, which could be transferred to other aspects of mathematics than viewing, were listed as priorities in teaching mathematics being misplaced. Many teachers seem to lay more emphasis on equipping students with facts and skills than on enabling them to build conceptual structures. Lessons on graphs for example, simply involve students plotting points or drawing bars without ever stopping to discuss or reflect on the meaning of what they are doing.

Speaking and listening

"Many mathematics teachers, unlike colleagues within out departments, are unused to handling classroom discussions. Oral work is often restricted to brief periods of questioning by the teacher followed by monosyllabic replies by students. Little opportunity is offered for students to describe and develop their ideas. Textbooks also place their main emphasis on the rehearsal and practise of facts and skills. They are written to minimise the demands on the busy teacher and their language, is therefore kept as simple as possible and tasks are fragmented into small isolated steps that can be mastered one at a time. Children are kept busy, and discussion is kept to a minimum." (Swan, 1990, p 67,58).

During discussions it is very difficult for a teacher to resist the urge to intervene and point out how answers are incorrect. The danger of succumbing too quickly to this form of "teacher-lust" has been demonstrated by Brekke (1987). He observed two teachers (among others) who had very contrasting styles. One always tried to help students in a positive way by directive teaching, explaining and correcting errors as they occurred. The other allowed the students to come to their own conclusions in discussions and in guided reflections, and did not pre-empt the discussion by stating the correct answer. A post test showed that much more substantial progress and interest had been generated by this second approach.

Teachers need to be aware that what they are saying may not be what students are comprehending. Consider the following example which is particularly relevant to teachers in Catholic schools. The mathematical term 'mass' can mean 'weight', it can mean 'church', and may be misheard to mean 'maths'. Consider the cartoon where a child states she is having trouble with eagles at school



I'm having trouble with eagles in school – one plus one eagles two, two plus two eagles four…."

Family Circus – Bill Keane

For some children the mention of eagles may be enough to have their minds soaring with the birds.

Writing:

Traditionally, writing and mathematics have been separated in the curriculum of schools, and in the minds of the general public. Most people identify writing with language, sometimes common tasks (like a letter to a friend) and sometimes not-so-common tasks (like an article in a medical journal). When considering written mathematics though, for many the stereotype of a formula with its unique symbolism comes to mind. Certainly this mathematical symbolism has credibility and should be a part of the contemporary mathematics curriculum. But is this the only form of writing mathematics? (Waters and Montgomery,1993)

Helen Pengelly (1990) notes that writing is a feature of the language curriculum, which is becoming incorporated into mathematics lessons. The catchcry "children learn to write by writing" has been interpreted into the mathematics classroom to mean children will learn mathematics by writing. Consequently, children are asked to write about the mathematics they do. How often are children expected to write about something they have just written? And the trend in mathematics classrooms influenced by language principles has been to get children to write about their mathematical experiences. This creates trivial responses, many of which have been illustrated in published articles and in papers presented at conferences. I did maths today and it was fun', or "In maths today we did polydrons. I like polydrons", or "I did maths with Sally today. I like doing maths with Sally" or, "Number 12 is the most fantastic number. I would say the number 12 is the best number in the whole wide world.

There is some evidence that the language of reasoning develops out of the language of reflection and only after the language of description and comparison is well established (Gawned, 1989). This knowledge could help teachers to scaffold children's writing in mathematics by including questions dealing with each of these aspects – describe, compare, reflect and reason.

If children are to learn to write mathematics, they need experiences writing mathematics. This is a distinctly different form of writing from the writing children do in language lessons. Finding ways to record and communicate mathematical ideas and processes encourages development in mathematical thinking. Writing mathematics, rather than writing about what was done in a lesson, will result in quite a different outcome.

As Clements (1984) has pointed out, children are likely to have more language-related difficulties in the mathematics classroom than they experience anywhere else. Teacher may not have thought about the level of reading skills necessary for comprehending technical text. Traditional school mathematics was designed for children who were learning English and Latin grammar, and who were proficient in English as their mother tongue. In Australian schools today there are large numbers of children who are learning English as a second language and many Australian-born children from non English-speaking backgrounds who may not be familiar with formal standard English. It can no longer be assumed by mathematics teachers that their pupils have acquired sufficient syntactic awareness and analytic reading skills in English to cope with the register of mathematical text. Learning how to read mathematical text needs to be part of the mathematics curriculum.

References:

Brekke, G (1987) *Graphical interpretation : a study of pupil's understanding and some teaching comparisons*. Shell Centre for Mathematical Education, University of Nottinngham, United Kingdom.

Clements, M.A (1984) Language factors in school mathematics. In Costello, P., Ferguson, S., Slinn, K., Stephens, M., Trembath, D., & Williams, D (eds) *Facets of Australian Mathematics Education.* Australian Association of Mathematics Teachers.

Department of Education, Queensland (1994) *English in Years 1 to 10. Queensland syllabus materials.* Government Printers : Brisbane.

Gawned, S (1990) An emerging model of the language of mathematics. In Brickmore-Brand J.(ed) *Language in Mathematics.* Australian Reading Association Inc : Carlton South.

MacGregor, M (1990) Reading and writing in mathematics. In Brickmore-Brand J.(ed) *Language in Mathematics.* Australian Reading Association Inc : Carlton South.

New South Wales Department of School Education (1997) *Teaching literacy in mathematics in Year 7.* Curriculum Directorate : Sydney.

Paulos, J.A (1988). Innumeracy: *Mathematical illiteracy and its consequences.* London: Penguin Books.

Pengelly H (1990) Acquiring the language of mathematics. In Brickmore-Brand J.(ed) *Language in Mathematics.* Australian Reading Association Inc : Carlton South.

Swan M. (1990) Becoming numerate: developing conceptual structures. In Willis S (ed) *Being Numerate : What Counts?* Australian Council for Education Research Limited : Hawthorn.

Waters, M & Montgomery, P (1993) Mathematics: Multiplying the Learning in Stephens, M, Waywood, A, Clarke, D & Izard, J (eds) *Communicating Mathematics.* Australian Council for Education Research Limited: Hawthorn.

The Glass Slipper

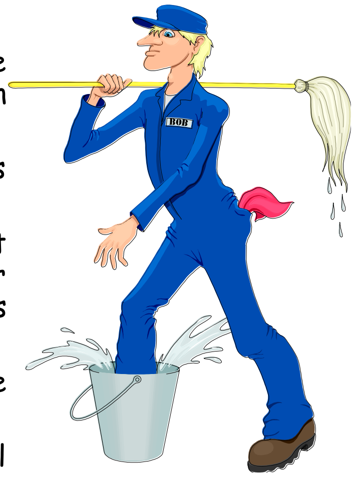
Merrilyn Goos
The University of Queensland

Introduction

Why is mathematics important? Many teachers would answer this question by pointing out that mathematics is *useful*; for example, mathematics can help us solve problems that arise in our everyday lives. This belief is echoed in the *National Statement on Mathematics for Australian Schools*, which argues that mathematics is used in daily living, in civic life, and at work (Australian Education Council, 1991). However, the *National Statement* also reminds us that mathematics is part of our culture, and, like art, music and literature, is the product of creative human minds.

The activity presented below allows junior secondary students to experience something of the creative nature of mathematics, by using a fairy tale as the context for stimulating mathematical thinking about a real life problem. The activity (adapted from Wright, 1996) makes use of graphing calculator technology to explore the relationship between shoe sizes and shoe lengths.

Cinderella puts her foot in it!



Cinderella heard the clock strike midnight. Frantically she freed herself from the Prince's romantic embrace and ran for the door.

"He must not see me in my ragged clothes," she gasped as she tumbled down the stairs.

All that remained as Cinderella vanished into the night was a tiny shoe which she had lost in her haste. The poor Prince was shattered but, thankfully, the glass slipper was not.

"I shall search my kingdom high and low until I find the foot that fits this shoe," he announced.

Next morning he took the dainty glass slipper to the royal cobbler.

"What can you tell me about this slipper which may help in my search for its owner?" inquired the prince.

"I'd look for someone who keeps falling over a lot. These are the smallest adult shoes I have ever seen!" exclaimed the shoemaker.

"Well, she wasn't a great dancer," replied the Prince, "She kept leaning on me all night. Hmmm ... perhaps she was not as fond of me as I thought? I will send a courtier around my whole kingdom with this slipper. Every woman must try it on until my true love is found."

"That would be terribly slow and the glass slipper might easily be broken by accident'" advised the shoemaker. "I can quickly make several leather shoes that are the same size. That way you can have several courtiers searching at once."

He took a measuring gadget and placed the glass slipper on it. "I knew it was small but a size negative five is unbelievable! Are you sure you want a Queen with deformed feet?"

"Yes I do. Now get to work!" commanded the Prince ...

Tasks

1. Investigate the relationship between shoe sizes and the length of feet.
2. Based on your findings from Task 1, estimate the length of Cinderella's foot if she wore a size negative five shoe. Be careful to identify any assumptions you make in arriving at your conclusions.

Possible Solutions

Task 1

This task is best tackled via a class discussion, which will probably bring forth the suggestion that students measure their foot lengths and record the corresponding shoe sizes in a table. However, there are some practical difficulties with this approach, not the least of which is the prospect of your students taking off their shoes in the classroom!

You could also point out that it will be difficult to obtain accurate data unless standardised techniques and special measuring instruments (such as those used in shoe shops) are used; for example, taking all measurements at the same time of day (feet swell in hot weather, and after long periods of standing), and when the person is standing with both feet firmly on the ground. Students will soon suggest measuring shoe lengths as an alternative, but again it is wise to agree on a method of measurement and stress the need for accuracy.

Once the data are recorded there is value in further discussion of any anomalies, such as obvious mis-measurements or the existence of different shoe size systems in the UK, USA and Europe. The size and length data can be entered into two lists via the STAT menu and Edit sub-menu. A scatter plot can then be produced through the STAT PLOT menu.

Since each class will generate a unique data set it is not possible to provide a definitive solution for this task; however, the scatter plot should suggest a linear relationship. If the relationship is not clear because there is a limited range of shoe sizes in the class, students could be asked to gather more data from their families for larger and smaller shoes.

You should be aware that there will be some variation in the measured lengths of shoes marked with the same size. Shoe manufacturers who were approached to supply realistic data for this task were reluctant to do so, because differences in styling affect the external length. This point can be elicited from the students themselves when you ask them to explain any trends in the data.

A sample solution is offered below, based on the published shoe manufacturer's UK size comparison chart in Table 1 (obtained from Berry & Green, 1983).

Table 1

Shoe size	1	1.5	2	2.5	3
Length of shoe (mm)	220.1	224.4	228.6	232.8	237.1

Shoe size	3.5	4	4.5	5	5.5
Length of shoe (mm)	241.3	245.5	249.8	254.0	258.2

Shoe size	6	6.5	7	7.5	8
Length of shoe (mm)	262.5	266.7	270.9	275.2	279.4

Figure 1 shows the data in list form, and Figure 2 the scatter plot mentioned above. This linear plot shows that shoe lengths increase by the same amount for each half size increase.

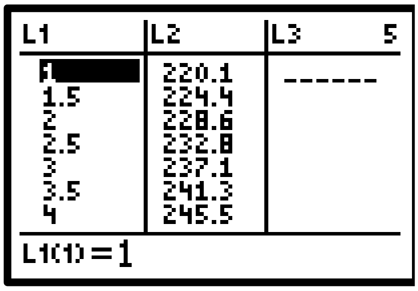


Figure 1

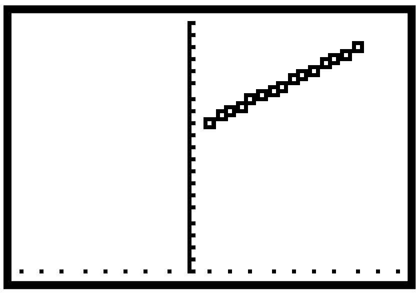


Figure 2

Task 2

A linear model appears to be appropriate, and the equation of a line of best fit can be found by choosing suitable data points and calculating the gradient *m* and y-intercept *c*. The resulting equation ($y = 8.47x + 211.67$ for the data shown above) can then be entered into the Y= editor and the line graphed over the scatter plot (Figure 3).

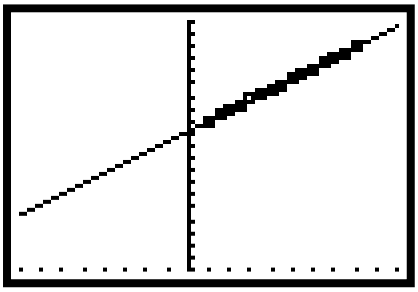


Figure 3

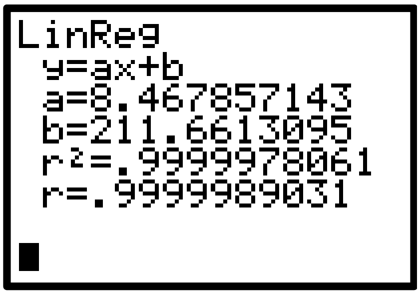


Figure 4

A linear regression analysis could also be carried out through the STAT menu and CALC sub-menu. Results for the data given in Table 1 are shown in Figure 4. The regression line can then be graphed by pasting its equation to the Y= editor and pressing GRAPH. One would not expect to obtain such a high correlation coefficient from actual measurements of students' shoes; however, a value of approximately 0.8 has been found by the groups of student teachers who tried this activity.

The length of Cinderella's shoe can be predicted by using the TRACE button (Figure 5), or by inspecting a table of values produced via the TBLSET and TABLE commands (Figure 6), or, most accurately, via the CALC menu and VALUE sub-menu (Figure 7).

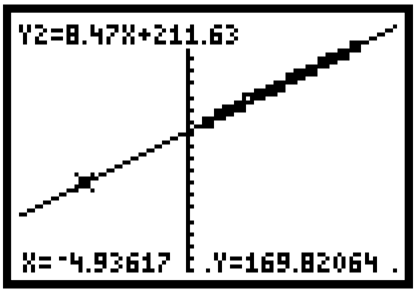


Figure 5

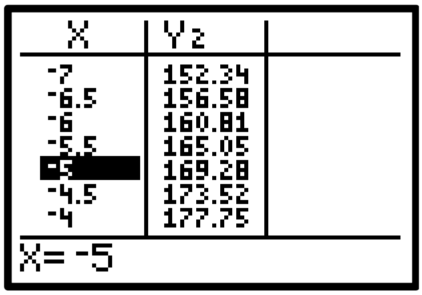


Figure 6

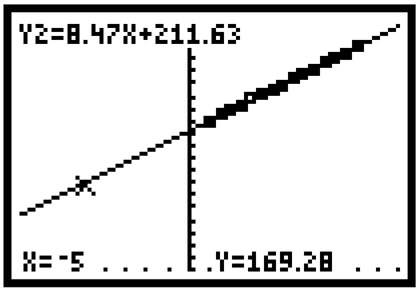


Figure 7

The shoe manufacturers' data supplied in Table 1 predicts the length of Cinderella's glass slipper to be 16.9 cm. However, since students' data will be less accurate than this, there is some room for error in extrapolating beyond the data points plotted. In addition, the question asks for Cinderella's foot length, which will be somewhat smaller. Students may now wish to carefully measure their own foot length and compare this with their shoe length to see whether there is a standard difference in these measurements. (A difference of about 1 cm is recommended by shoe retailers and manufacturers.)

Extension

What would be Cinderella's shoe size if European shoe measurements were used? (Hint: Try an Internet search for "shoe sizes", or shoe + size + measurement). An international shoe size conversion chart can be found at http://www.designersdirect.com/dd/shoes/alden/sizing.html.

References

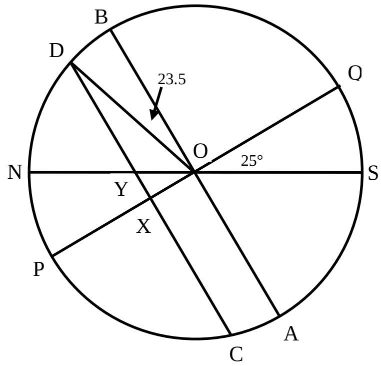
Australian Education Council (1991). *A National Statement on Mathematics for Australian Schools.* Carlton, Vic: Curriculum Corporation.

Berry, J. & Green, N. (1983). What shoe size? In J. Berry, D. Burghes and I. Huntley (Eds.), *Solving Real Problems with CSE Mathematics* (pp. 8-14). Cranfield: Cranfield Press.

Wright, V. (1996). *Traditional Tales with a Mathematical Twist*. Waikato: Mathematics Advisory Service, University of Waikato.

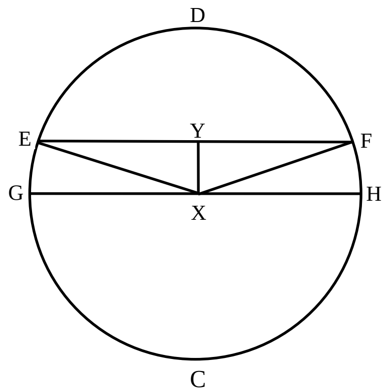
Page 15 Problem

Solution to ‘Sun up’



NS is a north-south line through Brisbane. It is horizontal with respect to Brisbane. Let the observer stand at O. The circle is the celestial sphere (the apparent dome of the sky) around O. The line parallel to the earth’s axis through O is PQ. At the equinoxes, the trace of the sun on the celestial sphere is the plane through O perpendicular to PQ. The projection of this plane on the diagram is the line AB. At the solstice, this trace is shifted to CD ($\angle BOD = 23.5°$). The intersection of PQ and CD is X; the intersection of NS and CD is Y.

The circle in the second diagram represents the trace CD viewed looking along QP. The plane of the horizon intersects the plane of this diagram in the straight line EF. GH passes through X and is parallel to EF.



The sun moves at constant angular velocity around the circle in the second diagram. The fraction of the day it spends above the horizon is the length of the arc EDF divided by the circumference of the whole circle.

Let the radius of the celestial sphere be 1.
From the first diagram, $OX = \sin 23.5°$; $XY = \sin 23.5° \tan 27.5°$; $XD = \cos 23.5°$
On the second diagram, $XF = XD = \cos 23.5°$; $XY = \sin 23.5° \tan 27.5°$
$\angle HXF = \angle XFY = \sin^{-1} (\sin 23.5° \tan 27.5° \div \cos 23.5°) = \sin^{-1} (\tan 23.5° \tan 27.5°)$.

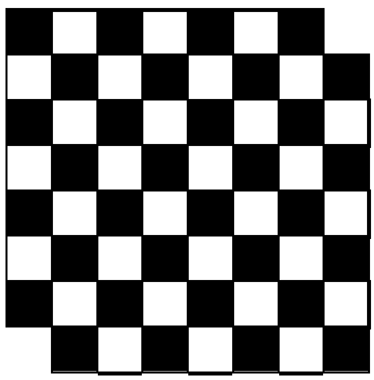
The time in hours between sunrise and sunset is $24 \times (180° - 2\angle HXF) \div 360°$. This is $12 - {}^{2}/_{15} \sin^{-1} (\tan 23.5° \tan 27.5°)$. This comes to 10.26 hours or 10 hours 16 mins.

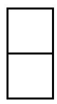
By assuming that the angle between the equatorial plane and the sun varies sinusoidally with time through the year, this result can be generalised to $12 + {}^{2}/_{15} \sin^{-1} (\tan \lambda \tan (23.5° \times \sin (0.986d°)))$, where λ is the latitude and d is the number of days since the March equinox.

Squares

A chess board has 64 squares. A domino has two squares.

It is quite easy to see how to cover a chess board with 32 dominos. But can you cover a chess board with two opposite squares removed with 31 dominos?





No specific mathematical knowledge required.

R(-squar)ED – DANGER

Helen MacGillivray
School of Mathematical Sciences
Queensland University of Technology

The educational problem

Regression is not in Maths A, Maths B or Maths C, but graphics calculators appear to make introductory regression easily accessible to high school students, particularly in Maths B or C. Unfortunately the appearance is illusory, and introducing the output of graphics calculators requires considerable care to prevent long term damage to the students' capacity to learn about regression. For example, students' belief that an R^2 of at least some given value (70% is a number often quoted to me by students; its source is both unknown and puzzling) means the model is "good" while an R^2 of less than the value is "bad", is highly inhibitive to their ability to learn about and use regression as a practical tool in data analysis. This article illustrates the dangers of R^2 without its statistical context. Although a variation of R^2 can be used judiciously in more advanced regression analysis, its interpretation for beginning students as "the % of variation explained by the model" is dangerous without its correct statistical context. In senior maths the line-fitting and curve-fitting capabilities of graphics calculators could possibly contribute to learning about functions and physical modelling, with emphasis that the statistics of line-fitting and curve-fitting is beyond the syllabus.

This article does not give examples or learning experiences for students, but is written to help provide background information for teachers in dealing with the output of graphics calculators and/or shareware software.

Example 1

Below is part of a typical computer output for regression involving two variables. The *y*-variable is the processing time in minutes for running an experimental program on a particular computer; the *x*-variable is the amount of input in kilobytes. The data are:

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y	8.7	7.6	6.5	4.4	11.1	8.4	6.6	10.9	7.1	6

x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
y	10.5	6.2	11.6	9.1	9.7	9.8	7.5	9.4	12.9	10

The regression equation is: time = 5.33 + 1.65 input

Predictor	Coef	StDev	T	P
Constant	5.328	1.663	3.20	0.005
input	1.6451	0.7811	2.11	0.049

S = 2.014 R-Sq = 19.8%

So R-sq is "only" 19.8%. However we shall see below that not only is the correlation here statistically significant, but in fact the straight line model for these data is THE correct model.

Example 2

Below is another example of a simple linear regression of y on x. The data are:

x	1	1.5	2	3	5
y	2.23	3.53	4.93	7.23	13.23
x	10	20	25	30	42.2
y	27.52	57.53	74.27	91.51	129.02

Part of the regression output is:

The regression equation is: $C2 = -1.91 + 3.08\ C1$

Predictor	Coef	StDev	T	P
Constant	-1.9126	0.5401	-3.54	0.008
C1	3.07893	0.02753	111.82	0.000

S = 1.199 R-Sq = 99.9%

With an R-sq of 99.9%, is this a "good" model? No, as we shall see below, it's a very poor model, whereas the straight line fitted in example 1, with an R^2 of only 19.8% is a "good" model.

Shouldn't we be looking at the contexts before making statements about how "good" the models are? Certainly we should be looking at contexts, but in these cases, as will be seen below, knowledge of the contexts will reinforce, emphatically, *what the statistical analysis can tell us*.

R^2 in simple linear regression

The model

For a variable denoted by some symbol, say ✉, a simple linear regression model is that ✉ varies around its mean (or expected) value, and that this mean (or expected) value of ✉ depends on another variable, denoted by some other symbol, say ☎, and in fact is a straight line in ☎. That is, the expected value of ✉ when we know ☎, = a constant + another constant multiplied by ☎. The model also usually includes the assumption that the variation, in contrast to the mean, of ✉ does not depend on ☎, and a further refinement is the assumption that the variation of ✉ around its mean is normal (sometimes called Gaussian). If we are dealing with variables in the physical sciences, the expected value can sometimes be the "true value", and the variation due entirely to experimental or measurement error. *But* this is just a special case of the general situation.

After the strain of the above paragraph, I will now revert to the (x, y) notation with which we are all more familiar (but reminding ourselves that over-done familiarity *in notation* in mathematics and statistics can sometimes produce only surface learning).

In terms of the familiar x, y notation, the model is that the expected value, or mean, of Y is a straight line in x. The next assumption is that the variation of Y around its expected value does not depend on x. This, plus data (n pairs of x, y observations), plus the principle of fitting the line by minimising the sum of squares of the distances of the y's from the line, give us the standard least squares estimates of the intercept and the slope of the "line of best fit" for our data. The assumption that the variation of Y around its expected value is normally distributed then gives us the standard statistical techniques for regression: the tests, the confidence intervals, and the prediction intervals. There are also other extra theoretical results that come from the normality assumption, but none of the formulae nor extra theoretical results are relevant to the discussion here.

Some of the most important aspects of regression analysis in statistics relate to diagnostics for the residuals. The residuals are the differences between the observed values of the Y variable and the fitted values on the fitted straight line. Some of these diagnostics also just need the above assumptions for the expected value and variance of Y, while others need the extra assumption of normal variation.

The examples and interpretation

In Example 1, R-sq is "only" 19.8%, but the p-value of 0.049 tells us that the slope of the line, and hence the correlation, is statistically significant at the 5% level. That is, if the slope of the line in the model, and hence the theoretical correlation, are zero, then the chance of getting "our data" or more extreme is only 0.049.

With less than 5% chance of getting our data or more extreme if there is no linear relationship between input and processing time, most people would agree that there is evidence that there is some linear relationship between the two, that is, that the slope of the line is non-zero.

What is the test statistic for testing if the slope is zero? To compare with t, it is 2.11 in Example 1 above. In terms of R^2, it is given by $(n\text{-}2)\ R^2/(1\text{-}\ R^2)$ and it is compared with $t_{n-2}^{\ 2}$ $(= F_{1,\,n-2})$.

In Example 1, n is 20.

The table below gives the cut-off values for R^2 that will give us statistical significance of a linear relationship at the (a) 5% (b) 1% level for different values of the sample size, n.

Cut-off values of R^2 that are significantly different from 0 at the 5% and 1% levels.

Sample size	10	20	30	40	50
5% level	40%	19.7%	13%	9.7%	7.8%
1% level	58%	31.5%	21.4%	16.2%	13%

Why do people like considering R-sq?

One of the forms for R^2 is that $R^2 = 1 - \text{RSS/SST}$, where RSS is the sum of the squares of the residuals (reminder – the residuals are the differences between the y-observations and the fitted line) and SST is the sum of squares of the differences between the *y*-observations and their overall average. In this form R^2 is often interpreted colloquially as the "percentage of variation explained by the least squares regression line".

Hence the above table shows that, for 50 observations, the simple regression line needs to explain only as much as 13% "of the variation" to conclude that there is substantial evidence of a linear relationship between *x* and *y*.

So is it a "good model"?

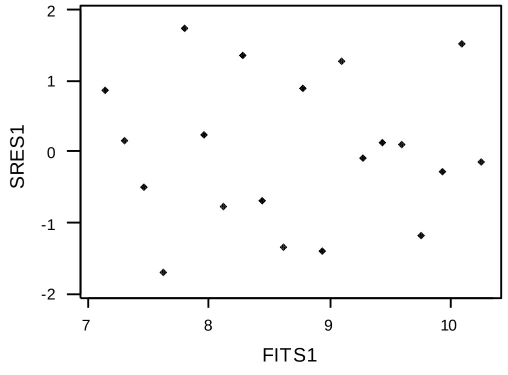
We have seen above that apparently "small" values of R^2 can provide quite strong evidence of linearity in the relationship between *x* and *y*. But how "good" is the relationship? Are there some values of R^2 that tell us whether the relationship is "good or not good"?

Unfortunately no – **by itself** R^2 does NOT tell us how "good" a relationship is.

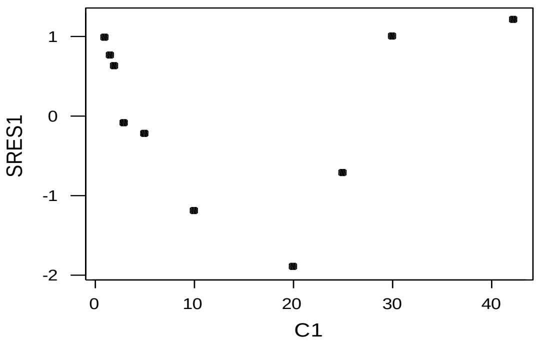
Statistically, how do we tell how "good" a model is? In simple linear regression, the testing of the significance of R^2 (or the slope) tells us if there is significant **linearity** in the relationship. Then diagnostics of the residuals can give valuable information. The residuals (standardised) should be randomly scattered around zero when plotted against the *x*-variable or the fitted values (these plots are different in multiple regression). The (standardised) residuals should have arisen from a normal distribution, but with only 10 or 20 observations this can sometimes require caution to check. Further diagnostics check other aspects of models.

Below are the plots of the (standardised) residuals in the above examples.

Example 1



Example 2



The plots above tell us that the model in example 1 looks "good" but the model in example 2 is not "good".

The contexts of the two examples.

What are these two examples? Is example 2 an "artificial" example? In fact, example 2 is very real data – *x* in example 2 is distance in kilometres in men's running events, and *y* is the corresponding record time in minutes as it stood in 1975. This explains why the R^2 in example 2 is so high and why a straight line model is absurdly wrong – just as the residual plot above tells us. Because the distances range from 1 to 42.2kms, the times are strongly dependent on the distances in a highly linear manner, that is, the longer the distance the greater the time. And because the times are record times, there is very little overall variation in the times once distance is allowed for. BUT just a straight line is obviously an absurd model because it says that speed is constant over the range of distances. We could add a quadratic term to the model to try to account for some of the curvature, but in this context, human movement considerations suggest that a more suitable model may be that the expected time is proportional to a power of distance. Taking logs of both distance and time then allows least squares to be used on the logs of the variables to estimate this power.

Although example 1 is based on a real example, the data above have been simulated so that the model that produced these data is known exactly. The data of example 1 above were produced by a random sample of 20 observations that were generated from a N(0, 4) distribution and added to a straight line in the chosen *x* values. That is, the R^2 of 19.8% is not only statistically significant, but is also definitely not indicative of a "poor" model. On the contrary, the straight line model is THE model, but *y* has a lot of variation around its expected value. In the jargon of signals, there is a lot of noise.

In real life, we won't know if the data of example 1 are just inherently noisy or if there are effects of other variables that we haven't observed or used. For example, perhaps the number of users on the system affects the processing time. Adding another term to the current model will *always* increase R^2. It's easy to get R^2 to 100% - we just fit as many parameters as there are observations – but it's a useless model. Statistical analysis will tell us whether the increase is big enough to recommend "keep the extra term in the model".

Other dangers – extrapolation

Although it is well-known that extrapolation "too far" beyond the range of the available data is not appropriate, there is often surprise at the results of the *statistical* analysis which clearly shows that "little information in, gives little information out". Below are confidence and prediction intervals in example 1 for an input (x-value) of (a) 5 (b) 2.

Predicted Values

(a)

Fit	StDev Fit	95.0% CI	95.0% PI	
13.553	2.348	(8.621, 18.485)	(7.054, 20.052)	XX

X denotes a row with X values away from the centre
XX denotes a row with very extreme X values

(b)

Fit	StDev Fit	95.0% CI	95.0% PI
8.618	0.452	(7.668, 9.568)	(4.281, 12.955)

An x value of 5 is so "far away" from the data that the range of expected y values in the confidence interval is greater than the original range of y-values!

Is there an educational solution?

Although the above examples consider only simple linear regression, they illustrate how *statistical* data analysis needs the statistical tools of interval estimation, tests, diagnostics and distributions. Does this mean we should tell school students to ignore R^2 on their graphics calculators? There is nothing educationally wrong with telling students to ignore parts of output from either graphics calculators or computer packages, until later in their learning. R^2 does not necessarily have to be ignored, but it is clear that considerable caution is needed, and, without the statistical tools, the emphasis on fitting with a graphics calculator should be on the functional and graphical aspects. And extrapolation should not be done without the statistics of distributions and interval estimation.

If we look at texts for senior physics, we see mention, not of statistics, but of least squares fitting of relationships in situations where any variation is due to experimental and/or measurement error. Note how "good" in this type of physical-relationship-with-measurement-error can refer to how "good" the experiments or measurements are – that is, keeping the measurement/experimental error under control. Perhaps that would be the best attitude in senior maths also – emphasis that it is not regression or statistics, and examples in which there is some underlying theoretical relationship between the variables and the variation is due to measurement or experimental error.

QUOTES

On two occasions I have been asked [by members of Parliament], 'Pray, Mr. Babbage, if you put into the machine wrong figures, will the right answers come out?' I am not able rightly to apprehend the kind of confusion of ideas that could provoke such a question.

Charles Babbage (1792-1871)

In mathematics you don't understand things. You just get used to them.

Johann von Neumann, (1903 - 1957)

REVIEWS

Hodder Maths 9, 668 pages, 1998

Hodder Maths 10, 480 pages, 1998

Katerina Katselas, Carol Osborne, Anthony George Priddle

Hodder Education, Rydalmere NSW

RRP $33 each

Any one who has used textbooks written by A.J. (Tony) Priddle will be familiar with the exercises in these textbooks. Add to this some interesting features and you have the Hodder Yr 9 and 10 Maths books.

The list of contents soon makes one aware that both texts address the National Statement on Mathematics for Australian Schools and more than adequately cover the content required in Years 9 and 10 by the Queensland Years 1-10 syllabus.

There is a summary of the Victorian Curriculum Standards Framework outcomes at the beginning of the books that shows which outcomes are related to which strand. All chapters have a list of CSF outcomes for levels 6 and 7, although it is made clear that these levels are not exhaustive.

Some of the interesting features of these texts are the:

- snippets of mathematical interest;
- speech bubbles that provide hints and suggestions, as well as challenging students' current perceptions of concepts;
- investigations, problem solving activities (including some open-ended tasks) and puzzles which involve individual and group work;
- cartoons and captions that apply to real life and are used to enhance concept formation;
- key sequences for scientific and graphic calculators (Casio FX-6300G and TI-82/83);
- computer tasks that supplement work and which are available on disk.

An introductory section provides a simple, clear explanation of what an investigation is and methods of investigating. It includes helpful hints and problem solving strategies. There is a simple blueprint for proceeding with an investigation in step by step form. A sample report and some possible investigation topics are included.

Throughout the texts, the cartoons and captions, investigations and problem solving activities provide an alternative for students to engage in a different mode of mathematics from textbook exercises.

The Year 10 text includes two Option topics –

1. Surveying
2. Modelling involving Linear Programming, Optimisation and Queues.

These texts contain an abundance of activities and exercises, and would be a worthwhile addition to any teacher's bookshelf.

I believe the texts are more suitable for advanced level classes rather than the general masses.

Dympna Rigby
Brisbane Catholic Education Office

SMQ10
Secondary Maths for Queensland

Ross Brodie and Stephen Swift

Moreton Bay Publishing

Second Edition 1998

RRP: $36.00 (629 pages)

This is the third of the SMQ textbooks covering the Queensland Junior Mathematics syllabus and has been written with emphasis on problem solving, open-ended activities and the use of technology. Topics include statistics, geometry, indices, functions, measurement, variation, probability, algebra, trigonometry and business mathematics, incorporating sections on modelling and problem solving, group work, as well as the use of scientific calculators, graphics calculators and computers in a variety of classroom activities. While SMQ10 can be used to follow SMQ8 and SMQ9, it can easily follow any of the Year 9 textbooks used in the majority of Queensland schools and will definitely enable students to undertake any of the Senior Mathematics courses.

Each chapter is attractively presented with interesting photographs, illustrations and comments about mathematicians and the practical application of mathematics, clear and colourful rules and formulae, examples, step-by-step instructions on scientific and graphics calculator and computer usage, exercises, problems, as well as diverse, frequently hands-on , individual, group or whole-class activities. A chapter summary, chapter review exercises and a memory map conclude each section. The number skills appendix may be used to revise basic skills, as appropriate to student needs.

Brodie and Swift have organised exercises in three sections, Skill Practice, Applications, Modelling and Problem Solving, with an easy-to-follow core and extension system catering for varying levels of development. There are more than sufficient exercises, problems and activities to enable teachers to guide students according to their individual or group needs. The introductory section (to be visited and revisited throughout the year) provides an excellent guide to the different methods that can be employed in solving theoretical and real-life problems encountered in situations such as shopping, building, sport and travel.

SMQ10 contains an introductory section listing mathematical symbols, abbreviations, the metric system and the Greek alphabet, a clearly presented answers section and a comprehensive index. Technical language is clearly defined and the reading level appears o be well below that of an average Year 10 student.

As SMQ10 embodies, in a spiral curriculum approach, both the content and the spirit of the Junior Mathematics syllabus and provides a solid depth of coverage of all topics, it can be recommended as an excellent teacher resource and student textbook.

Lyn Dempster
St Mary's College, Cairns

UNILEARN Senior Mathematics

Queensland Open Learning Network

This is a mathematics program designed for open learning situations. It would be suitable for senior students, taking them from a Year 10 Extension level to a level comparable in standard to Queensland Senior Mathematics B. It follows on from the Unilearn course *Introductory Mathematics* which was reviewed in the last issue of *Teaching Mathematics*.

The study materials consist of four books (modules) totalling about 800 pages. The topics covered are: functions and their graphs; coordinate geometry of the strait line and conics; trigonometry with special attention to periodic functions; an introduction to differential and integral calculus; financial mathematics; and applied statistical analysis relating to binomial, uniform and normal probability distributions.

As such it aligns with most of the Queensland Mathematics B syllabus and a few bits of Mathematics C. The networks and linear programming topics and parts of the applied geometry topic of Mathematics B are not represented.

The study materials are designed to take students through the course without the input of a teacher or other written materials. They contain detailed explanations of ideas along with plenty of examples and student exercises. The answers to the exercises in the backs of the books are backed up with worked solutions, so that, if students are stuck, they can see how the problem can be tackled.

The sequencing of ideas is logical; the language used would be accessible to the majority of students; and the pages have an uncrowded, ‘friendly’ appearance.

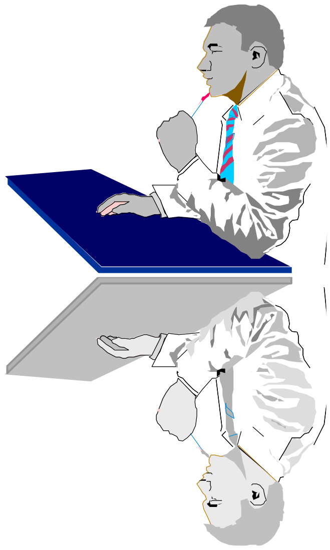
The cost of the study materials is $25 per book. Students may enrol in the course for $495. For this they get all the study materials, an HP38G graphics calculator, tutorial support by phone, fax or e-mail as they need it, marking of and feedback on Progress Tests and access to further materials at Open Learning Centres. A formal supervised examination is available on completion of the course. A Statement of Attainment is provided upon successful completion of the examination; a Statement of Completion is provided to students who complete the Progress Tests, but who elect not to sit the examination.

David Ilsley
Queensland School Curriculum Council

REFLECTION ON LEARNING

STRATEGIES FOR USE IN SENIOR MATHEMATICS

PART 2



Lyn Nothdurft
St Patrick's College, Gympie

This is the second part of a two part paper based on a presentation at the Queensland Association of Mathematics Teachers One Day Conference on 1 May 1999 in Brisbane.
Part 1 was published in the last issue of *Teaching Mathematics* Volume 24 No. 3, pages 16 – 19

In Part 1 of this article the idea of student reflection was examined and some of its advantages were discussed. Then some specific strategies for encouraging student reflection were described.

This second part of the article describes further strategies for encouraging student reflection.

Further strategies to encourage student reflection

Development of strategies for solving problems and reflecting on their use

The aim is to avoid having students carrying out procedures or algorithms without understanding why they are doing so: what Skemp (1976) called "rules without reasons". They are told that, at all stages, they should have a reason for using particular procedures. Fusco and Fountain (1992) have suggested students ask themselves the following questions as they seek to solve a problem:

- What am I doing? *(create a focus)*
- Why am I doing it? *(establish a purpose)*
- How does it fit in with what I already know? *(recognise context or interrelationships)*
- What questions do I have? *(discover what is still unknown)*
- Do I have a specific plan to understand this? *(design a possible method of approaching the topic)*
- How effective have I been in this process? *(evaluate progress)*
- Do I need to know more? *(monitor need for further action)*

Students can ask themselves these questions as they are working through questions, and can be encouraged to write notes to themselves in their exercise books about what they are doing and why. They are also encouraged to develop their own procedures for different types of questions. They might do this through the consideration of a worked example and the steps involved in it, through their discussion with the teacher or other students, or by working through a problem themselves and then reflecting on what they have done.

The class can also develop a list of problem solving behaviours: the questions they think that they should be asking themselves as they try to solve problems. The following list is derived from Polya's (1957) framework, and focuses on solving optimisation problems using calculus.

OPTIMISATION PROBLEMS

PROBLEM SOLVING BEHAVIOURS	**Questions you can ask yourself**
Set up the model	
UNDERSTANDING THE PROBLEM	*What do I know?*
Problem is correctly interpreted	*What do I want to know, to find?*
Goals and givens established	*What variables will I use?*
Goals and givens represented symbolically	*Can I draw a diagram?*
All relevant information used	*Can I write equations using these variables?*
PLANNING	
Identification of goals and sub-goals	*Can I think of a related problem?*
Use of strategies to explore and understand problem	*Can I restate the problem?*
Selection of strategies to carry out plans	*Can I solve part of the problem?*
Solve the mathematical problem	
MONITORING	*Am I still solving the problem I set out to solve?*
Monitoring of progress through problem	*What am I doing?*
Recognition of potentially fruitful solution pathways	*Why am I doing it?*
Recognition of solution pathways that will potentially lead to dead ends	*How does it help me?*
IMPLEMENTATION	*What am I trying to do? Will this do it?*
Comprehending problem statements	*Are the equations etc. in a form that I can use?*
Organising information and data	*Is the equation in terms of one variable?*
Executing plans	*What should I differentiate? Do I find SPs?*
Planning solution attempts	*What type of SP?*
Checking results	*Did I get the required minimum or maximum?*
Interpret the solution	*What does my answer mean?*
Check	*Are there any mechanical errors?*
VERIFICATION	
Evaluation of approach	*Does the result answer the original question?*
Evaluation of execution	*Does the answer seem reasonable?*

My students used this table when solving problems and continued to refer to it over the next few weeks. Some had never thought of explicitly seeking to understand the problem, plan a solution strategy, monitor their success as they implemented the strategy, and finally verify their solution. Students commented on how they were using this approach in other subjects such as Chemistry and Physics. They also found these questions helped them manage and reflect on solving unfamiliar problems for assessment. Whereas in the past, some students had not had any ideas about how to start and had submitted blank sheets of paper, this gave them a framework for approaching the question.

Students can also consciously reflect on their processes as they work through the questions, by doing the mathematics on the left half of the page and recording reflections and questions about the processes on the right half of the page. This requires them to clarify their thoughts about what they know and don't know, and helps them organise their knowledge. Emphasis needs to be placed on the processes being used rather than just on the correct solution. When a problem is being solved on the board, students are encouraged to focus on how it is solved and why particular strategies are used. They are asked not to copy the solution, but instead to concentrate on the problem solving processes. A student commented:

Another helpful strategy was the "Don't write down, watch and understand" routine. It is very easy to fall into the pattern of writing down everything on the board, but tuning out whilst doing it - not taking anything in.

With collaborative approaches, students are required to verbalise their thoughts, clarify their thinking, and consolidate their understanding by explaining ideas to others. Groups may be asked to solve problems, and then one or two of the group chosen to present the solution to the rest of the class. This places the onus on the group to ensure that all understand the solution processes and are able to explain them. Giving each group overhead projector transparencies on which to write their solution saves the time involved in their writing the solution on the board. Another collaborative approach is think-aloud problem solving. One student acts as the problem solver and the second as the listener. The problem solver vocalises what he or she is doing to solve the problem, while the listeners encourage and ask questions of the type: "What are you doing?", "Why are you doing that?", "What could you do now?". The comments of one student endorse the value of providing and listening to peer explanations:

When you have to explain something out loud so they can understand, it means that you are more clear in your own mind, and you work through the problem much more thoroughly and can then pick up any errors that you have made through "rushing" and not evaluating the steps you take and why. The constant asking of what you are doing and why keeps you on track.

Preparation for exams

Students can be encouraged to reflect on their depth of understanding of concepts as part of their preparation for assessment tasks. One way is to list the objectives to be examined, and ask the students to reflect on this to determine their depth of understanding. The students match each objective with one of "Know", "Understand, but need more practice", or "Don't understand". They then decide what they need to do to review the work. This helps them focus on their particular learning needs. They may also be asked to review their goals from earlier in the year, and then plan their preparation for the assessment task. This will make them more conscious of what they want to achieve.

Review of assessment outcomes

By reflecting on assessment outcomes, students are better able to accept responsibility for their own learning and learning outcomes, and to relate their results to their use of learning strategies. After a test, they write in their journals about their preparation and assess their learning behaviours. This is an opportunity for those who have been implementing effective strategies to realise how they have helped, and to acknowledge their success to themselves. It also enables those who were disappointed with their results to understand what they have not done and to commit themselves to action to overcome the problem. The following questions might be used:

- Were you happy or disappointed with your results?
- What did you do to prepare for this test: in the last week, in the last fortnight, during the whole term?
- What do you consider to have been effective strategies in your preparation?
- What could/should you have done more of?
- What did you do during the actual test, eg. monitor time, choose easier questions first, read the whole paper first, indicate incomplete questions to return to etc?
- Did you run short of time? What could you do to overcome this problem?
- What strategies will you use to prepare for the next exam: during term, in the one or two weeks preceding the exam?
- What strategies will you use in doing the exam?
- What are your goals for next term concerning study?

Review of goals and progress made

At the end of a term, students can evaluate their learning as a class. The focus is on what the students have found helped them with learning, what did not help, and what different strategies they consider worth trying. This allows an open consideration of students' concerns and current difficulties. These discussions are followed up with individual journal writing to allow more reflection time, and the opportunity to make comments that students might prefer not to make in public. The following is an example of prompts given to the students to use to focus the discussion:

Consider the teaching strategies being used:

- what have you found contributed to better learning
- what did you find did not help learning

Consider your learning:

- what have you done differently from how you have previously learnt maths
- what have you done that contributed to better learning
- what have you done that did not contribute to better learning

Consider the approach to the next term:

- what teaching and learning strategies should continue
- what ones should not continue
- what different approaches could be introduced to enhance learning

Class feed-back

Feed-back comments permit the teacher to focus on the effects of particular teaching strategies. The class is asked to reflect on the teaching approach, generally using their journals. The teacher is exposed to potential criticism and needs to be prepared to accept the students' comments and act upon them or to be able to explain why changes will not occur. Questions could be of the type:

- What is happening in maths this year that is helping you learn better?
- What do you like about how maths is taught?
- What do you dislike about how maths is taught?
- What would you like to be different in how maths is taught?

Brookfield (1995) has discussed a similar approach which he calls a Critical Incident Questionnaire. This is given to the students at the end of the last class for the week. The students anonymously respond to questions about how they are experiencing their learning and the teacher's teaching. It provides a way to ascertain the effects that the teacher's actions are having on students.

General journal writing

The students can also be encouraged to write in their journals whenever they wish to reflect on some aspect of their learning and to use the journal as an avenue of communication. It gives each student the opportunity to communicate individually with teacher. Students who will not ask questions in class may prefer to express their concerns in writing and have this answered by the teacher in their journals. While the teacher cannot spend five minutes focused on each individual student each lesson, an equivalent amount of time can be spent reading and responding to the student through the journal. Students appreciate the time spent by the teacher in answering their questions, commenting on their observations, or raising questions and possible approaches for the student to try. Perusal of past entries in their journals should enable students to find indicators of their own self-growth and maturity in learning.

Use of questionnaires about strategies

Another approach to helping the students reflect on their learning is to have them answer questionnaires about learning. Responding to the statements can make them conscious of learning strategies and whether or not they use them. The Motivated Strategies for Learning Questionnaire, developed by Pintrich, Smith, Garcia, and McKeachie (1991), with some changes to make it more suitable for the Australian context has been used. It asks students to respond to statements concerning intrinsic goal orientation, extrinsic goal orientation, task value, control of learning beliefs, self-efficacy for learning and performance, cognitive learning strategies, metacognitive self-regulation, time and study environment management, effort regulation, peer learning, and help seeking. Students have commented that this questionnaire focused their attention on how they went about learning. It has also stimulated discussion about learning strategies.

Difficulties encountered

While I feel strongly about the advantages of students' reflection, this is not always matched by the students' attitudes. Sometimes only the most committed, articulate and thoughtful students will bother with journal writing (Brookfield, 1995, p. 97). Teachers can easily overestimate students' willingness and ability to reflect on how and what they are learning. As students are generally not used to writing in mathematics, there is often an initial reluctance to write about their learning. Some may lack the language with which to articulate their thoughts. Students may also be reluctant to be too honest: it could be risky for them to criticise their teacher. Some will passively resist writing by always forgetting to bring their journals. Students who do not believe that reflecting in their journals is of assistance to their learning have stated that they are wasting "valuable" lesson time. We also expose ourselves to criticism with these reflective activities, and must be able to consider the effects of various teaching strategies from the students' viewpoint.

Teacher reflection

As we encourage students to reflect on their learning, we also need to reflect on our practice and respond to the effects of our actions. Reflective teachers take

> time to reflect on what they are learning, confront assumptions and ways of doing things, and solve problems about new information. Reflective practitioners value taking time to think about their work, about what they are doing and why, and about what they might do differently. The process of reflection requires learning what questions to ask, how to ask them, and what to do with the information that results from asking questions. (Tabor, 1997, p. 67)

Through reflection, we can see our practice through our students' eyes, focus on the real problems of the classroom, investigate multiple perspectives and courses of action, help overcome the problems, and consider possible consequences of the action. A different sense of the learning enterprise can develop through reflection. Learning becomes more collaborative with a community of teacher and students working together to enhance the students' learning instead of the teacher providing information and skills for the students to receive. Both teachers and students can be empowered through reflective practice, as:

> a definite link exists between the thoughtful performance of teachers and the thoughtful performance of their students. As a result, there is a greater pay-off for both teachers and students when time is given to the thought processes related to teaching and learning. (Tabor, 1997, p. 63)

References

Brookfield, S. D. (1995). *Becoming a critically reflective teacher.* San Francisco, CA: Jossey-Bass.

Fusco, E., & Fountain, G. (1992). Reflective teacher, reflective learner. In A. L. Costa, J. Bellanca, & R. Fogarty (Eds.), *If minds matter* (Vol. 1) (pp. 239-255). Palatine, IL: Skylight Publishing.

Pintrich, R. R., Smith, D. A. F., Garcia, T., & McKeachie, W. J. (1991). *A manual for the use of the motivated strategies for learning questionnaire (MSLQ)*. Ann Arbor, MI: National Center for Research to Improve Postsecondary Teaching and Learning.

Polya, G. (1957). *How to solve it* (2nd ed.). Princeton, NJ: Princeton University Press.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*, 20-26.

Tabor, M. (1997). A process-oriented paradigm: Implications for professional development. In A. L. Costa & R. M. Liebmann (Eds.), *Supporting the spirit of learning: When process is content* (pp. 55-72). Thousand Oaks, CA: Corwin Press.

Tanner, H., & Jones, S. (1998, July). Dynamic scaffolding and reflective discourse: Successful teaching styles observed within a project to teach mathematical thinking skills. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times* (pp. 596-604). Proceedings of the twenty-first annual conference of the Mathematics Education Research Group of Australasia, Gold Coast.

Journal Writer's Lament

The verse below makes sense if you read it the right way.

Eye border whirred prose cess sir
Sew why curd right this tough.
Its poster cheque mice belling,
Butt-eyed own thin kits were king prop lee.

Or maybe one should say it makes cents if you reed it the write weigh!

Linking 2 & 3 dimensions – a challenge for students and teachers

John Butler,
Queensland Board of Senior Secondary School Studies

Too often our teachers of mathematics find even our most talented students struggle to solve problems that in essence, only involve relatively simple, straightforward mathematical concepts and techniques. How would your students fair at calculating the volume of a regular tetrahedron, given the length of its edges? Why is it that most students are not able to apply simple trigonometry and the theorem of Pythagoras to situations like this? Success in solving problems like this depends so much on getting started, being able to illustrate the situation diagrammatically and consequently, making the underlying concepts visible.

We live in an age when our curriculum documents place high value on mathematical applications. In teaching our students mathematics we strive to contextualise the concepts where possible in real world situations. Our daily interaction with man-made constructions is testimony to a growing need for each of us to engage with three dimensional space and applied geometry.

Curriculum documents

Queensland's Board of Senior Secondary School Studies (1992) syllabuses in Mathematics A and Mathematics B articulate a need to develop the capacity of students to engage with three-dimensional space. In the Mathematics A syllabus, the focus statement for *Elements of Applied Geometry* (p.13) states that "Students are encouraged to develop a working knowledge of some geometrical concepts and relationships in two and three dimensions …" In that same document, the focus statement for *Linking Two and Three Dimensions* (p.17) states that students are encouraged to develop a working knowledge of the practical considerations inherent in a variety of construction areas. Particular emphasis should be given to the representation of three-dimensional constructs in two dimensions."

The focus statement for *Applied Geometry* in the Mathematics B syllabus (p.12) reads "Students are encouraged to develop skills in applying geometry in the real world by using and extending their knowledge of spatial relationships in two and three dimensions."

The need to pay particular attention to the development of spatial visualisation is emphasised in "A National Statement on Mathematics for Australian Schools" (Australian Education Council and Curriculum Corporation, 1991):

> "Having a developed ability to manipulate images in the mind (spatial visualisation) is useful. …. Visualisation is of obvious importance in geometry and many find it a powerful tool in mathematics generally. Whether through play and 'tinkering' at home or through the study of subjects such as technical drawing, some students will have been in a better position than others to develop their spatial visualisation skills. For equity reasons, the development of spatial visualisation skills should not be left to chance." (p.80).

What happens in classrooms?

Is there a mismatch between what is espoused by the curriculum documents and what actually happens in the classroom? Do teachers of mathematics generally have the necessary tools and skills to provide appropriate learning experiences to foster the development of visualisation? At the moment I suspect the honest answers to these questions are "yes" and "no" respectively.

Let's examine a simple task. Suppose you wanted to construct a right square based pyramid with a specific length for the side and a specific axial length. If you issued each of your Year 11 mathematics students with a sheet of card, a drawing compass, a ruler, a pair of scissors, some glue and a calculator, what proportion of them would be able to complete the task with reasonable accuracy?

The problem is essentially a mathematical one requiring no more knowledge than the theorem of Pythagoras and a basic understanding of nets. Some of your students would have studied Technical Drawing (Graphics nowadays in Queensland) and would be able to complete the task routinely providing they had access to sufficient drawing equipment. Very few I suspect, would be able to provide a mathematical solution - Why?

I believe the answer to this question lies in the learning experiences offered to students. Assuming that the majority of our students realise that they need to determine the length of the edges joining the apex to the vertices of the base, I believe it is likely that generally these same students are not able to develop a suitable strategy to calculate that length.

Regardless of whether students study Mathematics A or Mathematics B, the most likely learning experiences provided to them relating to linking two and three dimensions within Applied Geometry would involve:

- drawing and interpreting scale plans, and
- calculating the surface area of, and costs of finishing for walls, floors and ceilings of buildings from scale drawings (plans).

My observations of students' responses to assessment tasks and teachers' model responses to the same tasks, lead me to deduce that it is unlikely that much effort is channelled into developing links between three dimensional figures or constructions and their representation in two dimensional views or sketches. It seems to me that students' understanding and facility with three dimensional geometry would be significantly enhanced if some time were devoted to developing some understanding of orthogonal projections.

The tetrahedron problem

Consider the problem of calculating the volume of a regular tetrahedron given the lengths of the edges of each face. My hypothesis is that students generally are not able to provide mathematical solutions to tasks like this, not because they lack facility with the theorem of Pythagoras and basic trigonometry, but because they haven't the tools to illustrate and model the problem. Any three dimensional figure can be illustrated by linked orthogonal projections with each providing information about space in two dimensions. Perhaps insufficient time is devoted to learning experiences like:

- research how radiologists use two-dimensional X-ray photographs to assess three dimensional parts of the body such as hip joints (Mathematics A syllabus, p.19), and
- investigate the shape of three dimensional structure by considering a number of two dimensional plans (sic) taken from different perspectives (Mathematics B syllabus, p.14).

The tetrahedron can be illustrated easily as a plan (top view) and elevation (front or side view) as in Figure 1.

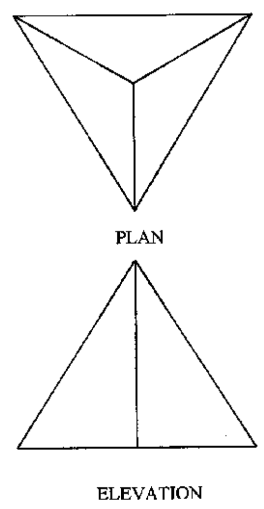


Figure 1

At times however, the orientation of the views can inhibit access to the underlying concepts. The orientation in Figure 2 provides better access by illustrating the right-angled triangle AOB in elevation "square on", meaning that it is parallel to the vertical plane onto which the view is projected.

Students who are able to interrelate three dimensional images with their two dimensional orthogonal projections are in my mind significantly advantaged when it comes to solving problems involving three dimensional geometry.

The advantage is of course in the student's capacity to individually analyse the component parts in 2D whilst visualising them in 3D.

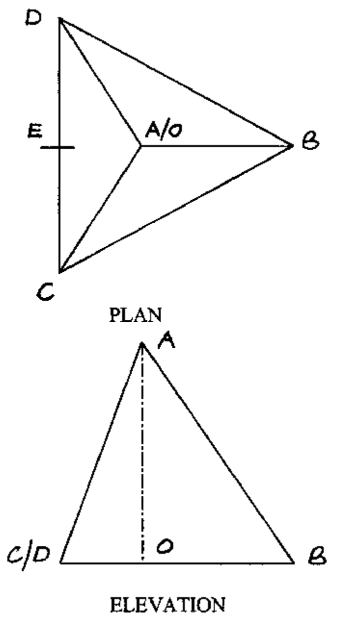


Figure 2

In this particular problem, determining the volume is reasonably straightforward once the perpendicular height BE of the triangular base BCD and the axial length (height) AO have been found. These linked 2D views have the potential to make the pathway to solution much more obvious to students, thus allowing them to concentrate on selecting and applying the techniques required to solve the problem.

From the plan view the application of some basic trigonometry will determine expressions for the lengths BE (needed to find the area of the base) and BO (needed to find AO). After transferring the value for BO into the elevation, application of the theorem of Pythagoras will reveal the axial length (height) AO (needed for the volume).

Other problems

It is my firm belief that of the problems that we offer our senior students, solutions to most of those relating to geometry in three dimensions would prove to be significantly more accessible to students if we took the time to develop these skills in spatial visualisation. Examples of such problems sometimes offered to students of Mathematics A, Mathematics B and Mathematics C are outlined below. You might like to try these with your classes.

Mathematics A:

A rectangular house 14 m × 8 m has 600 mm overhangs on each of the four sides. If the house is to have a tiled hip roof with a pitch of 20°, determine the length of the timbers needed for the four hip rafters and the ridgeboard.

Mathematics B:

From the deck of a ship sailing due north the skipper notes that the peak of a mountain due west from him is at an angle of inclination of 12°. After maintaining his heading for 10 minutes at a constant speed of 12 knots, he notes that the angle of inclination has decreased to 10°. Given that a knot is one nautical mile per hour (approximately 1.852 km per hour), determine the height of the mountain.

Mathematics C:

The woodchop events at Queensland's Royal National Show (the Ekka) always seem to draw a substantial crowd. The master axemen cut what appears to be a 90° wedge from each side of the log. If the logs are 300 mm in diameter, what volume of wood is cut from each log in each event?

Implications for schools

It is fairly clear to me that having good spatial visualisation empowers students and builds their confidence in mathematics. It is also fairly clear that our male students have better spatial visualisation skills than their female counterparts. In terms of mathematics education, this highlights a significant social inequity. For me, the source of that inequity is not as important as the need to find ways to reduce that imbalance.

A solution might be to allocate more time in the senior mathematics courses to develop these skills and less time on applications relating to area. My view is that we tend to overdo the carpeting of the lounge room and the tiling of the bathroom. More time could be allocated to sketching two dimensional orthogonal views of 3D figures and less on applying scale to floor plans of cottages.

An alternative strategy might be to address the issue in the junior school and maintain the skills throughout the senior years. Mathematics teachers would do well to seek some advice from the teachers of Graphics. The emphasis however in Mathematics should be on sketching rather than on accuracy and the quality of line work. It would be a tragedy to invest our valuable time in pursuit of refined drafting skills at the expense of opportunities to develop skills in spatial visualisation. There are ample commercial resources available to help build and maintain the skills.

An inexpensive way to get started is to haul out the "centi-cube" blocks from your store of mathematics resources and procure some square grid paper and some isometric grid paper. A great activity that gets the students involved is as follows:

- have students work in small groups (pairs or threes) with the number of groups in multiples of four organised so that they are able to pass the products of their work on to the next group in a circuit of four stations;
- each group should build a shape with the blocks and pass it on to the next group in the circuit;
- each group should then sketch the three orthogonal views of the shape (front, side and top views) on the square grid paper and then pass the sketch on to the next group;
- each group should then sketch the 3D view on the isometric grid paper from the orthogonal views and pass on the sketch;
- finally each group should build the shape from the 3D sketch given to them and compare it with the original.

Concluding Comments

We often express disappointment in our students' apparent inability to apply their mathematics to novel situations. Noting the deficiency is the first step in remedial action. The next step is to identify any causal relationships and develop strategies to counter the situation. If your own research in your school identifies a deficiency in spatial visualisation skills, then you can do something about it.

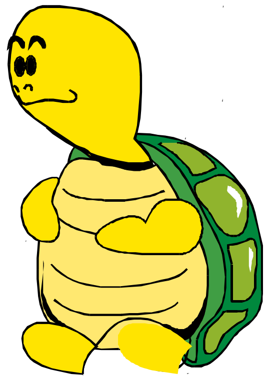
References

Australian Education Council and Curriculum Corporation (1991): *A National Statement on Mathematics for Australian Schools*, Australian Education Council, Victoria

Board of Senior Secondary School Studies (1992): *Senior Syllabus in Mathematics A*, Queensland

Board of Senior Secondary School Studies (1992): *Senior Syllabus in Mathematics B*, Queensland

MSWLogo



Paul M. Dooley
Fairholme College, Toowoomba

A re/introduction to the Logo environment for Junior and Senior High School students. The Activities mentioned in this article and a link to the FREE software are on the internet at **http://www.mathgym.com.au/htdocs/down.htm**

The Logo programming language has been around for close to 25 years. It has a large following of admirers world-wide, especially in the United Kingdom. There are many articles and web-sites which have examples of how Logo is used in the modern Mathematics classroom. The workshop I presented at the Annual Conference used a collection of Activities that I have written for an excellent, new, and FREE Logo called MSWLogo for Windows. In addition to the usual Logo functions, MSWLogo supports, 3D drawings and perspectives, GIF animation, control interfacing, and networking.

At my school we presently use Logo in years 8 and 9 and will be expanding into year 10 next year. In year 8 we introduce the students to the language with Activities 1 through 7. Year 9 continues on to Activity 13. Year 10 will continue with the second volume yet to be written which will require the students to write procedures to investigate mathematical constructs. (I envisage some activities like investigating primes, Pythagorean triples etc.)

The attraction of Logo for me is the intuitive way in which a student is introduced into programming. Also, along the way they develop very rich concepts of two and three dimensional space, as well as number, logical thinking, and working mathematically. I personally believe that once you have used Logo, you will be another convert to this powerful environment.

QAMT MATHEMATICS PROBLEM SOLVING COMPETITION 1999

The 1999 competition was held on Saturday 24th July or a nearby school day at one hundred and thirty different centres throughout Queensland, with many schools being able to hold the competition during school hours, usually on Friday 23rd. Over 3400 students from about 160 schools entered the competition.

We acknowledge with pleasure the generous support of our sponsors, M.I.M. Holdings, the five departments of the School of Engineering at The University of Queensland, and the Department of Mathematics at The University of Queensland. Their donations covered most of the prizes and administrative expenses. Total prize money this year was $1850. This year saw the University of Queensland Mathematics Department increase its support by offering the top prize in Year 10, in addition to its prizes in Years 8 and 9. Queensland Newspapers, after generously sponsoring our competition for over thirty years, has undertaken a major rationalisation of its sponsorship commitments, and is no longer able to support the competition.

The papers each consisted of five questions, all of which needed some ingenuity and thought for their solution, and it is pleasing to see how many students were willing to accept the challenge. Students should feel a sense of achievement in obtaining a good solution to even one question. About the top twenty percent of contestants in each year level were awarded a merit certificate, and a Certificate of Participation was available to all those taking part.

I record my thanks to the people who assisted in the setting and moderating of the question papers and in the marking of the students' scripts. I thank the teachers who gave up their time to supervise at the competition centres, and of course, I thank the many teachers who encouraged their pupils to enter for the competition.

The prize winners and the highly recommended students are listed opposite.

Neil Williams
8 September 1999

QAMT MATHEMATICS PROBLEM SOLVING COMPETITION 1999

PRIZE WINNERS and HIGHLY COMMENDED STUDENTS

YEAR 12

U.Q. Chemical Engineering Dept Prize	$200	Kieran Rowe	Anglican Church Grammar
Q.Q. Civil Engineering Dept Prizes	$80	Bronwen Fairbairn	Somerville House
	$60	Mu Yan	Mt Gravatt State High
	$60	Esther Yun	Somerville House

HIGHLY COMMENDED

David Anh (MacGregor State High, Jason Kwong (Rockhampton Grammar), Ryan Kelly (Townsville Grammar), Burton We (The Gap State High), Matthew Morrison (Malanda State High), Nelson Tam (Brisbane Boys' College), Kaori Misawa (St Rita's College), Samantha Watt (Warwick State High)

YEAR 11

U.Q. Comp. Sci. & Electrical Eng. Dept Prize	$200	Terry Yu	Indooroopilly State High
M.I.M. Holdings Prizes	$80	Chiari Kameyama	All Saints Anglican School
	$60	Jay McDowall	St Patrick's College Shorncliffe
	$60	Leigh Brincat	Iona College

HIGHLY COMMENDED

David Berting (Dalby State High), Conan Wang (Anglican Church Grammar), Steve Lee (Ipswich Grammar), Robert Brier (South Burnett Catholic College), Mitsuhiro Sumiya (Somerset College), Zane Smith (All Saints Anglican School), Herris Xiao (John Paul College), Glen Maddern (Brisbane Boys' College), Sam Jones (All Saints Anglican School)

YEAR 10

U.Q. Mathematics Dept. Prize	$150	Ben Roberts	Toowoomba Grammar
Q.A.M.T. Prizes	$80	Zeo Yu	Redeemer Lutheran College
	$60	Franky Wong	Marist College Ashgrove
	$60	Paul Wu	MacGregor State High

HIGHLY COMMENDED

William Pettersson (Grace Lutheran College), Marciej Debowski (MacGregor State High), Trevor Gudenswager (Albany Creek State High), Akari Kameyama (All Saints Anglican School), Brett Peterson (Dalby State High), Li-wen Yip (Tully State High), Jane Olsen (Ipswich Girls' Grammar), George Ma (Brisbane Grammar), Goh Ikeda (Iona College), Joel Beckett (Somerset College).

YEAR 9

U.Q. Mechanical Engineering Dept Prizes	$130	Charlotte Lau	Somerville House
	$70	Jay Huang	Anglican Church Grammar
U.Q. Mathematics Dept Prizes	$50	Stuart Roberts	Downlands Sacred Heart College
	$50	Mimi Yue	MacGregor State High
	$25	Billy Kim	Brisbane Grammar
	$25	Joseph Kung	St Laurence's College

HIGHLY COMMENDED

Andrew Chai (Redeemer Lutheran College), Julian Connell (Iona College), Matthew Monaghan (Iona College), David de Weger (Iona College), Michael McLaughlin (Iona College), Jane Ziao (Centenary Heights State High), Judy Kuo (Redeemer Lutheran College), Mark Flegg (Brisbane Grammar), Michael Burchill (Mercy College), James Friend (Gladstone State High), Miranda Meyer (Southern Cross Catholic College), Jack Ren (Brisbane State High), Ayako Yuasa (All Saints Anglican School).

YEAR 8

U.Q. Mining, Minerals & Materials Eng. Dept Prizes	$130	Fay Chen	Somerville House
	$70	Emily Balfe	Redeemer Lutheran College
U.Q. Mathematics Dept Prizes	$50	Angela Sheu	Brisbane Girls' Grammar
	$50	Chris Yu	Brisbane Grammar
	$25	Fiona Skerman	Ipswich Girls' Grammar
	$25	Natalie You	Somerset College

HIGHLY COMMENDED

Ashley Darvill (Redeemer Lutheran College), David Kalinowski (St Laurence's College), Helena Wu (Craigslea State High), Lucy Manderson (Somerville House), Kate Wilson (Warwick State High), Edward Pietsch (Toowoomba Grammar), Rachel Trigger (St Aidan's School), Raymond Wu (Brisbane Grammar), Rebecca Girard (Brisbane Girls' Grammar), Jonathan Keller (Biloeal State High), Nick Martinez (Townsville Grammar), Tom Kearney (St Patrick's College Gympie), Jeremy Lim (Brisbane Grammar), Dalveer Singh (Brisbane Grammar).

PYTHAGORAS REVISITED
Lessons from the Past

Paul Dooley, Fairholme College, Toowoomba. e-mail pmdooley@mathgym.com.au

This is the last of four essays where the author has attempted to show how historical / cultural / social / philosophical contexts can be used in the teaching of some pretty familiar, and sometimes dry mathematical concepts to secondary students. The articles which have appeared in this Journal are shortened extracts from a collection of Essays and Activities on the authors web site http://www.mathgym.com.au. Space does not permit in this medium to give each topic justice. To read a more expansive treatment and to obtain a copy of any of the Activity sheets - Click on the "history" button at the site and then follow the link to the Activity you want. You can then print out the page to get a black-line master for reproduction.

Introduction

In the three essays prior to this one, I outlined the influence of the Pythagoreans on Western science and philosophy and in particular their contributions to mathematics in the areas of number theory and algebra. The interested reader will have recognised the significance of geometry in all these discoveries. This final essay on the Pythagoreans will attempt to describe some of their contributions to and influence on the body of geometry.

Brief History of Geometry to the time of Pythagoras

As has been discussed in the earlier essays, there are few surviving works from this period. Much of what follows is generally accepted as the history of geometry at this time, but the reader should be aware that there are differing accounts about the contributions of Pythagoras and the Pythagoreans to mathematics.

The principal source of information concerning very early Greek geometry is the Eudemian Summary in the introduction to Proclus' (500 A.D.) *Commentary of Euclid, Book I*. Unfortunately the original reference, *The History of Geometry* by Eudemus (370 B.C.), a pupil of Aristotle, is lost, though Proclus had reference to it for his summary. This book apparently gave a full account of the developments in geometry to about 335B.C. What follows is Proclus' summary of Eudemus' brief history of geometry up to Pythagoras as described in Allman [1]:

Proclus states, that in the Eudemian Summary the origins of geometry are attributed to the Egyptians. It claims that the Egyptians invented geometry to restore the land boundaries which had been destroyed by the annual inundation of the Nile. Eudemus credits Thales of Miletus (ca. 640 - 548 B.C.) as having brought geometry into Greece :

"that he discovered many things himself, and communicated the beginnings of many to his successors, some of which he attempted in a more abstract manner and some in a more intuitional or sensible manner."

A review of available references to Thales credits him with the following geometrical results:

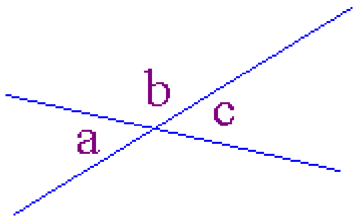
(i) *A circle is bisected by any diameter.*
(ii) *The base angles of an isosceles triangle are equal.*
(iii) *The vertical angles between two intersecting straight lines are equal.*
(iv) *Two triangles are congruent if they have two angles and one side equal.*
(v) *An angle in a semicircle is a right angle*

Babylonian and Egyptian mathematics was basically "intuitional and sensible" , there being little evidence to suggest that mathematics was valued as anything more than as an approximate, useful tool for census, commerce, calendar construction, building and surveying. While Eudemus asserts that Thales discovered and transmitted many things in an "intuitional and sensible manner" he also credits Thales with making arguably the most important contribution to Western thought - the discovery (creation?) of the value of abstraction in proof. While rhetorical proof and concepts of truth had existed in some of the epic poetry of the times such as in Homer's *The*

Odyssey, Thales appears to be the first person in Western history to value logical discourse free of qualification.

One can only guess at the stimulus for such a revolutionary insight. It could possibly be that Thales' first glimpse of this idea occurred with the vertical angle theorem. That both angles are congruent is easily demonstrated by folding about the intersecting point of both lines so that the two angles lay upon each other. The use of superposition to "prove" statements was a regular practice across the Middle East at this time.

Thales may have realised that this demonstration was in fact only an illustration of one instance of the theorem, and that this theorem might be a "universal truth". It is possible that Thales' proof was similar to the one used in mathematics classes today.



In the figure above:

angle **a** plus angle **b** equals a straight line.
but angle **c** plus angle **b** equals a straight line.
so angle **c** must equal angle **a**.

Q.E.D.

The logical device used here was later recorded by Euclid [2] as *common notion 1* in Book I of *The Elements*.

"1. Things that are equal to the same thing are also equal to one another."

Whether or not the example above was the first abstract proof, the significant point is that the mathematical intellect was wakened. The secrets of the physical universe were exposed to clear analysis - this proof (or a similar proof), was the first abstract "truth", pure and immutable, in a dusty, rhetorical and changing world. It is as if the human mind, released from the drudgery of everyday survival (Thales was thought to be a wealthy merchant with ample leisure time) naturally seeks out and is seduced by the beauty and purity of abstract entities.

This idea, that truth can only be sought in the purity of abstraction and therefore is mathematical in nature, was developed by Pythagoras and reached its zenith with Plato and Aristotle. It is recorded that the motto over the door to Plato's school *The Academy* was *"Let no one unversed in geometry enter here"*. It is Aristotle who is credited with putting the final touches on the Postulational Method as it appears in *The Elements.*

After discussing Thales, Proclus next mentioned Ameristus for *his "zeal in the study of geometry"* and then claims:

"Then Pythagoras changed it into the form of a liberal science, regarding its principles in a purely abstract manner, and investigated its theorems from the immaterial and intellectual point of view…"

Thales was about 50 years senior to Pythagoras and Miletus and Samos are very close geographically, so it may be that Pythagoras actually studied under Thales. Whether this was the case or not, most agree that Pythagoras was aware of and built on the discoveries and teaching of Thales. It is also generally accepted that the Pythagoreans contributed the bulk of the first two books of *The Elements* (on angles, triangles, areas, and geometric algebra) together with some of Books IV (polygons), VI (proportions) and XIII (polyhedrons).

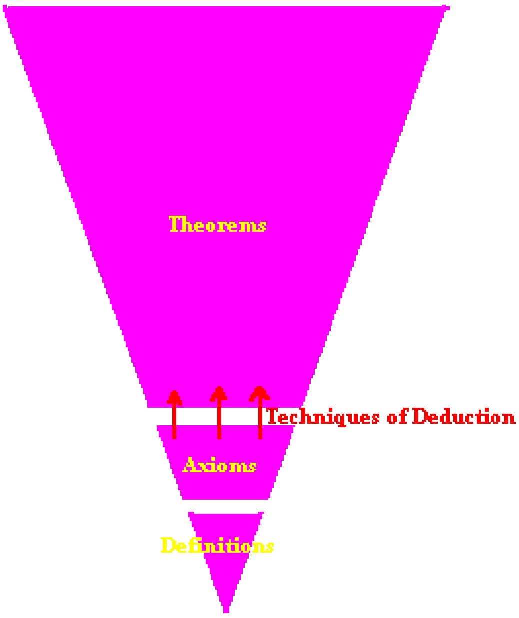
Much of these have been loosely described in the earlier essays. What remains to be described is the development of the postulational method and some of the Pythagoreans' more interesting geometrical discoveries.

The Postulational Method

Eves [3] claims:

"Sometime between Thales in 600 B.C. and Euclid in 300 B.C., the notion was perfected of a logical discourse as a sequence of rigorous deductions from some initial and explicitly stated assumptions. This process, the so-called ***postulational method****, has become the very core of modern mathematics; undoubtedly, much of the development of geometry along this pattern is due to the Pythagoreans."*

The Postulational method (sometimes called the axiomatic or logico-deductive method) is shown in the following diagram.



- Initially a few key terms are introduced and *defined*.
- Next some assumptions are declared which the reader is expected to accept without proof - these are called *Postulates* or *Axioms*.
- All other facts and relationships called *theorems* are derived from these definitions and axioms using acceptable *deductive processes*.

Early Greek mathematicians and philosophers made a distinction between an axiom and a postulate, and it is generally thought that there were three different distinctions made by the different groups at the time. Eves claims that Euclid considered an axiom as an "assumption common to all sciences" (the "common notions" in set A below) and a postulate as "an assumption peculiar to the particular science being studied" (set P below) as described in Book I of *The Elements*:

A1 Things which are equal to the same thing are also equal to one another.

A2 If equals be added to equals, the wholes are equal.

A3 If equals be subtracted from equals, the remainders are equal.

A4 Things which coincide with one another are equal to one another.

A5 The whole is greater that the part.

P1 (It is possible) to draw a straight line from any point to any other point.

P2 (It is possible) to produce a finite straight line continuously in a straight line.

P3 (It is possible) to describe a circle with any centre and distance.

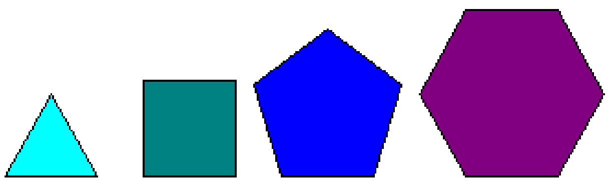
P4 (It is true) that all right angles are equal to one another.

P5 (It is true) that, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

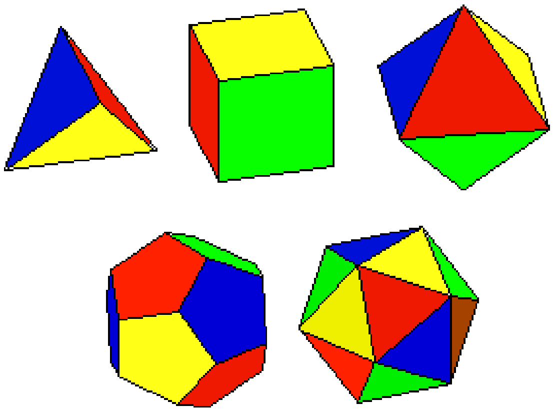
Building on the definitions, common notions and postulates, the Pythagoreans were able to produce deductive proofs of a number of important geometric propositions (theorems). These axioms and proofs were further developed by several mathematicians up to Euclid, and have been a useful set of tools for many subsequent generations of mathematicians, showing how powerful and beneficial deductive reasoning can be! It would be misleading though, not to mention at this point, that there are considerable limitations of deductive reasoning in an axiomatic system; the major one being that any proof is only as good as the assumptions and axioms it starts off with! It took over 2000 years after Pythagoras before alternative geometries were found, ones that started from different assumptions and resulted in different propositions.

The Regular Polygons and Polyhedra

The **regular polygon** is a convex plane shape (fills a flat surface) that has all sides the same length and all internal angles the same size. The simplest regular polygon is the equilateral triangle, then the square. Following this, each polygon is named for the number of sides it possesses, the 5 sided pentagon, the 6 sided hexagon etc.



A **regular polyhedron** is a convex solid (fills space) that has all faces the same shape and size, and all internal angles between adjacent faces are the same. There are only 5 regular polyhedra – the 4 faced tetrahedron (the tetractys of a previous essay), the 6 faced hexahedron (cube), the 8 faced octahedron, the 12 faced dodecahedron, and the 20 faced icosahedron as shown in the following figure.)



The regular polyhedra are commonly called the Platonic Solids because it is thought that Plato's *Academy* was the first to establish the geometrical methods to construct them (as in *Book XIII of The Elements*). It is generally accepted though that Pythagoras "produced" them and probably investigated some of their properties as in Allman:

"It is well known that the Pythagoreans were much occupied with the construction of regular polygons and solids, which in their cosmology played an essential part as the fundamental forms of the elements of the universe."

The interested reader should refer to an earlier essay which discusses the cosmological importance the Pythagoreans assigned to the regular solids.

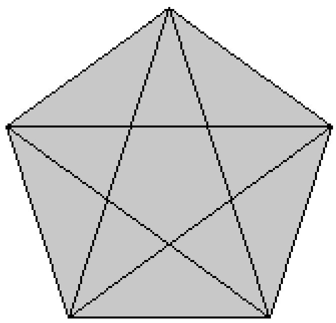
It is thought that Pythagoras arrived at the regular polygons by investigating the properties of regular shapes which "filled the plane" around a point, and was probably influenced by the art of the day. Certainly the Babylonians and Egyptians had developed some ideas about the properties of tessellating shapes as evidenced in the tiling of their floors and walls etc. Also the presence of the crystalline shapes of pyrite and others was known at this time, and Heath claims that:

" It appears that dodecahedra have been found, of bronze or other material, which may belong to period's earlier than Pythagoras' time by some centuries."

The regular polygons (up to the hexagon) were constructed by the Pythagoreans with the use of inscribed circles as is represented in Books I and IV in *"The Elements"*. In fact the very first Proposition (Book I Proposition 1) shows how to construct the equilateral triangle using congruent circles each intersecting the other's centre. The remaining constructions are generally those which also appear in mathematics text books today.

The web site has an Activity on the regular polygons and the regular polyhedra. The Activity shows how the Pythagoreans may have investigated tessellating (tiling) these regular shapes around a point to arrive at the conclusion that only the equilateral triangle, square and hexagon can "fill the plane" at a point, and that there are only five regular polyhedra.

The Section



If you join the vertices of the pentagon internally you arrive at the "star-pentagram" (above). The pentagram is claimed to have been the symbol of the Pythagorean Order and was given by them the name "HEALTH". As mentioned in an earlier essay, the pentagram is still claimed to have magical properties in modern mystic cults.

The construction of the regular pentagon is given in *The Elements* in Book IV Proposition 11. It makes use of the previous Proposition (10) to construct the "isosceles triangle whose base angles are twice the other" , that is the 72-72-36 triangle. There are 10 such triangles in the figure above.

The construction of the triangle in Proposition IV. 10 relies on Proposition II. 11 which essentially divides a line *in extreme and mean ratio*, that is, the construction of the **golden section**. Heath claims that there is considerable justification for crediting the discovery of ***the section*** to the Pythagoreans considering that II. 11 is an example of *application of areas* which we have shown (in an earlier essay) to be Pythagorean. Johannes Kepler, in the sixteenth century claimed (in Boyer [4]):

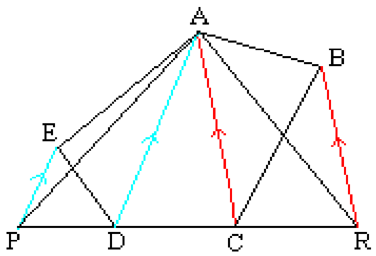
"Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a

line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel"

The web site has an Activity which looks at some of the properties of the golden section.

Transformation of Areas

The Pythagoreans were interested in finding the area of any polygon. They did this by transforming the area of the given polygon into another plane shape whose area they could find, a square. To do this, they used what appears as Proposition 14 in Book II of *The Elements*, which in turn draws on II. 5, I. 42, I. 44, and I. 45. To give an idea of the method Eves claims; *"A simpler solution, probably also known to the Pythagoreans, is the following."* He then describes the technique as set out below.



Consider any polygon ABCDE (above) Draw BR parallel to AC to cut DC (extended) in R. Then, since triangles ABC and ARC have a common base (AC) and equal altitudes on this common base, these triangles have equal areas (I. 38). It follows that polygons ABCDE and ARDE have equal areas. But the derived polygon has one less side than the given polygon. By a repetition of this process, we finally obtain a triangle APR having the same area as the given polygon. By dropping a perpendicular from one vertex (A) of the triangle to the opposite side (PR), they could use straightedge and compass to find the square whose area is congruent to the triangle. This is shown in an Activity on the web site.

Conclusion

Over the past four issues of Teaching Mathematics I have proposed that Western mathematics essentially began with the genius of Pythagoras. Where possible I have resisted the accounts of the more "romantic" versions of the achievements of Pythagoras and the Pythagoreans. There are some recent books on this theme, written for school students, which represent the man as a quaint mystic and follower of bizarre rituals. This is unfortunate for many reasons, not only because it gives an historically inaccurate picture of the period, but also because the opportunity is lost to share with students the common bond (socially, culturally and intellectually) we have with this man and his ideas.

As the interested reader will already have noticed, the primary sources for these essays are about 100 years old. I am aware that there are more recent accounts of this period which credit Pythagoras with little more than forming shapes in the sand with pebbles. In my essays I have preferred to tell the more traditional version. In so doing I too may have succumbed to the romantic, but even so, I trust that this version has been worthy of the telling, and that it has provoked some reflection on the part of the interested reader.

The reader who is knowledgeable of mathematical history is probably surprised that I have ended this series without an in-depth discussion on the theory of proportion, so central to mathematical development at this time. I have instead, chosen to hint at some of the challenges proportion caused to mathematicians of the time, and have reserved an in-depth discussion on the theory of proportion for another essay.

References:

[1] Allman, George J., -*Greek Geometry from Thales to Euclid* , Dublin University Press 1877 (also on-line at "http://moa.cit.cornell.edu")

[2] Heath, Sir Thomas L., *Euclid - The Thirteen Books of The Elements Second Edition Vol i*, N.Y. Dover Publications (orig 1908) (also on-line with java applets at "http://www.perseus.tufts.edu/" , and with java applets at http://aleph0.clarku.edu/~djoyce/java/elements/elements.html")

[3] Eves, Howard, *An Introduction To The History of Mathematics Fifth Edition*, Saunders College Publishing 1983

[4] Boyer, Carl B. *A History of Mathematics*, N.Y.: John Wiley and Sons, 1968

STUDENT PROBLEM PAGE

Cheryl Stojanovic   Forest Lake College
Garnet Greenbury   Publications Committee

Question 1

A set of five natural numbers has it's mode, median, mean and range all exactly equal to 6. What is the set of numbers?

Question 2

A solid cube with side length 5 cm is made up of individual one centimetre cubes. How many of the cubes are face-to-face with exactly four others?

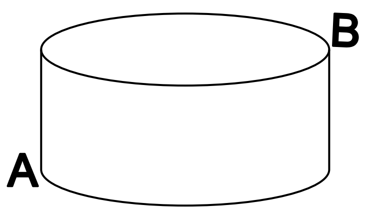
Question 3

The contents of a 2 kg packet of sugar are poured into two identical jars. The individual jars of sugar are then found to weigh 1.1 kg and 1.7 kg. How much sugar is in each jar?

Question 4

My uncle, who is now retired, once told me that his age was the square root of the year in which he was born. In what year did he say this?

Question 5



A cylinder has a circumference of 48 cm and height 10 cm. Point A on the bottom rim is diagonally opposite point B on the top rim. What is the shortest distance (in centimetres) from A to B along the surface of the cylinder?

Submitting solutions

Students are invited to submit solutions. Include your name, the problem number, your school and year level (clearly printed). Send them to Garnet Greenbury, Unit 14 Greenleaves Village, Upper Mt Gravatt, 4122. Closing date: 10 February 1999.

Solutions to the problems, Vol. 24 No. 3

1. 2520
2. 6.25 sq. m
3. 1320
4. 6 cm
5. 526 rooms

Prizes

Prizes for solutions to the student problems in *Teaching Mathematics* Vol. 24 No.3:

Craig West of Pimlico State High School is the winner of the Penguin book prize.

Jing Xiao of Centenary Heights State High School is the winner of the annual subscription to *Tenrag*.

Solutions were received from:

Thulimbah State School
Pimlico State High School
Centenary Heights State High School
Kingaroy State High School
All Hallows' School
Gladstone State High School
Blackheath and Thornburgh College
Kalkadoon State High School
Stanthorpe State High School
Sienna Catholic College
Ipswich Girls Grammar School

INDEX TO VOLUME 24 OF TEACHING MATHEMATICS

Author	Title	No.	Page
Baturo, Annette & Cooper, Tom	Spreadsheets and upper primary mathematics	2	25
Boggs, Rex	Understanding r^2	1	32
Butler, John	Linking 2 & 3 dimensions – a challenge for students and teachers	4	27
Commins, Lydia	The Mayday Conference	2	4
Dooley, Paul	Pythagoras revisited – lessons from the past	1	35
		2	36
		3	34
		4	34
	MSWLogo	4	31
Ford, Janelle & Cheng, Michael	Investigating exponential growth and decay with graphics calculators	2	11
Grace, Neville	Ten myths plus one	3	26
Griffiths, Rachel	Using information texts to integrate numeracy and literacy learning	1	7
MacGillivray, Helen	R (-squar) ED DANGER	4	16
Goos, Merrilyn	Technology education for pre-service mathematics teachers	1	16
	The glass slipper	4	12
Hartnett, Judy	The literacy of numeracy	4	8
Hekel, Adrian &Gills, Simon	Basketball maths	3	31
Hitchings, Owen	Double your money – a resolution to the paradox	2	42
Ilsley, David	Teaching *e* without calculus	1	13
	Algebra as the study of relations	2	16
McDowell, Jan	Maths in the mall	3	14
Morgan, Geoff	Strategies for calculating exact answers mentally	3	5
Mousley, Judy	Teaching mathematics for understanding – some reflections	2	6
Nesbit, Steven	Displaying statistical data	1	29
Norton, Stephen	The Queensland senior syllabuses – roots of conflict?	2	31
	Effects of the senior mathematics syllabuses upon teachers' goals and practices	3	22
Nothdurft, Lyn	Empowering students to accept responsibility for their own learning	1	22
	Reflections on learning – strategies for use in senior mathematics	3	16
		4	22
Oliver, Jack	What is a billion (or a trillion …)?	3	13
Shield, Mal	Applications in geometry for Mathematics A and B	3	10
Sullivan, Jeremy & Daniels, Matthew	Fibonacci sequences on the graphics calculator	1	17