May 2012 Volume 37 No 2

# TEACHING MATHEMATICS

## Regulars



## Features



## Front Cover

## QAMT Executive & Committee



## QAMT Office

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## QAMT Mailing List
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*Teaching Mathematics* is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, September and November) and is provided as a right to all members of the association.


The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- to speak and act on matters related to mathematical education.

Advertising material published in and accompanying this journal does not imply endorsement by the association.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the Editor, Rodney Anderson. The preferred way is by email. Contact details are as follows:

Rodney Anderson
Moreton Bay College

Phone: 07 3390 8555
Fax: 07 3390 8919
Email: andersonr@mbc.qld.edu.au

Microsoft Word is the preferred format. All receipts will be acknowledged - if you haven't heard within a week, e-mail Rodney to check. Copy dates are: mid-February; mid-May; mid-August; mid-October.

The views expressed in articles contained in Teaching Mathematics belong to the respective authors and do not necessarily correspond to the views and opinions of the Queensland Association of Mathematics Teachers.

If you have any questions regarding Teaching Mathematics, contact Rodney Anderson. Publications sub-committee members are listed below. Feel free to contact any of these concerning other publication matters.

| | | |
|---|---|---|
| Rodney Anderson (Convenor) | Moreton Bay College | 07 3390 8555 |
| Gaynor Johnson (Newsletter) | QAMT Office | 07 3365 6505 |

Books, software etc. for review should be sent to Richard Porter; information to go in the newsletter should be sent to Gaynor Johnson at the QAMT Office. Newsletter copy dates are the beginning of each term.

Contact the QAMT office for advertising enquiries.

| Advertising Rates | 1 Issue | 2 Issues | 3 Issues | 4 Issues |
|---|---|---|---|---|
| Quarter page | $44 | $88 | $110 | $132 |
| Half page | $66 | $132 | $176 | $220 |
| Full page | $110 | $220 | $308 | $396 |
| Insert (single A4 or folded A3) | $220 | $440 | $660 | $770 |

QAMT also now offers a colour advertising service for the covers.

# From the President

**Rodney Anderson**

If you have not yet registered for the annual conference, please do so. It is shaping up to be a wonderful conference with a wide range of presenters and trade displays. In addition, if you are attending the conference, please book your accommodation (at SeaWorld Resort) as soon as possible as the number of rooms available are decreasing daily.

As members of QAMT, what can you do for other members of QAMT? Maybe you could establish a network of local teachers. Maybe you could mentor 1st and 2nd year teachers. Maybe share resources. Some people say what you get in return is proportional to what you put in.

All of the above is a great way to help members. **Being a member of QAMT is an investment in your career.**

In regards to the journal, what parts do you turn to first? What do you want to add? Is there anything that should be deleted? Please e-mail me (andersonr@mbc.qld.edu.au) with any suggestions.

To enable the Journal to be as successful as it is, consider submitting articles based upon

1. Practical Teaching Ideas
2. Differentiation Mathematics in the Classroom
3. Trends in International Mathematics Education
4. Research/Theoretical Paper

It is professionally rewarding and this Journal is on International Registers and your article can be internationally searched and cited. And yes, there is a financial reason to also submit articles.

The author of every article selected for publication will receive a fully paid annual membership to the Queensland Association of Mathematics Teachers, along with the Quarterly Journal, conference discounts and other benefits.

In addition, two prizes of will be awarded of $200 to spend on mathematics teaching resources of the author's choice from the Australian Association of Mathematics' Teachers. These prizes will be for the Best Practical Teaching Article and the Best Research Article.

Another executive member of the QAMT, Michael Bulmer, has created a website that displays the covers of the past 24 journals. If you select the journal on the website, you will also see a larger view of the journal and also a description of the cover and how it was created.

http://www.maths.uq.edu.au/~mrb/qamt/

A pdf of the Student problems is on the QAMT website, www.qamt.org/resources.

**A reminder**. Samples of graph and grid paper can be found at www.qamt.org/resources

**Rodney Anderson**
**President, QAMT**

# From the Editor

**Rodney Anderson**

A reminder that we encourage contributions from members for the Journal, since after all, it is your Journal. This is a chance to share your ideas and practices with other members. We also welcome suggestions for particular topics that you would like to read about.

E-mail suggestions and submissions to andersonr@mbc.qld.edu.au

## Incentive to contribute articles/teaching ideas to the journal

### QAMT MEMBERSHIP DRAW

1 For every article/teaching idea contributed, the author will receive a ticket in a Membership Draw

2 If you are contributing an article/teaching idea for the first time, the author will receive two tickets in a Membership Draw.

3 The next winner of the Membership Draw will be announced in journal V37 N4.

## Early Years Conference Feedback

**The following comments/feedback from the Early Years Conference have not been edited.**

Hi, today I went to the annual Mathematics Convention held at Jindalee SS. One word... AWESOME!!! If you didn't go, make sure you go next year! The presenters really knew their stuff and were very engaging! A big thank you to the organisers Sue Allmond and Jill Wells. I went last year it was awesome and you did an awesome job this year! I certainly will be going to next year's convention too!

I am sorry but I didn't fill in an evaluation form but I just wanted to say how I enjoyed the whole day. All the speakers were great. I thought they had a fantastic mix of theory, practical and humour. I really enjoyed the Keynote speaker. I would love if possible a copy of the powerpoint or some notes. I also attended Active Maths, Tick and Flick and the number strand of ACARA. They were all great as I said they all had theory, practical application and things we could take away and humour. It was great to have the stalls and especially the sales table. I really want to encourage the organisers as a teacher I really value the opportunity to hear about the current thinking with such practical ideas for application. This is my second year in Year Two having moved from prep after 5 years there and I have found it quite challenging especially with ACARA.

# Annual Conference

## Motivating Mathematics

The annual conference which is being held at SeaWorld resort on the Gold Coast is looking like being a wonderful (as all QAMT conferences are) conference. Charles Lovitt is the keynote speaker and Rex Boggs is giving the plenary by a practising teacher.

**The home page (there are holiday deals)**
http://seaworldresort.myfun.com.au/default.aspx

You can navigate the tours (on the right hand side of the page) or to save you time the following links will help.

**Resort Map**
http://seaworldresort.myfun.com.au/Resort-Info/Resort-Map.aspx

**Resort Facilities (many photographs)**
http://seaworldresort.myfun.com.au/Resort-Info/Resort-Facilities.aspx

**Take a Room Tour**
http://seaworldresort.myfun.com.au/Accommodation/Resort-Room.aspx

**Things to Do**
http://seaworldresort.myfun.com.au/Resort-Info/Things-To-Do.aspx

Have we (QAMT) got a deal for you. If you wish to stay at the resort (before or after the conference), there is a cheaper rate. 3 days before and 3 days after at a rate of $175 per night for a double room (i.e. two double beds per room), Breakfast is an additional $20 per adult but this is discounted from $29.50, deal includes 25% discount on theme park entries (SeaWorld, Movie World & Wet'n'Wild, 10% off beauty treatments & massages, use of water park, lagoon pools and tennis courts). For the accommodation deal use the form available at
http://www.qamt.org/qamt/qamtac-12-motivating-mathematics

**Also go in the draw to win one of these great prizes:**
Dolphin Encounter for 2 adults to be used during the conference
1 Night's Accommodation & Sea World entry (2A & 2CH - lucky door prize)

**Plus experience Shark Bay as part of your conference registration....**

Hope to see you at SeaWorld in June.

# QAMT Dates
# Professional Development
# Planning for 2012

**22nd to 24th June**
**Annual Conference**
**"Motivating Mathematics"**

| | |
|---|---|
| **Welcome Function:** | Friday, 22nd June 7-9pm |
| **Dinner:** | Saturday 23rd June |
| **Conference**: | Saturday 23rd and Sunday 24th June |
| **Venue:** | SeaWorld Resort, Gold Coast |
| **Cost:** | $330 presenters, $440 members, $550 non-members includes membership |

Keynote speakers followed by presentations, breakout sessions and workshops. Sponsorship & trade are welcome.

Please contact Rodney Anderson andersonr@mbc.qld.edu.au to provide or suggest workshops and presentations.

Visit www.qamt.org for updates.

**Friday, 20th July**
**Problem Solving Competition**
The UQ/QAMT PSC is open to all students of secondary schools in Queensland.

| | |
|---|---|
| **Cost:** | $2 per student |
| **Contact:** | Michael Bulmer - m.bulmer@uq.edu.au<br>www.maths.uq.edu.au/qamt |

**Thursday, 2nd August**
**Australian Maths Competition**
Australian Maths Trust
Details www.amt.canberra.edu.au

**August and September**
**DET & QAMT Year 8 Quiz**
A quiz style competition with 3 members to a team suggested dates are

| | |
|---|---|
| **Round 1:** | Week beginning 30th July |
| **Round 2:** | Week beginning 19th August |
| **State Grand Final:** | Friday, 14th September |
| **Entry fee:** | $22 per team |
| **State Co-ordinator:** | Peter Cooper, spcooper@uq.net.au |

**Saturday, 25th August**
**Implementing the Australian Curriculum**

| | |
|---|---|
| **Time:** | 9am – 4pm |
| **Venue:** | Somerville House |
| **Cost:** | $44 members & students, $99 non members |

To provide or suggest workshops and presentations contact Gaynor Johnson at qamt@uq.net.au

# University of Southern Queensland

## Bachelor of Education

Dr Barry Fields (Pre-Service Co-ordinator) presents the Outstanding Achievement by a graduate of the Bachelor of Education in Mathematics to Esther Wearmouth.

# Teaching Ideas

## Putting the 'Cool' Back into Mathematics

**Megan Munckton**
**Kepnock State High School**

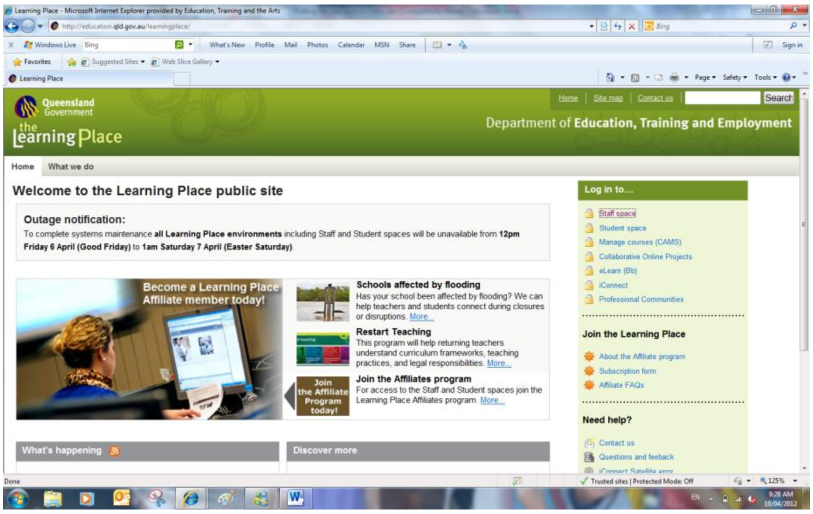
It has to be stated as a fact that maths is not most students' favourite subject at high school. In fact it does not even come a close second for ranking students most loved subjects. When we have subjects like art, physical education, agriculture and film and television to compete with…need I say more?

For 5 years now I have been teaching mathematics and every year find myself taking one step forward and two steps backwards trying to evoke excitement in my students for a pending maths lesson. Every year I develop a wonderful rapport with most students and even have some students appearing at my classroom door awaiting their maths lesson with a glow of enthusiasm on their faces, but it remains a mission of mine to get the odds back in my favour…'our favour', of ranking mathematics as the 'cool subject'.

A couple of years ago I started hearing more and more about ICT use in the classroom. Apart from the fact that I honestly didn't know what ICT stood for, I believed this concept was just another agenda from the powers above who really had very little idea of the behaviours I was dealing with daily from my 120 or so students. Not to mention teaching the mass of content by exam time, communicating with parents, reporting, making lesson plans etc.

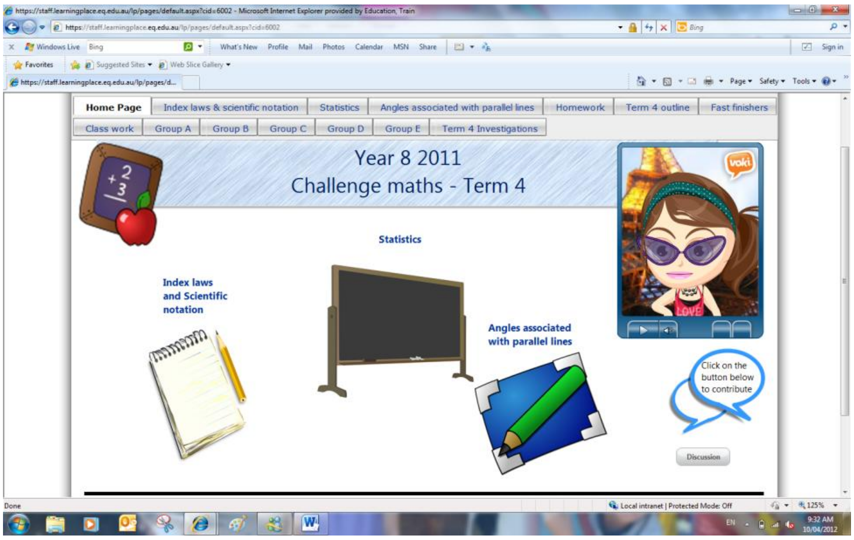
Have you noticed how your eye is drawn to information that is presented digitally rather than plain text? As an adult I take more notice of a roadside digital billboard with the many bright colours and patterned movement, compared with the standard printed billboards. I knew that my students are growing up in a fast paced technology age and they utilise various electronic devices with ease daily, it is the way of the future and unless I jumped aboard I was not going to progress with effective teaching and most importantly bettering my student learning outcomes. By incorporating ICT's into my teaching not only for the extensive educational purpose, I knew I would be engaging and exciting my students, hence promoting an effective learning environment.

*The Learning place* was something I had heard a little about and seemed to be associated with the term' ICT '. I made the decision over the summer holidays in 2010/2011 to look into *the learning place (www.learning place.eq.edu.au)* and find out what it was all about and more specifically if it would be able to help me on my mission. It was on this site that I was drawn to what is called 'edstudio'. This is basically a digital learning space that teachers are able to compile websites, images, videos and other teaching resources and also allow for collaboration between students and the teacher.



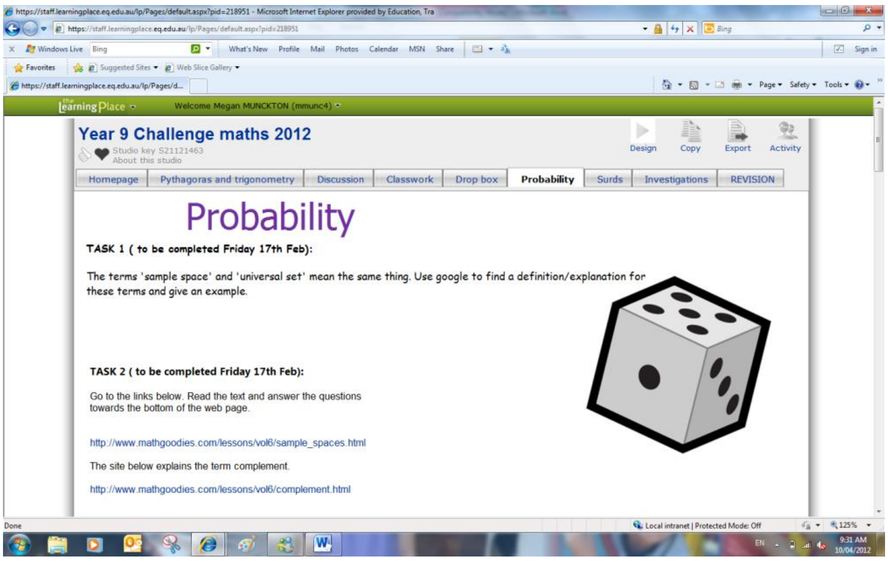
I designed and implemented my very first edstudio to trial with my year 8 mathematics class. I customised the edstudio with various colour and fonts, backgrounds, images and videos. In amongst the 'pretty' design is the underling mathematics. I was able to:

- Select relevant websites to direct the students to read text, diagrams or even watch a video to be summarised and evaluated.
- Direct the students to an interactive game relevant to the current topic or perhaps to use as a form of revision.
- Upload recorded audio of myself explaining a new concept as well as via a Voki (*www.voki.com*) for the students to listen to, which is a great alternative to me teaching the entire class from the front of the room. The fantastic advantage is the capability for the individual student to replay the audio until they have mastered the new concept.
- Insert a block of text, directing the students to set textbook activities, typed questions for the students to answer, or perhaps have the students use Google to define words/terms.




As edstudio is an internet site, my students have access to the edstudio using a computer from anywhere (granted they have internet connection). I found this is invaluable for many reasons:

- Student absence is no longer a problem as the student is able to access most of the work covered.
- The edstudio is used as a revision tool.
- If a student is having problems with the classwork/homework/assignments, they can post a comment in the discussion for their peers (a great way to achieve peer mentoring) or myself to answer.
- Assignments/investigations can be accessed on the edstudio.
- Students can post their completed assignment/investigation via the 'dropbox' (a digital method of submitting work to me).



Students collaborate and publish content on the edstudio such as; defining terms or concepts, answering maths problems, explaining how to solve a problem, listing assumptions associated with a problem, or I even have them design their own maths problem relating to the current topics to have visible for their peers to attempt solving.

Unfortunately when starting anything new there is always hiccups along the way. This was true for my journey through designing and implementing my edstudio, however with perseverance and seeking assistance from the right people I have over-overcome each hurdle I encountered (and I am sure there will be more hurdles to come). It also needs to be said that taking on this new teaching format has certainly involved a lot of time outside our standard work hours, searching for great internet sites and designing the edstudio. The bonus for putting in all the extra time is I then have a complete digital folder of all my lessons ready to go next year!

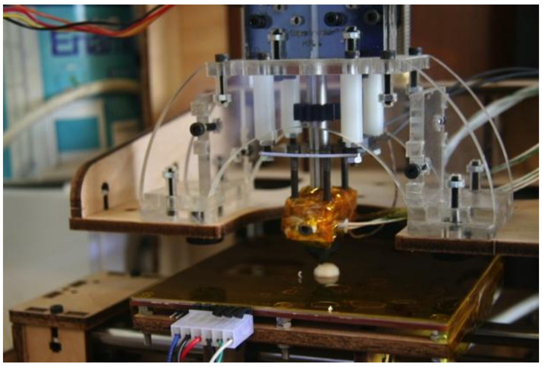
Today, sixteen months after discovering the learning place, the majority of my year 9 mathematics lessons are based around the edstudio. I continue to improve my knowledge and skills of various ICT's and strive to implement new technologies in my classroom.

The impression I hope my article has on you is this…..try something new in your classroom. Step out of your comfort zone and learn something new that you can utilise for teaching and learning. If it doesn't go so well, try again.

I am proud to say that I have accomplished my mission! Maths is definitely cool again!

## 3D Printers in the Primary Classroom

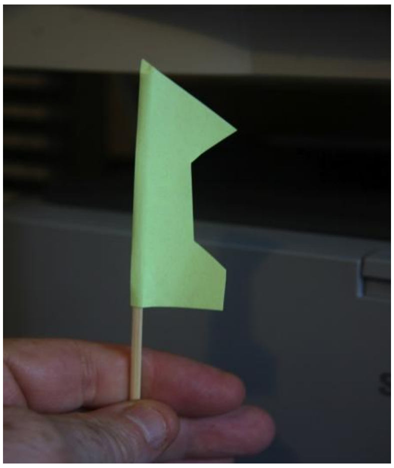
3D printers are a reality and are becoming more and more available to the everyday person. The arrival of units is being heralded on Google, YouTube, and TED with basic home/handyperson units now below $1000.

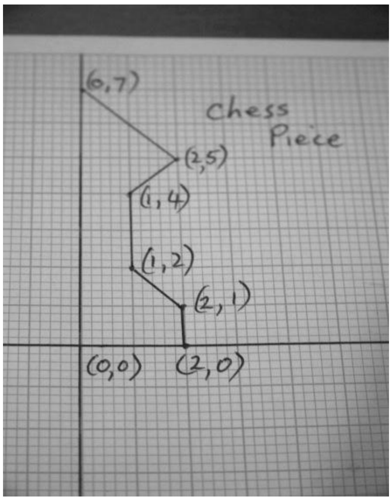


Exemplars of how people are using them in the home are becoming more readily available and some examples include designing new light brackets to replace broken ones to identifying broken parts in a vacuum cleaner, going on line to download the part specifications then printing out the offending article. How does this assist in the primary classroom?

Whether talking of translations, rotations, flips, slides turns or locating points on a graph the 3D printer can make maths more exciting.

Imagine that your classroom has an old chess set with a pawn missing. The children do not enjoy it as much using a button as a replacement.






Ask the children to draw the silhouette of a similar piece. Using the pawn allows for more groups of children to do this at the same time using one chess set. Next draw the silhouette onto graph paper with an ‘x’ and ‘y’ axis. Mark the key points and record the ordered pairs. Cut out the design and glue to a bamboo skewer. Using the rubbing of hands rotate the design quickly and notice the 3D shape formed and see if it matches the desired shape.

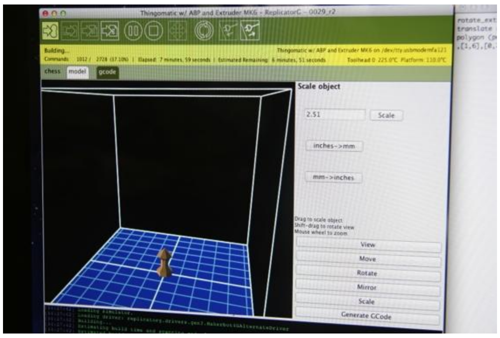
If it does copy the co-ordinates to software for creating 3D models.

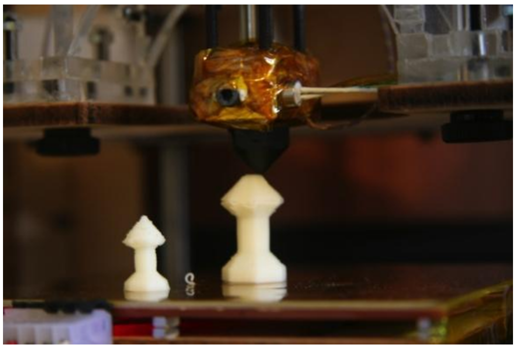
{this is the program for the chess piece illustrated, notice the ordered pairs;

```
rotate_extrude (convexity=10)
translate ([0,0,0])
polygon (points=[[0,0],[2,0],[2,1],[1,2],[1,4],[2,5],[1,6],[0,7],[0,0]]);}
```

I use either OpenSCAD or Google Sketchup (because they are free and talk to my 3D printer, a Makerbot Thing-o-matic) then hey presto push the print button and watch the design appear.

Mathematics language used included shape, translations, coordinates, ratio, scale and time.





# Patterns in Numbers and the Cartesian Plane

Ian Lindsay
Runcorn SHS

The Runcorn SHS Mathematics Department's logo is

and in this unit we will look at how mathematicians use pictures to explore patterns in numbers.

But first, why do we bother with exploring patterns in numbers? There are three main reasons:

- It is a fun problem solving activity
- Patterns enable us to understand how things in the world 'work'
- We use patterns to predict what will happen in the future

**<u>Some simple patterns</u>**

**<u>Task 1</u>:** Can you 'pick the pattern' in the following lists of numbers and hence write down the three numbers that would follow?

1. 2, 5, 8, 11, _____, _____, _____

2. 17, 13, 9, 5, _____, _____, _____

3. 4, 9, 16, 25, _____, _____, _____

4. $-1.2,\ -0.7,\ -0.2,\ 0.3,$ _____, _____, _____

5. 1, 1, 2, 3, 5, 8, 13, 21, 34, _____, _____, _____

The last of these patterns you can see on the logo above and is one of the most famous patterns in mathematics. What is the name of this pattern?

________________________________________

**<u>Rules for patterns</u>**

If the letter "t" is used to represent the word "term" and "$t_n$" represents the value of the n'th term (e.g. when n=3 then $t_n$ is $t_3 = 8$), then we could write for the first pattern above:

n= 1, $t_1$= 2; n= 2, $t_2$= 5; n= 3, $t_3$= 8; n= 4, $t_4$= 11

Now doesn't that look confusing? This would be better presented in a table:

| | | | | |
|---|---|---|---|---|
| n | 1 | 2 | 3 | 4 |
| $t_n$ | 2 | 5 | 8 | 11 |

The question that would then be asked is: "Can we find a rule that would enable us to work out the value of any term in this pattern?"

For example, what is the value of $t_{17}$? Ok, you could work this out by simply adding 3 until you get the $17^{th}$ term but that isn't clever. Would you do the same if asked for the value of $t_{124}$, or how would you decide if 231 is the value of one of the terms in this pattern?

The rule for the above is: $t_n = 3n - 1$

**<u>Task 2</u>:** Can you use this rule to work out the value of $t_{17}$ and decide if 231 is the value of one of the terms of this pattern? Try it!

**<u>Task 3</u>:** Now write up tables for the patterns 2, 3 & 4 above and see if you can come up with a rule for each. I <u>don't</u> recommend you try 5 as you probably won't get there – though it is possible!

**<u>Coming up with the Rules</u>**

Now, you may well ask: "But where did you get the rule for the first pattern from?" In mathematics there are various ways of working out patterns in numbers but the one we are going to explore is where we draw 'pictures' of the patterns on what is called the <u>Cartesian Plane</u>.

To do this we will first relabel the 'n' and '$t_n$' in our table above to get:

| | | | | |
|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 |
| y | 2 | 5 | 8 | 11 |

We can now represent this table as a set of <u>ordered pairs</u> which would look like:

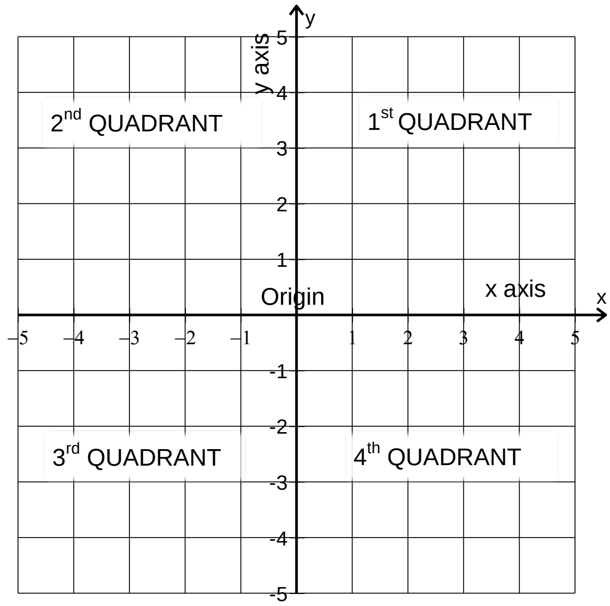
$$\{(1,2), (2,5), (3,8), (4,11)\}$$

The next thing we need to do is <u>plot</u> these ordered pairs on the Cartesian Plane, but first we need to introduce the Cartesian Plane.

**<u>The Cartesian Plane</u>**

The Cartesian Plane (which I will abbreviate as CP from now on) is named after its inventor, the Frenchman Rene Descartes (1596-1650). With it Descartes invented a branch of mathematics called Analytical Geometry which can be used to solve geometric problems using algebra. Doesn't that sound like fun? It is!

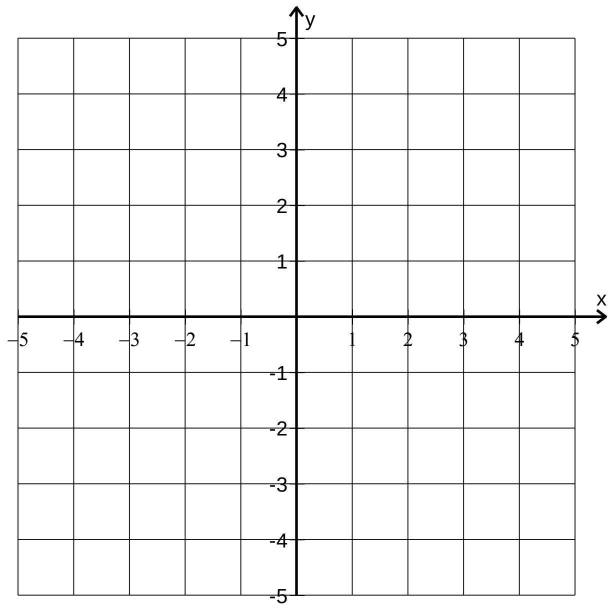
Anyway, the following is a labelled picture of the CP. You need to know the vocabulary of the CP.




So, when we plot a point on the CP we first find our ‘x’ value on the x-axis and then our ‘y’ value on the y-axis and then the point will be somewhere in one of the quadrants. Points can also find themselves on an axis and hence between two quadrants. The point in the middle of the CP is called the origin and has the ordered pair, or coordinates, (0,0).

**Task 4:** Plot the following points on the CP below and describe where the points are located.

$$A(1,3),\ \ B(-2,5),\ \ C(3,0),\ \ D(0,-4),\ \ E(-3,-1)$$




The point:

A is located ______________________________

B is located ______________________________

C is located ______________________________

D is located ______________________________

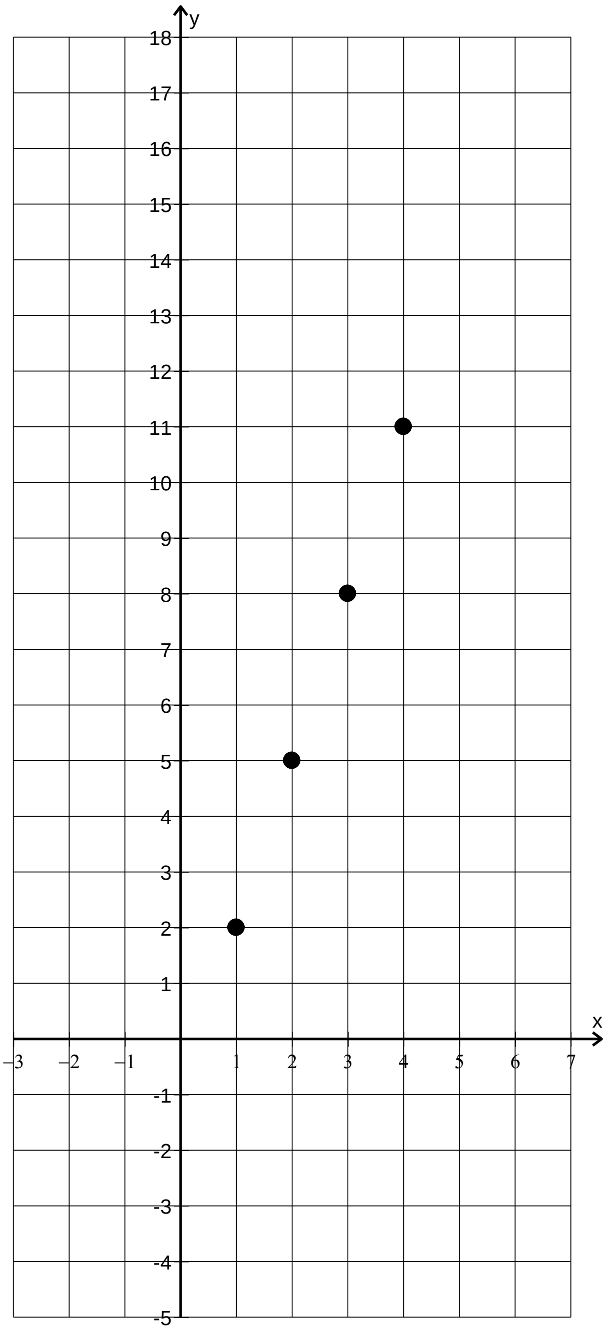
E is located ______________________________

**Back to patterns**

So, let's now plot the ordered pairs for our first pattern; i.e. the set:

$$\{(1,2),(2,5),(3,8),(4,11)\}$$

This is done below and you should notice something about how these points are arranged. It is possible to draw a straight line through all four of these points. When a set of points lie on one line we say the points are <u>colline-ar</u>.




Once we realise that a line can be drawn through the *given* points we can use this line to come up with other points or ordered pairs that lie on the line. For example, the point (5,14) lies on this line.

**Task 5:** Write down the coordinates of at least four other points that lie on this line:

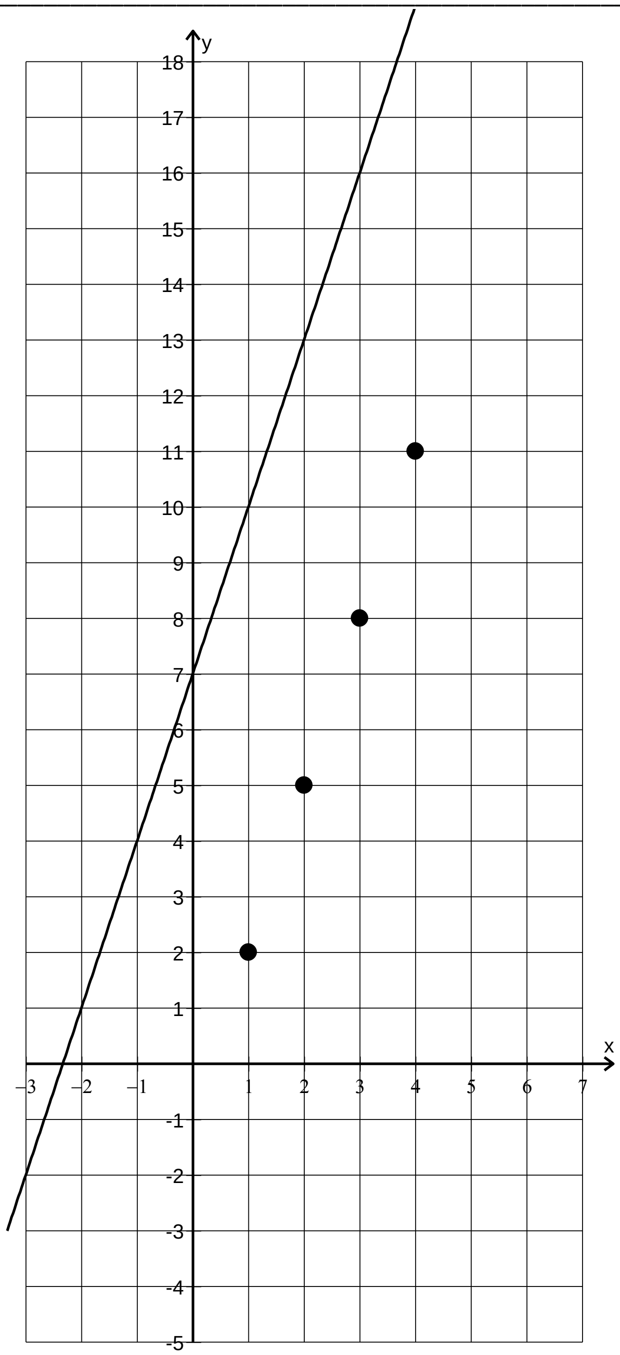
___________________________________________________________________________

Now, you may have had trouble finding a fourth ordered pair. This is because there are only three other points on this part of the CP that have 'nice' values; i.e. integer values for 'x' and 'y'.

**Task 6:** Can you come up with four more ordered pairs of points that lie on this line that don't have 'nice' x and y values? The CP below with the line drawn on it might help. Try it:

___________________________________________________________________________

___________________________________________________________________________



**Equations**

Remember that this all began with the set of numbers: 2, 5, 8, 11, ____, ____, ____ from which we formed the table:

| | | | | |
|---|---|---|---|---|
| n | 1 | 2 | 3 | 4 |
| $t_n$ | 2 | 5 | 8 | 11 |

We then relabelled the 'n' values as 'x' and the '$t_n$' values as 'y' and formed the set of ordered pairs: $\{(1,2), (2,5), (3,8), (4,11)\}$ which we then plotted as points on the CP.

Now, using our 'x' and 'y' labels, the rule that we 'saw', i.e. $t_n = 3n - 1$ can instead be written as $\mathrm{y} = 3\mathrm{x} - 1$.

$\mathrm{y} = 3\mathrm{x} - 1$ is an example of an equation or the 'name' that we give to the line that passes through the set of points.

It is extremely important that you understand that:

> An equation summarizes the pattern or relationship between the 'x' and 'y' values of the ordered pairs that lie on a line (or some special curves).

Much earlier we asked the questions:

1. what is the value of $t_{17}$,
2. what is the value of $t_{124}$, and
3. is 231 the value of one of the terms in this pattern?

Each of these questions can now be reworded. They become: If $\mathrm{y} = 3\mathrm{x} - 1$:

1. what is the value of 'y' when 'x = 17',
2. what is the value of 'y' when 'x = 124', &
3. what must the value of 'x' be for 'y' to equal 231?

The answers to these become quite simple.

1. If x = 17 then $\mathrm{y} = 3 \times 17 - 1 = 50$
2. If x = 124 then $\mathrm{y} = 3 \times 124 - 1 = 371$
3. This one becomes: find the value of 'x' such that 'y = 231'. This means we have to solve the equation: $231 = 3\mathrm{x} - 1$. If we do this we get x = 77.333…

Notice that this is not a 'nice' answer, i.e. an integer answer, but it is still a valid answer. Also, notice that none of these answers can be 'seen' on our picture of the line but if we could extrapolate (i.e. stretch out the line) we would eventually reach these points.

The answer, x = 77.333… would not be a valid answer in the context of the original question because there we were only 'allowed' integer values of 'x' (i.e. 'n') but in the context of points that lie on the line x = 77.333… is a valid answer.

**Task 7:** Using $y = 3x - 1$ one last time, determine either the 'x' or 'y' value as asked:

1. Find the value of $y$ given that $x = 1.5$

____________________________________________________________________________

2. Find the value of $x$ given that $y = 32$

____________________________________________________________________________

3. Find the value of $y$ given that $x = -6.2$

____________________________________________________________________________

4. Find the value of $x$ given that $y = -19$

____________________________________________________________________________

5. Write down that coordinates of FOUR other ordered pairs of points that would lie on this line (be daring and creative!):

____________________________________________________________________________

<u>But hang on…</u>

"You still haven't told me how you got the rule $t_n = 3n - 1$ or $y = 3x - 1$." That is next…

**<u>The Equations of Lines or Linear Relationships</u>**

We saw above that the pattern for the set of ordered pairs $\{(1,2), (2,5), (3,8), (4,11)\}$ is summarised by the equation of the line that passes through these points, $y = 3x - 1$. But, other than 'it works', how do we come up with this or similar equations?

Firstly, we need to understand that:

> If a set of points are collinear, then the equation of the line that passes through them can always be written in the form:
>
> $$y = mx + c$$
>
> where: '$m$' and '$c$' are fixed numbers called <u>constants</u>.

Note that 'x' and 'y' are <u>not</u> constant, that is they don't have 'fixed' values. 'x' and 'y' vary (change in value) and so we call them <u>variables</u>.

Let's see if we can plot other sets of ordered pairs and deduce what the meaning of the constants 'm' and 'c' might be.

To do this investigation we will use six sets of ordered pairs and then plot various pairs of them on the one Cartesian Plane. The sets of ordered pairs we will use are:

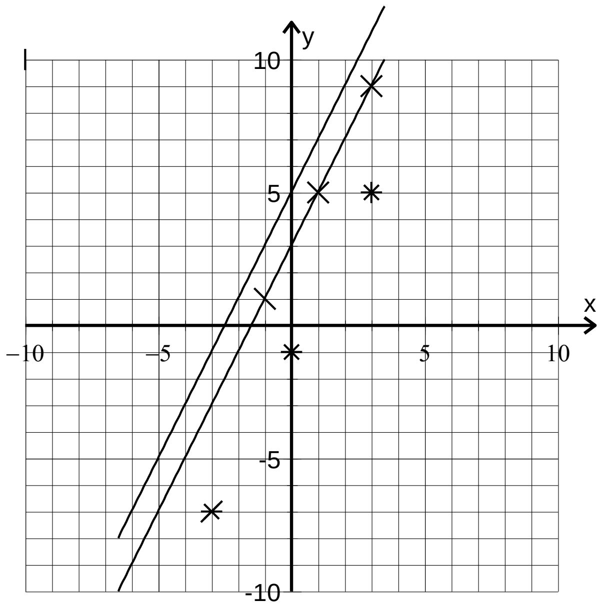
Set A: $(-1,1), (1,5)$ & $(3,9)$
Set B: $(-3,-7), (0,-1)$ & $(3,5)$
Set C: $(-2,-7), (1,2)$ & $(3,8)$
Set D: $(-3,-4), (-1,2)$ & $(1,8)$
Set E: $(-1,7), (2,1)$ & $(6,-7)$
Set F: $(-3,9), (0,3)$ & $(3,-3)$

**Task 8:** Before we begin, can you find a pattern in each of the sets of ordered pairs above? That is, what do you do to the 'x' values to get the 'y' values in each set expressing your answer in the form: $y = mx + c$.

This is what I call "crystal ball gazing" and those who know their number facts and can do mental arithmetic well are better at doing this. Hint: one of them you have seen before ( $y = 3x - 1$ ).

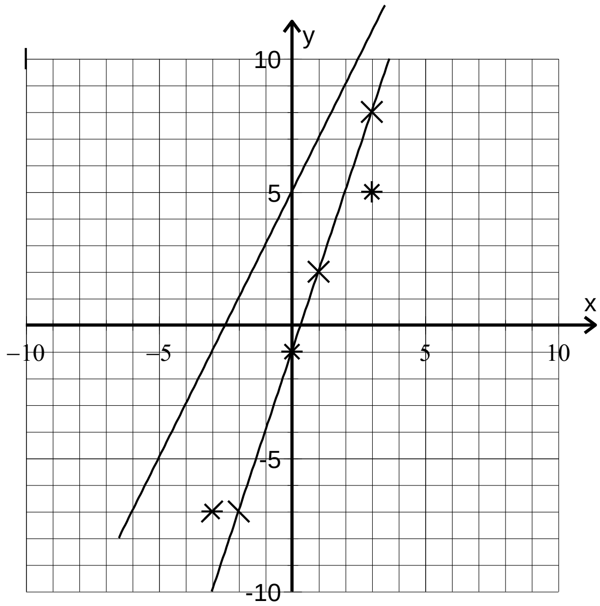
**The Meaning of 'm' and 'c'**

Let's begin by plotting Sets A and B and drawing a line through them. Doing this we get:



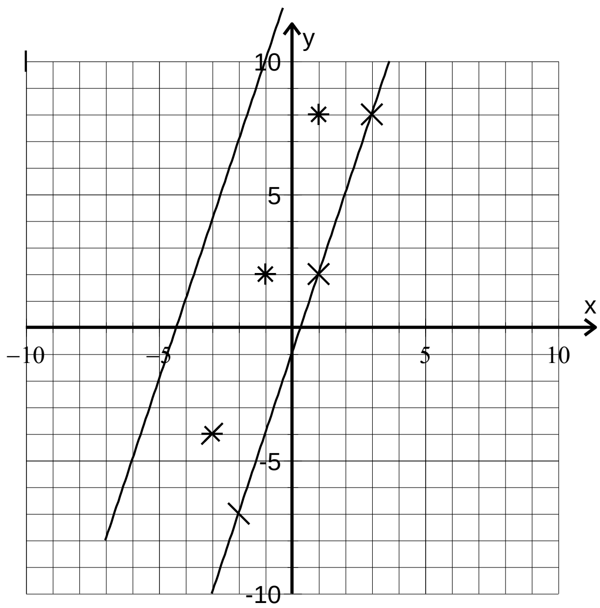

Notice that these lines are parallel which could suggest that maybe their equations will have something in common.

If we plot Sets B and C together we get:




This time the lines are not parallel but they do share the same y-axis intercept. That is, both lines cross the y-axis at the same point (0, -1).

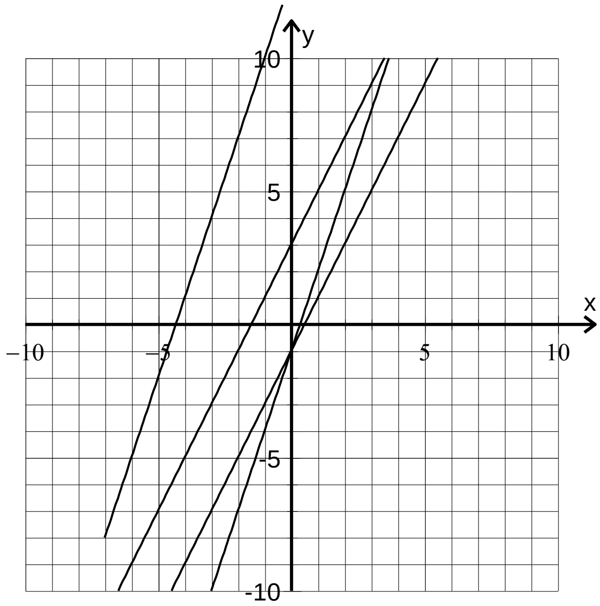
If we now plot Sets C and D together we get:




and again we see that these lines are parallel but notice that they are 'steeper' than the lines for Sets A and B. That is, they go up quicker.

If your crystal ball didn't work very well, these are the equation for the first four sets of ordered pairs:
Set A: $y = 2x + 3$
Set B: $y = 2x - 1$
Set C: $y = 3x - 1$
Set D: $y = 3x + 5$

Can you see the significance (i.e. the meaning) of the constants ‘m’ and ‘c’ from these equations and their graphs? To help you, here are all four of these lines draw on the one CP:

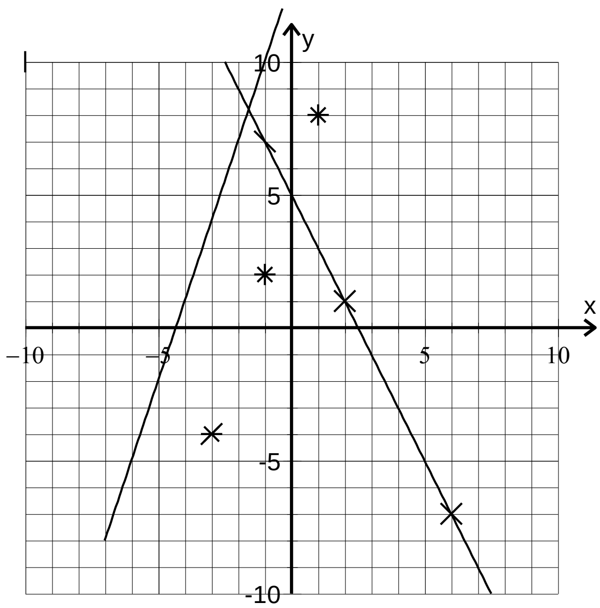



**<u>Task 9</u>:** Can you work out which line has which equation?

Hopefully you can see that:

> The ‘c’ in the equation $y = mx + c$ is the <u>y-axis intercept</u> of the line.

This might be clearer if we look at the potting of <u>Sets D and E</u>:



Notice that they have the same y-axis intercept of (0,5) but this time <u>the line for Set E goes down</u> (if we look from left-to-right).

Now, the equations of these lines are:

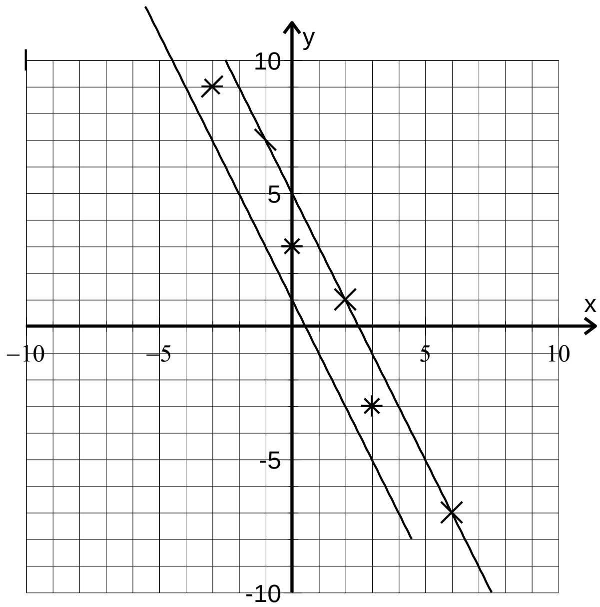
Set D: $y = 3x + 5$
Set E: $y = -2x + 5$

As expected they have the same constant 'c' as they share the same y-axis intercept but the 'm' for Set E is '-2'.

This would suggest that:

> The 'm' in the equation $y = mx + c$ is related to the steepness of the line and whether is 'goes up' or 'goes down' when looking from left-to-right.

As a final check, let's look at the plotting of <u>Sets E and F</u>:




You can see that they have different y-axis intercepts of (0,3) and (0,5) but they are parallel and both go down.

The equations of these lines are:

Set E: $y = -2x + 5$
Set F: $y = -2x + 3$

You should clearly see that our summaries in the boxes above are correct. The constant 'c' is clearly the y-axis intercept and the 'm' is related to the steepness and direction of the line.

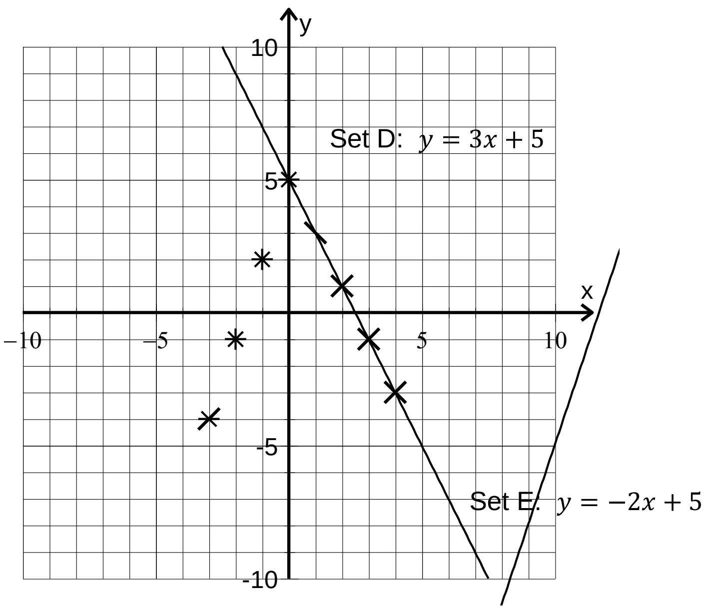
Now, if we have a line (or we draw a line through a given set of ordered pairs), we can very easily find the ‘c’ as we simply read it off the graph (i.e. where does the line cross the y-axis?), but how do we work out the value of the ‘m’?

Well, we can easily decide if it is positive or negative:

> The ‘m’ in the equation $y = mx + c$ is:
> $+'ve$ if the line goes up, and
> $-'ve$ if the line goes down

To work out the actual value of the constant ‘m’ let’s compare more closely the equations and graphs for the Sets D and E.

To help us do this I have marked particular points or ordered pairs on each line. REMEMBER, there are an infinite number of ordered pairs that fit these patterns – every point that is on the line!




As we know, the value of ‘c’ for both lines is 5 because the y-axis intercept for both lines is (0,5).

If we write a table for the ordered pairs for Set D we get:

| | | | | |
|---|---|---|---|---|
| x | -3 | -2 | -1 | 0 |
| y | -4 | -1 | 2 | 5 |

The thing to notice is that as the ‘x’ values increase by one (from -3 to -2 to -1 to 0) the ‘y’ values increase in increments of 3. This is why the value of the constant ‘m’ for Set D is +3. That is $m = 3$. Can you see this on the graph?

Similarly, if we write a table for the ordered pairs for Set E we get:

| x | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| y | 3 | 1 | -1 | -3 |

This time, as the 'x' values increase by one (from 1 to 2 to 3 to 4) the 'y' values decrease in increments of 2. This is why the value of the constant 'm' for Set E is -2. That is $m = -2$. Can you see this on the graph?

So we can summarize our finding as follows…

> The value of 'm' in $y = mx + c$ is how much the y-value changes as the x-value increases by 1. If the y-value:
>
> - increases the 'm' will be $+'ve$
> - decreases the 'm' will be $-'ve$

But, I still hear you asking, so…

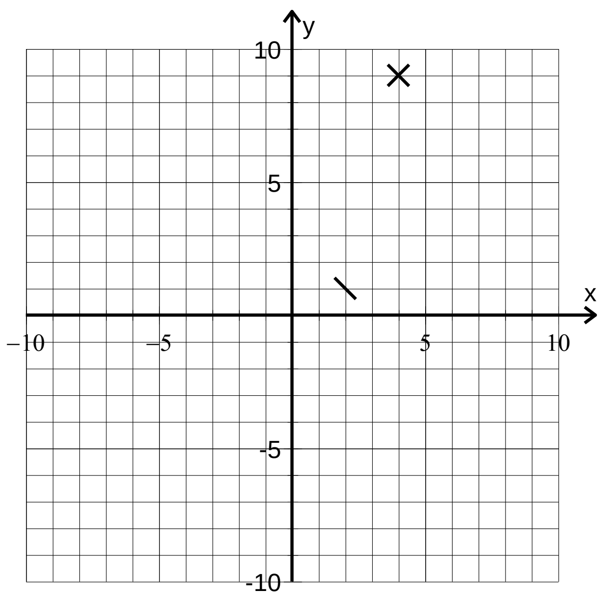
**How do we use this to 'work out' the equations?**

We will use two more sets of ordered pairs to illustrate.

**Task 10:** Can you 'see' the pattern in the following set of ordered pairs before you look ahead?

Set G: $(-3,-19)$, $(2,1)$, $(4,9)$ & $(7,21)$

So let's plot these points and (hopefully) draw a line through them:



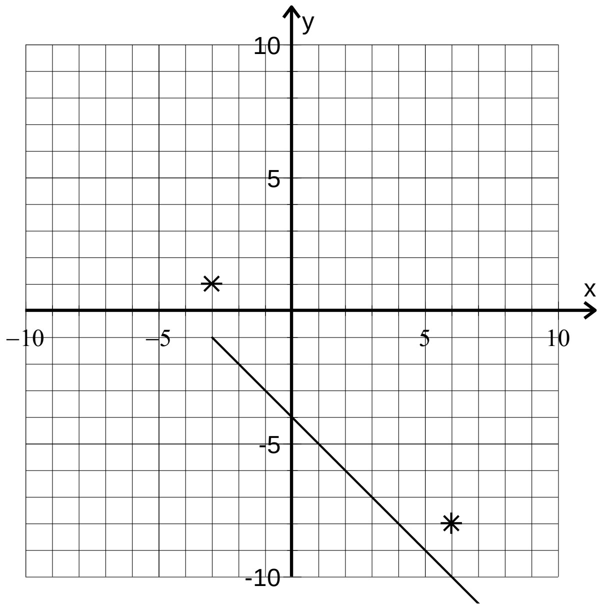

Notice that two of the points do not fit on this Cartesian Plane because of the scale we have used. We will have to trust that the other two points are collinear with these for now. We will be able to check.

It should be clear that the value of the constant 'c' must be -7 since the y-axis intercept of the line is (0,-7). To work out the 'm' look at the point (2,1). If we go across one unit from this point (i.e. increase 'x' by 1) what do we have to do to get back on the line? The answer is that we have to 'go up 4'. Therefore 'm' must equal 4. This tells us that the relationship for the Set G of ordered pairs or the equation of the line through them must be: $y = 4x - 7$. Did you get this?

Finally, let's look at Set H:

Set H: $(-11{,}9),\ (-3{,}1),\ (6,-8)\ \&\ (10,-12)$

Again we find that two of the points 'don't fit':




This time the 'c' constant must be -2 and the 'm' will have to be negative because the line is 'going down'. If we look at the point (-3,1) and do what we did for Set G we see that as we increase 'x' by one unit the y-value decreases by 1. This implies that m = -1.

So the relationship for the Set H of ordered pairs or the equation of the line through them must be: $y = -1x - 2$. Did you get this? Note: since the coefficient is '-1' we don't write the '1' but we must have the negative sign. So we would write this equation as: $y = -x - 2$.

To finish we need to check if the other points are actually on the lines. I will illustrate using the point (-3,-19) from Set G. That is, does the point (-3,-19) lie on the line whose equation is $y = 4x - 7$ ?

To check we <u>substitute</u> into the equation the x-value we are given (i.e. -3) and see if the y-value generated is -19.

So:

$$y = 4x - 7 = 4 \times -3 - 7 = -12 - 7 = -19$$

which is what we expected and so (-3,-19) is on the line.

# The General Case of the "Monty Hall" Problem

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Students who have seen the movie "21" may recall the "Monty Hall" problem as presented by the maths professor played by Kevin Spacey. The problem was originally created by Steve Selvin in 1975 and was based on a game show with a host named Monty Hall. Basically it can be described as follows:

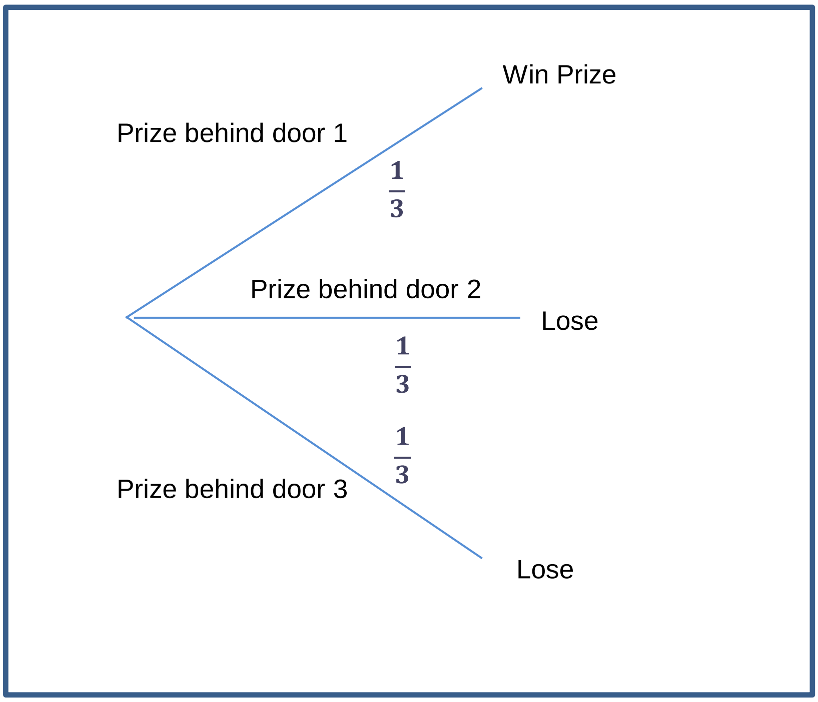
A contestant on a game show is given the choice of three doors to open. Behind one door is a prize; behind the other two doors is nothing. The contestant picks a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has nothing behind it. He then offers the contestant the option to switch to door No. 2. This is the problem: is it to the contestant's advantage to switch the choice from door 1 to door 2? The answer is "yes" although there were many who disagreed when the problem was first published in the journal American Statistician. The most common (erroneous) response is that it makes no difference which door is chosen, that it is equally likely that the prize is behind door 1 or door 2. The famous number theorist, Paul Erdös was reported to have been convinced that the prize was equally likely to be behind either door until he was shown a computer simulation demonstrating the correct solution.

There are several ways to solve the problem. Here are two of them:

1. Using the logical complement

The prize is equally likely to be behind any one of the three doors, therefore the probability that the prize is behind the door chosen by the contestant is $\frac{1}{3}$ and the probability that it is behind one of the doors **not** chosen is $\frac{2}{3}$. The number of doors is reduced by one when the host opens the door with no prize. If the contestant chooses to switch doors, there is only one left, therefore the probability that the prize is behind that door is $\frac{2}{3}$.

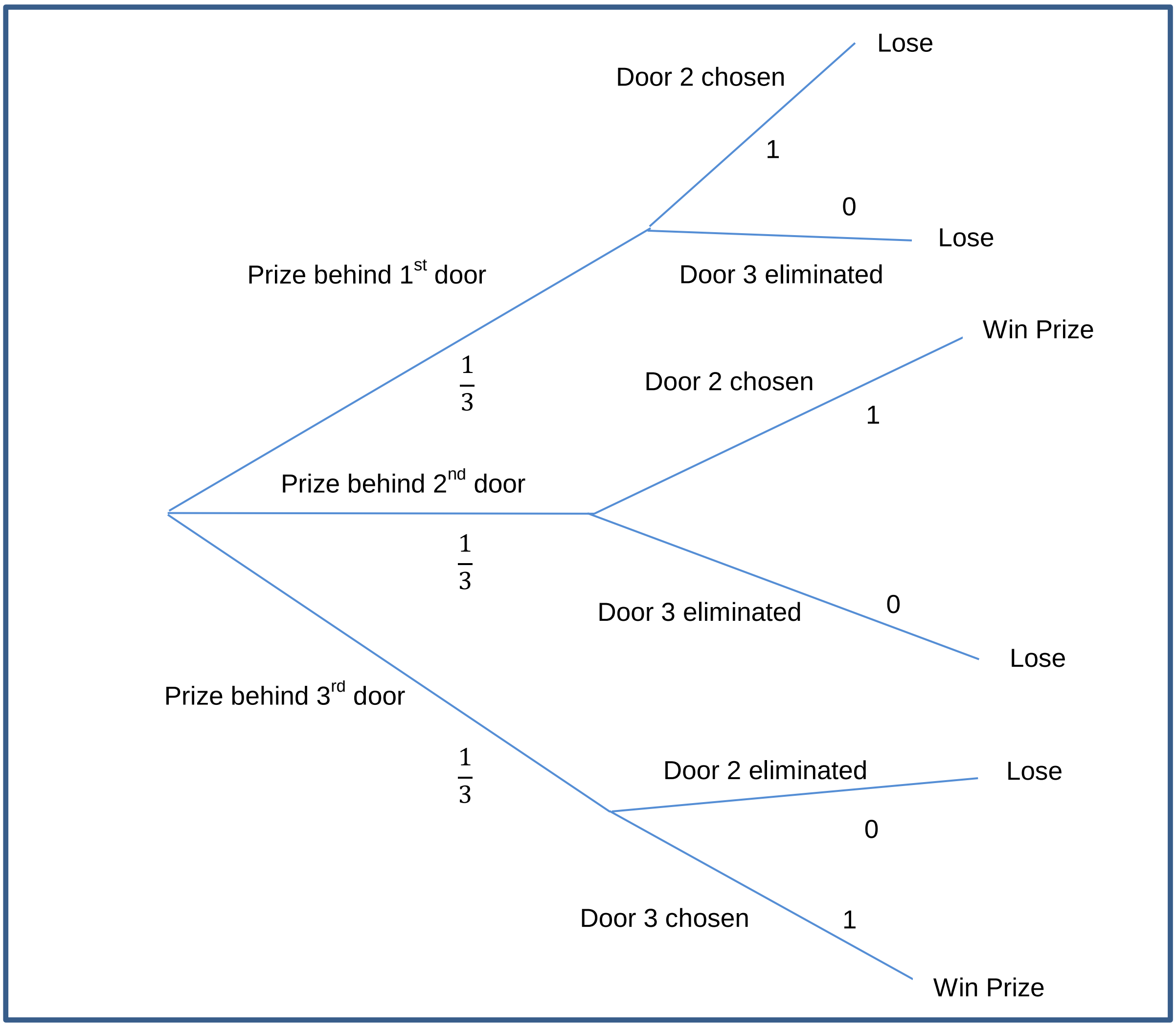
By Tree diagram
Condition 1: The contestant chooses door 1 and **does not** switch



The top branch of the tree diagram indicates that the probability of winning by choosing door 1 is $\frac{1}{3}$.

If the contestant chooses door 2 or 3 and does not switch, then the probability of winning is also $\frac{1}{3}$.

Condition 2: After the contestant chooses door 1, Monty opens one door showing no prize and the **contestant switches** their choice to the only remaining unopened door.




According to the tree diagram, the probability of winning the prize if the choice is switched from door one to door 2 or 3 is P( win|door is switched ) = $\frac{1}{3} \times 1 + \frac{1}{3} \times 1 = \frac{2}{3}$

Therefore the contestant is twice as likely to win if they switch their choice to the other door.

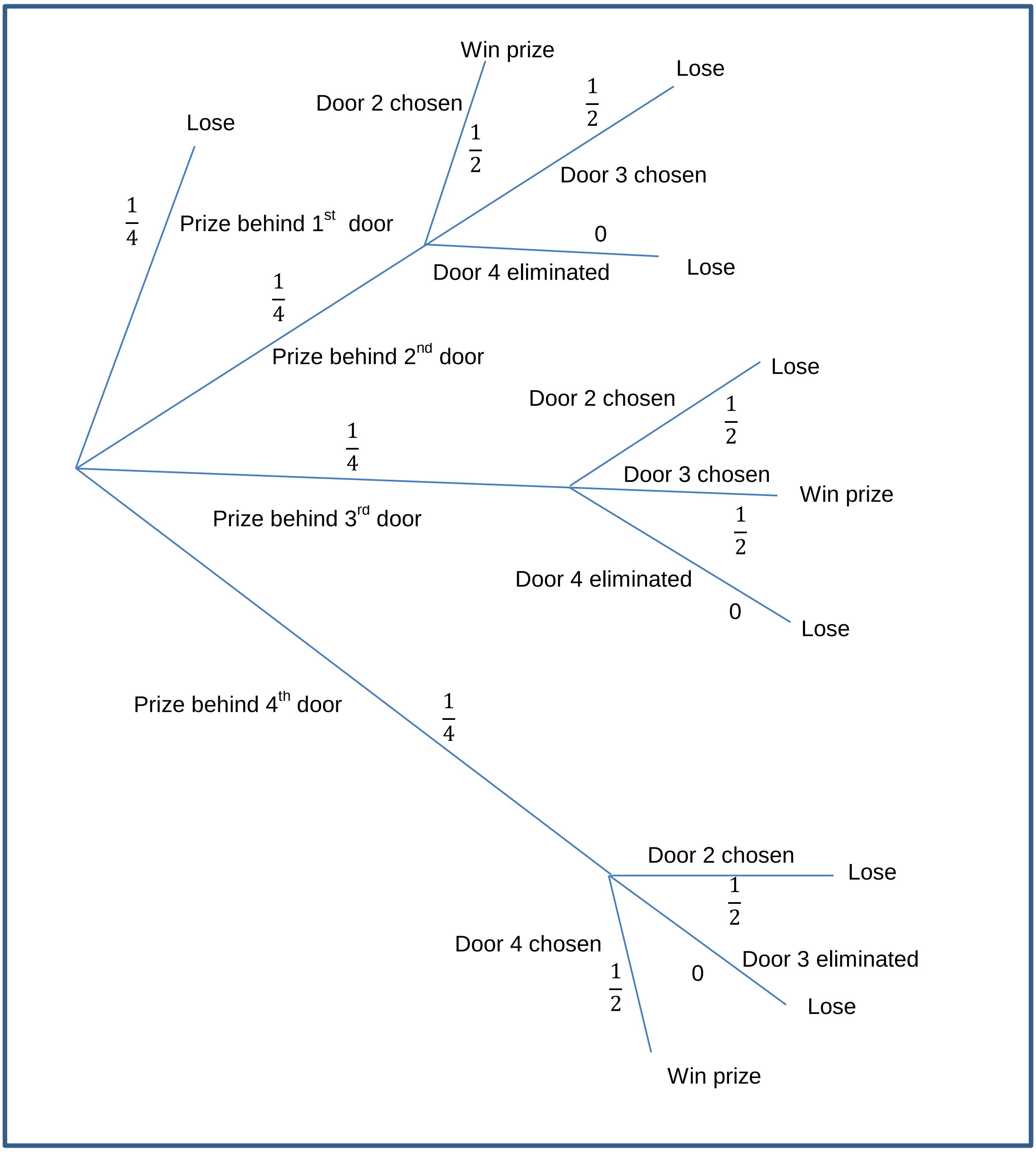
The natural extension of this problem is to apply it to the general case for n doors .

We begin by letting n=4 .

Condition 1 : The contestant **does not change** their choice of doors

The probability of winning is $\frac{1}{4}$

Condition 2 : The contestant chooses a door, say door 1, and after Monty opens a door without a prize, switches their choice to one of the 2 remaining closed doors.




According to the tree diagram, the probability of winning the prize if the choice is switched from the first door to one of the two remaining doors is

P( win|door is switched ) = $\frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} = \frac{3}{8}$

The contestant is therefore 50% more likely to win if they abandon their first choice and chose one of the two remaining doors. This advantage is diminished each time a door is added to the game.

For n = k doors we have

<u>Condition 1</u> : The contestant **does not change** their choice of doors then P(win) = $\frac{1}{k}$

<u>Condition 2</u> : The contestant **changes** their choice to one of the other unopened doors

$$P(win|door\ is\ switched) = (k-1)(\frac{1}{k})(\frac{1}{(k-2)} = \frac{(k-1)}{k(k-2)}$$

The advantage of switching doors expressed as a ratio ( switched to not switched ) is

$$\frac{(k-1)}{k(k-2)} : \frac{1}{k}$$

Clearly, this advantage is lost when k becomes very large, i.e. the limit of the ratios as k→∞ is one to one.

An extension of this topic could be a project for Maths C students to find a formula for the probability of winning when ***n*** doors are opened by Monty out of the original ***k*** doors.

# The Pole Vault and Usain Bolt

**Stephen Broderick**
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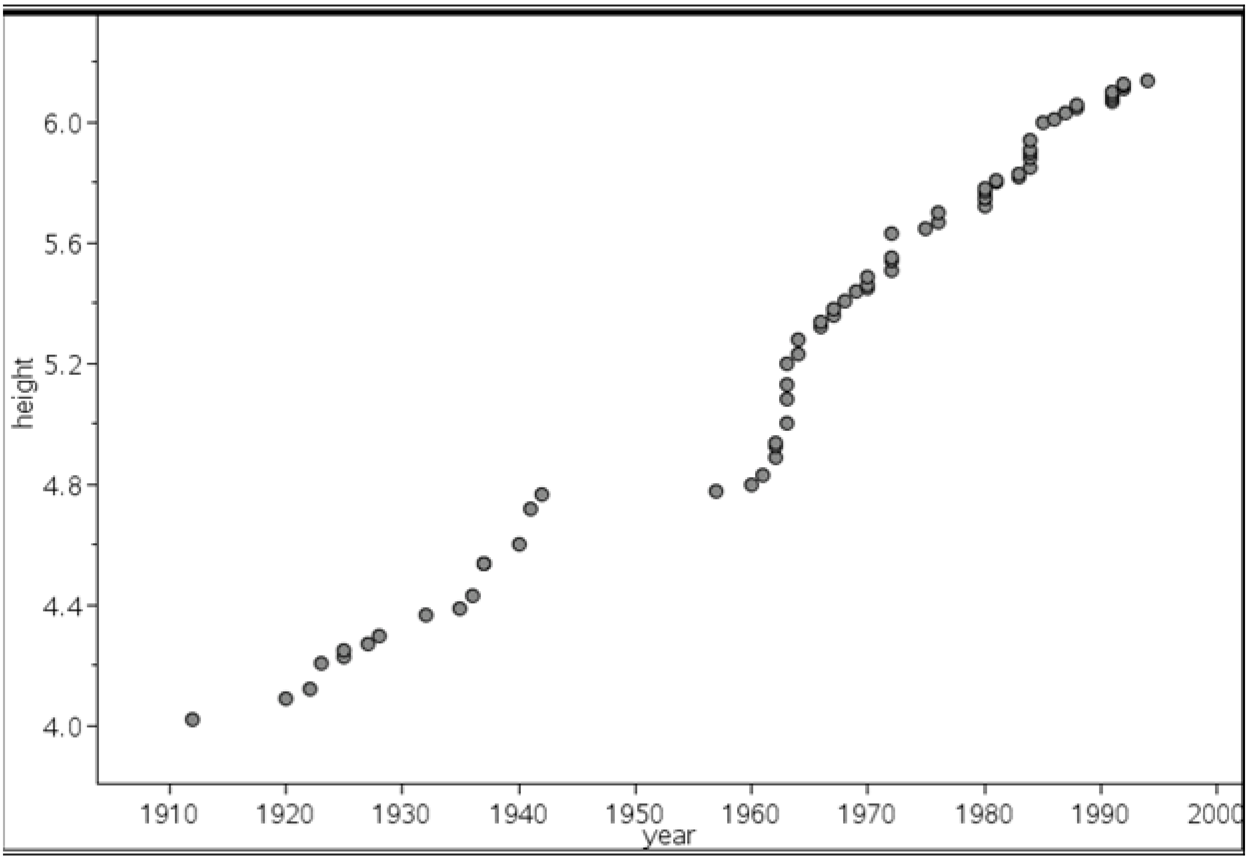
To qualify for the London Olympics, pole vaulter Stephen Hooker must first clear 5.72 metres, which is the A-qualifying height. Hooker, the current world indoor and outdoor Continental Cup, Olympic and Commonwealth champion has battled problems since withdrawing from a qualifying competition in February this year. Fortunately he qualified on Saturday 12th May at his second attempt in Perth and will now defend his Olympic crown. The current world record stands at 6.14 metres and was set by Ukrainian Sergey Bubka on 31st July 1994.
The graph below displays the record heights (in metres) and the corresponding year for the record.




The graph has two distinct regions- (1910 to 1940) and (1955-1994). [The gap in the middle represents World War II] These regions correspond to the construction of the pole which was originally made from solid wood. Then in 1910 bamboo was used until the 1960's, after which glass fibre was used and finally carbon fibre poles were introduced in the 1990's. In fact during the 1960's there was a marked improvement in record heights. The IAAF regulations for the construction and length of the pole are as follows:

"The pole may be of any material or combination of materials and of any length or diameter, but the basic surface must be smooth"

What is the main factor which results in a record height? Will using a long pole or a light pole help? Is the speed of the vaulter more important?

In terms of physics the sprint run-up of the vaulter produces kinetic energy which is ultimately converted into potential energy. The vaulter's pole allows the maximum possible momentum to be converted to upward acceleration. Not all of the energy of the run up is converted to vertical acceleration; some energy is lost to friction and the bending of the pole. The conversion of kinetic energy to potential energy is summarised mathematically as:

$$\frac{1}{2}\, m\, v^2 = mgh$$

where m = mass in kilograms,
v = velocity in metres/second
g = acceleration due to gravity (9.81 m/sec$^2$)
h = height in metres

After rearranging the formula and cancelling out mass (m), it is clear that the height (h) depends only on the velocity (v) of the vaulter and the force of gravity. The mass is not a factor.

h = $\frac{v^2}{2g}$

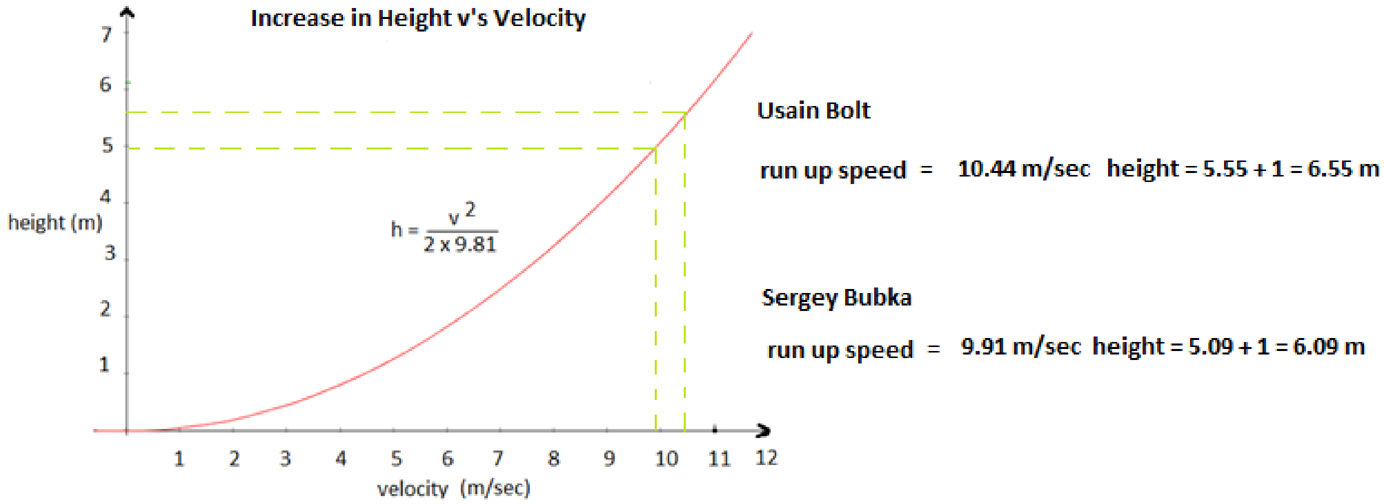
Since gravity is a constant, the main factor contributing to the height achieved by the vaulter is $v^2$, the run up speed of the vaulter. Increasing the length of the pole will result in more weight to carry and will ultimately lead to a reduction in speed and height. In fact a 1% increase in speed will result in a 2% increase in height due to the $v^2$.

So the introduction of glass fibre poles in the 1960's allowed for a faster run-up (since they were lighter than bamboo) and glass fibre poles were engineered to bend under the load of the athlete-storing elastic strain energy. The bending allowed athletes to change vaulting technique, going over the bar with their feet upwards.

Suppose a vaulter on their approach can run at 10 metres/second; then the increase in height will be 5.097 metres ($\frac{10^2}{2x9.91}$ = 5.097). Since the centre of mass of the vaulter is approximately one metre above the ground, then the total height reached will be 6.097 metres.

Usain Bolt who holds the 100 metre world record at 9.58 seconds would achieve an increase in height of 5.55 metres, resulting in a total height of 6.55 metres.

The graph below represents the increase in height versus velocity for a pole vaulter. Sergey Bubka's world record of 6.14 metres is indicated on the graph as an increase in height of 5.14 metres (since his centre of gravity is approximately one metre above the ground) and based on the intersection of his record and the equation for height, his required velocity is approximately 10 m/sec. In fact video evidence indicates that Sergey does reach this speed in his last couple of strides.




Triple Olympic champion Usain Bolt wants to amaze the world at the London Games by running 9.4 seconds for the 100 m. He holds the record at 9.58 seconds for the 100 m. He has told the BBC he wants to dominate the Games and become a living legend.

The table below represents the winners of the 100 metres from 1896 to 2008. Usain Bolt's world record time (9.58 seconds) set in 2009 has also been included for a better prediction.

| Year | Winner | Time (seconds) |
|---|---|---|
| 1896 | Thomas Burke (US) | 12 |
| 1900 | Francis Jarvis (US) | 11 |
| 1904 | Archie Hahn (US) | 11 |
| 1906 | Archie Hahn (US) | 11.2 |
| 1908 | Reginald Walker ( S Africa) | 10.8 |
| 1912 | Ralph Craig (US) | 10.8 |
| 1920 | Charles Paddock (US) | 10.8 |
| 1924 | Harold Abrahams (UK) | 10.6 |
| 1928 | Percy Williams (Canada) | 10.8 |
| 1932 | Eddie Tolan (US) | 10.38 |
| 1936 | Jessie Owens (US) | 10.3 |
| 1948 | Harrison Dillard (US) | 10.3 |
| 1952 | Lindy Remigino (US) | 10.78 |
| 1956 | Bobby Morrow (US) | 10.62 |
| 1960 | Armin Hary (FRG) | 10.32 |
| 1964 | Robert Hayes (US) | 10.06 |
| 1968 | James Hines (US) | 9.95 |
| 1972 | Valeriy Borzov (USSR) | 10.14 |
| 1976 | Hasely Crawford (Trinidad) | 10.06 |
| 1980 | Allan Wells (UK) | 10.25 |
| 1984 | Carl Lewis (US) | 9.99 |
| 1988 | Carl Lewis (US) | 9.92 |
| 1992 | Linford Christie (UK) | 9.96 |
| 1996 | Donovan Bailey (Canada) | 9.84 |
| 2000 | "Maurice Green (US) | 9.87 |
| 2004 | Justin Gatlin (US) | 9.85 |
| 2008 | Usain Bolt (Jam) | 9.69 |
| 2009 | Usain Bolt (Jam) | 9.58 |

A linear model of the data can be used to predict the winning time(hopefully Bolt wins) for the 100 metres in 2012. (See graph below)

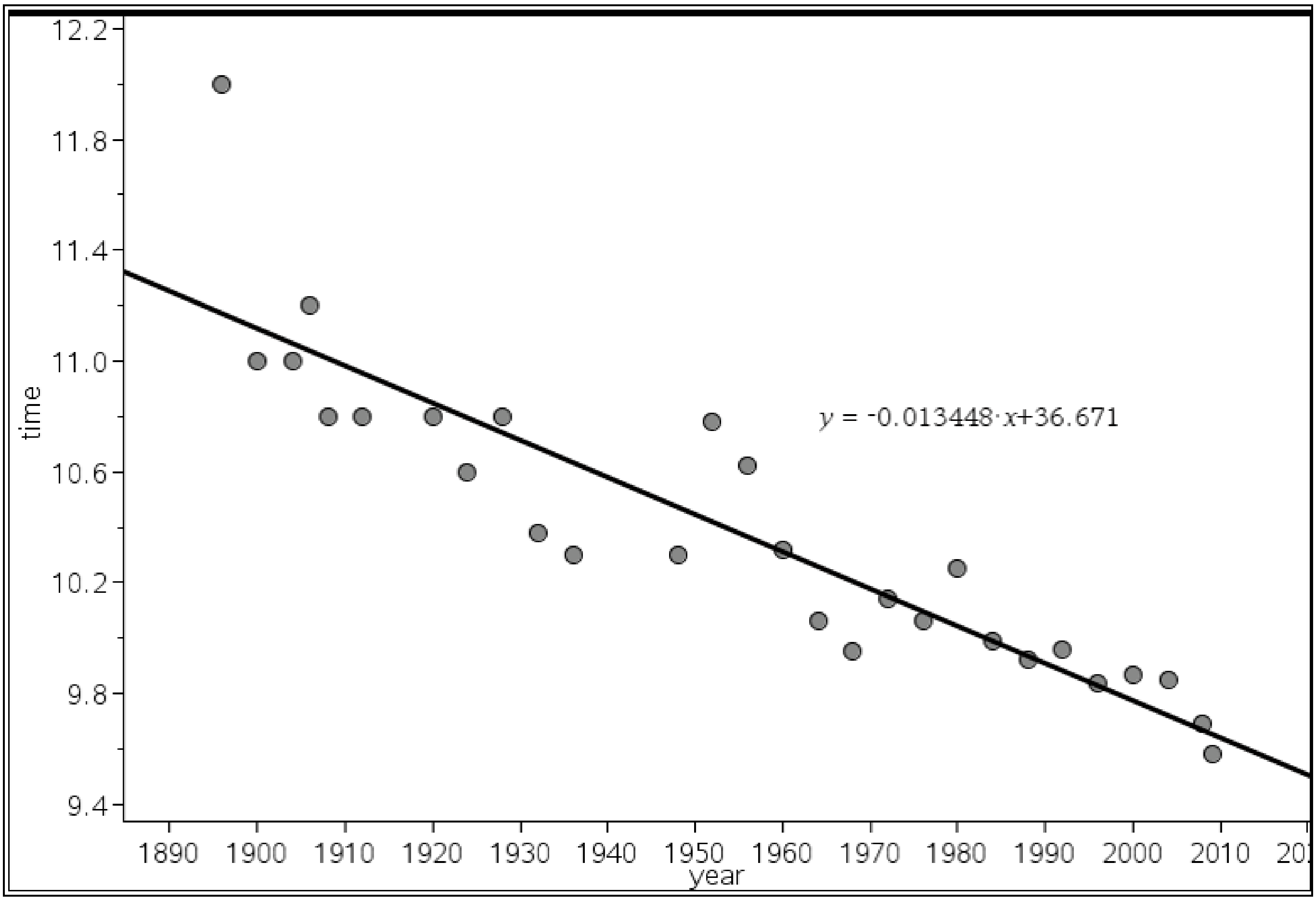



Fig 1

Time = -0.013448 × year + 36.671
= -0.013448 × 2012 + 36.671
= 9.61 seconds

Based on the linear model, a time of 9.4 seconds won't occur until 2028.

**References**


Farndon. J *Do You Think You're Clever* (2009)

# QAMT prize for Best Practical Teaching Article and the Best Research Article

The Queensland Association of Mathematics' Teachers is running a writing competition in order to recognise the outstanding talents of teachers of mathematics and mathematics education post-graduate students.

QAMT is looking for articles of between 2 and 4 pages that address one of the following topics at any level of schooling from P-12. In recognition of the importance of linking research and practice, categories from both fields are being sought:

**Practical Teaching Categories**

1. **Practical Teaching Ideas**: Held a lesson that went really well? Found a great way of teaching a concept? Know how to get students reasoning in class? Have a great warm-up idea for the start of a lesson? Then share it with us. These articles should contain enough information and/or resources, diagrams etc. to enable readers to replicate the activity.

2. **Differentiating Mathematics in the Classroom**: Differentiation is a practice that must be carried out in every classroom. In reality, we know that it can be tough meeting the needs of a cross section of abilities and developing understandings. That is why we are looking for practical ideas from the classroom. If you have ways of extending or broadening the curriculum or assisting those who might find the curriculum challenging, share them with your fellow teachers.

**Research Categories**

3. **Trends in International Mathematics Education**: With a number of international studies comparing (and ranking) the success of Australian students and their international counterparts, there is increased interest in mathematics teaching outside Australia. If you have conducted research, or have practical experience, in international settings, this is a great opportunity to share them with an interested audience.

4. **Research/Theoretical Paper**: Have you been conducting research or a review of literature in a field associated with mathematics education? If so, then you have something to share with the mathematics teaching profession, and this is your chance to get it published and into the teaching community.

**Prizes**

The author of every article selected for publication will receive a fully paid annual membership to the Queensland Association of Mathematics Teachers (valued at $95) along with the Quarterly Journal, conference discounts and other benefits.

In addition, two prizes will be awarded of $200 to spend on mathematics teaching resources of the author's choice from the Australian Association of Mathematics' Teachers. These prizes will be for the Best Practical Teaching Article and the Best Research Article.

**Entry**

Entries must be received at QAMT by 31 August 2012. Publication of articles will be progressive so publication may occur before that date. Further terms and conditions are available on the submission form and all entries must have a signed submission form attached. This form can be downloaded from the QAMT website www.qamt.org. Please email all entries to qamt@uq.edu.au. Submission forms can be scanned and emailed also or faxed to: 07 3365 6505.

If you have any questions regarding entry, please contact Jill Wells on j.wells2@uq.edu.au.

## QAMT prize for Best Practical Teaching Article and the Best Research Article

**Submission Form**

Article Title: ____________________________________________________________

Author's Name (Title, First, Last): ___________________________________________

Author's Affiliation (School, University etc.). Please leave blank if you do not wish this information to be published: _______________________________________________

Author's Email address (not for publication): _______________________________________________

Category for submission (check one only):

- ☐ Practical Teaching Ideas [eligible for Best Practical Teaching Article]
- ☐ Differentiating Mathematics in the Classroom [eligible for Best Practical Teaching Article]
- ☐ Trends in International Mathematics Education [eligible for Best Research Article]
- ☐ Research/Theoretical Paper [eligible for Best Research Article]

**Terms and Conditions:**

1. Articles should be between 2 and 4 pages when using 'normal' margins and Arial size 11 font.
2. Entries must be received at QAMT by 31 August 2012.
3. Entries will not be accepted without a signed submission form attached.
4. Submission of the article gives QAMT the right to publish the article in QAMT's quarterly journal with appropriate acknowledgement of the author and the author's affiliation (unless otherwise indicated above).
5. Publication of articles will be progressive and publication of some articles may occur before the closing date.
6. The QAMT Executive's decision regarding the awarding of prizes is final.
7. All articles must be appropriately referenced, with ideas and work of others acknowledged.
8. Members of the QAMT Executive are ineligible to enter.
9. Two prizes of $200 of resources from the Australian Association of Mathematics' Teachers (to be selected by the winner) will be awarded: one to the best paper from the category of Best Practical Teaching Article and one to the best paper from the category of Best Research Article. Paper's published in the QAMT journal *Teaching Mathematics* will receive a fully paid annual membership to the Queensland Association of Mathematics Teachers (valued at $95).

**Author's Declaration:**

I have read the Terms and Conditions contained herein and agree to comply by these. The work which I have submitted is solely my own unless otherwise acknowledged in the document.

Signed: _______________________________________________________ Date: ________________________

Answers to Student Problems

## Volume 37, Number 1

1.

$$\bar{x} = \frac{\sum x}{n}$$
$$2.38 = \frac{\sum x}{20}$$
$$\sum x = 47.6$$

Total of the scores must be reduced. Hence, $\sum x = 47.6 - (9.56 - 5.69)$

$$\sum x = 43.73$$

$$\bar{x} = \frac{\sum x}{n}$$
$$= \frac{43.73}{20}$$
$$= 2.1865$$

2.

$$x = \sqrt{2 - x^2}$$
$$y^2 = 2 - x^2$$
$$x^2 = 2 - y^2$$
$$x = \sqrt{2 - y^2}$$

3.

a + b = 5 (1) b + c = 6 (2) a + c = 7 (3)

Rearrange

b = 5 - a c = 7 - a

Substitute into (2)

5 - a + 7 - a =6

12 - 2a = 6

-2a = -6

a = 3

Substitute into

b = 5 - 3 c = 7 - 3

b = 2 c = 4

Solutions are a = 3 , b = 2 , c = 4

4.

$$\frac{2x + 1}{x + 7} = -4$$
$$2x + 1 = -4(x + 7)$$
$$2x + 1 = -4x - 28$$
$$6x + 1 = -28$$
$$6x = -29$$
$$x = \frac{-29}{6}$$

5. Let the angles be x, y and z. x is the smallest angle and z the largest angle.

$x = \frac{y}{2}$ $x = \frac{z}{3}$

y = 2x z = 3x

Sum of angles = 180°

x + y + z = 180°

x + 2x + 3x = 180°

6x = 180°

x = 30°

Hence, y = 2 × 30° = 60° z = 3 × 30° = 90°

The angles are 30° , 60° and 90°

## Entries

Solutions for the student Problems were submitted by Brisbane Boys' College, Kepnock SHS, King's Christian College, Marist College Ashgrove, St Joseph's College Gregory Terrace, St Luke's Anglican College, St Paul's School (Bald Hills), Tamborine Mountain College,

## Winners

Congratulations are extended to Matthew Rose of Brisbane Boys' College and Mary Mineo of Tamborine Mountain College.

They received prizes provided by our generous sponsor, The University of Queensland.

## Submitting Solutions

Students are invited to submit solutions to the Student Problems.

Please photocopy the problem page and clearly print your name, your school, and your year level.

Write your solutions (with working) next to each question, or by filling in the appropriate boxes.

Send your solutions to:

"QAMT Student Problems"
C/- Rodney Anderson
Moreton Bay College
PO Box 84
WYNNUM QLD 4178

Closing date is 1st August, 2012

Student Problems

**Name: ........................................................ School: ........................................................ Year: ...........**

Write your solutions (with working) next to each question or on a separate sheet of paper or by filling in the appropriate boxes.

A pdf copy of the student problems is on www.qamt.org/resources.

**Question 1.** If $a - b = 3$, evaluate $a^2 - 2ab + b^2$

**Question 2.** Complete the magic square

| | | |
|---|---|---|
| x + 1 | | |
| x - 5 | x - 2 | |
| x - 3 | | |

**Question 3.** If the following pattern continues,

STUDENTPROBLEMSSTUDENTPROBLEMSSTUDENTPROBLEMS…

what will be the 2 653$^{rd}$ letter in the pattern?

**Question 4.** Two friends, who live 4km apart, arrange to meet. They both leave home at the same time, one walking at 6 km/h while the other runs at an average speed of 14 km/h. After how many minutes will the two friends meet?

**Question 5.** Emily spent one quarter of her allowance at the school tuckshop purchasing lunch Monday and two thirds of the remainder at the bookstore on Wednesday. If she had $4.50 left in her wallet, how much is Emily's weekly allowance?

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