TEACHING MATHEMATICS

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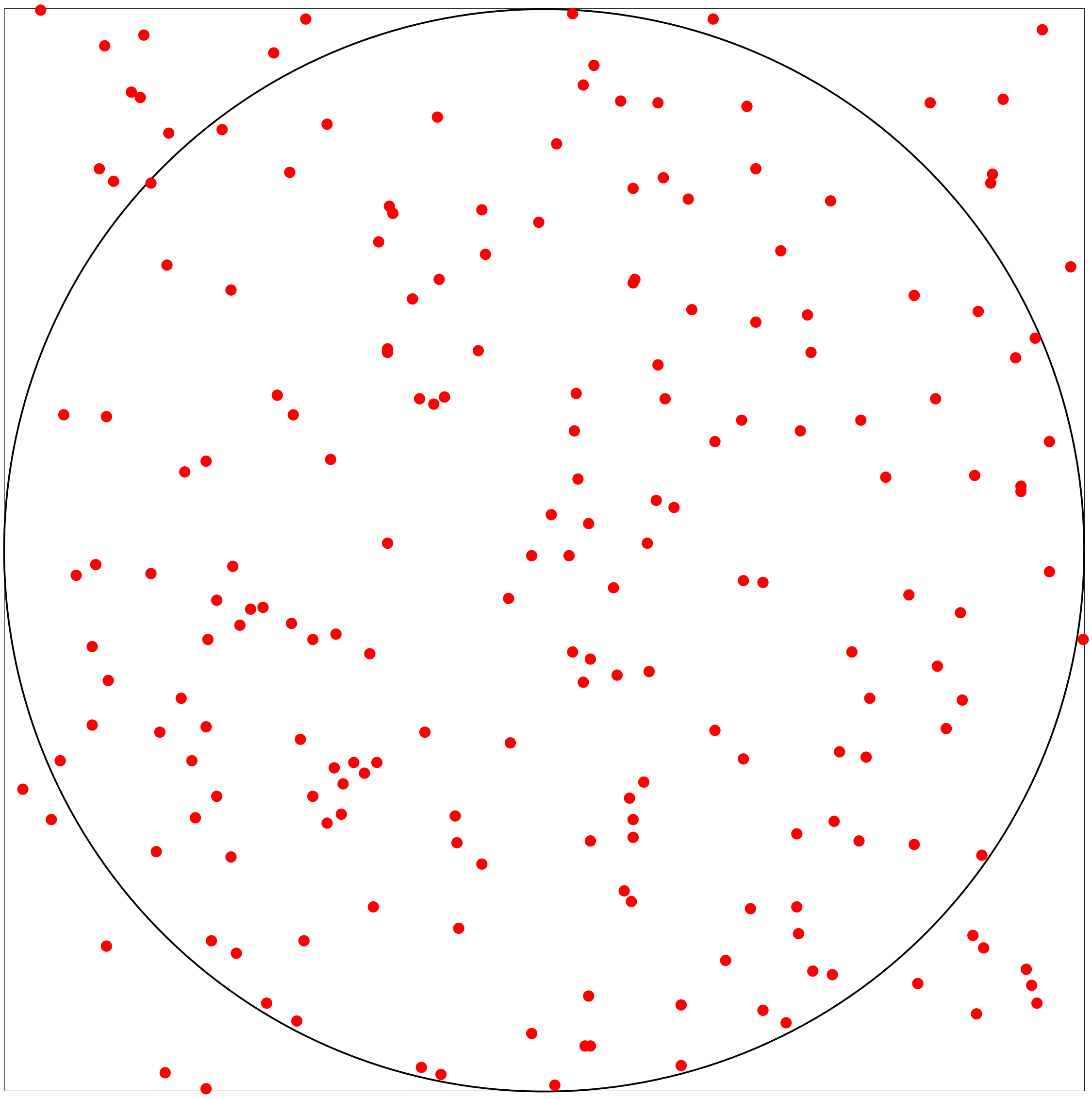

The Journal of the Queensland Association of Mathematics Teachers Inc

June 2016 Volume 41 No. 2

TEACHING MATHEMATICS

Regulars

Features

Front Cover

The cover shows 200 points that have been randomly generated in a square, similar to the activity on Page 27. How many points lie within the circle shown? Since we know the ratio of the area of the circle to the area of the square, we can use this random process to estimate a value of pi. What value do you get here? Are you convinced by it?

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Teaching Mathematics is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, September and November) and is provided as a right to all members of the association.

The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- to speak and act on matters related to mathematical education.

Advertising material published in and accompanying this journal does not imply endorsement by the association.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the Editor, Rodney Anderson. The preferred way is by email. Contact details are as follows:

Rodney Anderson
Moreton Bay College

Phone: 07 3390 8555
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Microsoft Word is the preferred format. All receipts will be acknowledged - if you haven't heard within a week, e-mail Rodney to check. Copy dates are: mid-February; mid-May; mid-August; mid-October.

The views expressed in articles contained in Teaching Mathematics belong to the respective authors and do not necessarily correspond to the views and opinions of the Queensland Association of Mathematics Teachers.

If you have any questions regarding Teaching Mathematics, contact Rodney Anderson. Publications sub-committee members are listed below. Feel free to contact any of these concerning other publication matters.

Rodney Anderson (Convenor)	Moreton Bay College	07 3390 8555
Gaynor Johnson (Newsletter)	QAMT Office	07 3365 6505

Books, software etc. for review should be sent to Rodney Anderson; information to go in the newsletter should be sent to Gaynor Johnson at the QAMT Office. Newsletter copy dates are the beginning of each term.

Contact the QAMT office for advertising enquiries.

Advertising Rates	1 Issue	2 Issues	3 Issues	4 Issues
Quarter page	$44	$88	$110	$132
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Full page	$110	$220	$308	$396
Insert (single A4 or folded A3)	$220	$440	$660	$770

QAMT also now offers a colour advertising service for the covers.

From the President

Rodney Anderson

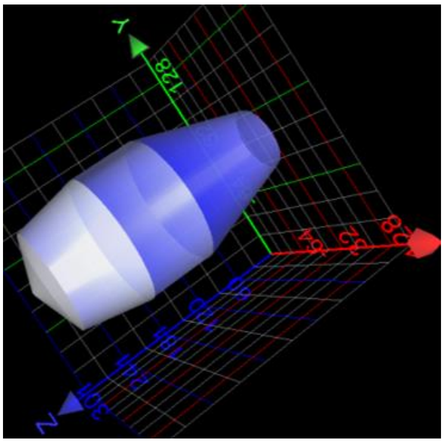
By the time you read this the trial of marking for the Mathematics B external exam has finished. If you were a marker, the members would be interested in your feedback on the process. Also, if your students were part of the trial, feedback would be also welcomed.

Mathematics Talent Quest – Hosted by Mathematical Association of Victoria

Three Queensland students won prizes in the 2015 competition. They include Clint Therakam (Queensland Academy for Science, Mathematics and Technology), Jed Hoo (Craigslea SHS) and Min Hoo (All Hallows School).

Clint has submitted a report of his entry and is on page 7.

Pursuing my project in integration was quite a challenge, yet fascinating and a unique learning experience at the same time. In the past, I was always amazed by the fact that we can easily measure the "area under the curve" of complex functions. However, I was even more astounded by the idea of integration in more than just two dimensions and through this appreciation came about my project involving 2D and 3D integration in various scenarios involving fluids. The Maths Talent Quest itself, provides students the freedom to explore absolutely any topic in mathematics and thereby places no restrictions on creativity. Hence, not only did I look at simpler real life examples such as integrating the cross section of a partially filled pipe, I also extended the example to 3D modelling of a cement truck barrel using cylindrical coordinates. By doing so I not only dwelled deeper into integration but also connected mathematics to graphics/modelling and simultaneously proved the formulae for cones, cylinders, etc. Lastly, I would like to thank Mr Overland, Mrs Gorman and Mr Hams from QASMT for their continual support in my project. It is fantastic to see such competitions increasing the interest and general perception of maths in the youth community, and it has been a privilege to be an award recipient.



National Mathematics Summer School

The National Mathematics Summer School (NMSS) is a program for the discovery and development of mathematically gifted and talented students from all over Australia.

Who runs NMSS?

The current Director of the NMSS is Associate Professor Leon Poladian (University of Sydney) who conducts the Summer School on behalf of the joint sponsors, the Australian National University and the Australian Association of Mathematics Teachers. A highly able and committed team of lecturers, tutors and other support staff volunteer their time to an important contribution to mathematics talent development.

Who is it for?

The NMSS is aimed at students completing Year 11, but others are included from time to time. Approximately 60 students attend each year. Potential participants can be suggested to the relevant State/Territory selector for consideration. The Queensland Selector is Michael Bulmer m.bulmer@uq.edu.au

When and where is it held?

The NMSS is a two-week residential school held in January each year at the Australian National University in Canberra. Dates for 2017 are 8-21 January but applications close in July 2016.

What is the program?

Students participate in a series of lecture courses from mathematicians in a number of branches of mathematics at a relatively advanced level. They attend tutorials under the guidance of a range of staff – postgraduate students, mathematics teachers and academic mathematicians. There is also an extensive social and cultural program which takes advantage of the attractions in Canberra.

What are the costs?

Students are expected to contribute to the costs of their attendance, but these are subsidised by sponsorship at the national and local levels. This year UQ's School of Mathematics and Physics have offered $5 000 towards Queensland's successful applicants to help with travel and accommodation costs. For more information visit www.nmss.edu.au

A pdf of the Student problems is on the QAMT website, www.qamt.org/resources.

A reminder. Samples of graph and grid paper can be found at www.qamt.org/resources

Rodney Anderson
President, QAMT

From the Editor

Rodney Anderson

A reminder that we encourage contributions from members for the Journal, since after all, it is your Journal. This is a chance to share your ideas and practices with other members. We also welcome suggestions for particular topics that you would like to read about.

E-mail suggestions and submissions to andersonr@mbc.qld.edu.au

Incentive to contribute articles/teaching ideas to the journal

QAMT MEMBERSHIP DRAW

1. For every article/teaching idea contributed, the author will receive a ticket in a Membership Draw.

2. If you are contributing an article/teaching idea for the first time, the author will receive two tickets in a Membership Draw.

CHOOSEMATHS

CHOOSE**MATHS** is a five-year project with the aim of increasing participation (particularly from girls) in the advanced levels of maths at school, and then hopefully into university. The project has a number of components and there are CHOOSE**MATHS** teacher and student awards which has just been launched.

The teacher awards have a particular emphasis on rewarding teachers who have been good at mentoring students and inspiring them to continue and persevere with mathematics whilst the student awards are based on students competing in teams to create a short creative video about mathematics. The competition will culminate in an awards presentation to be held during the day in Melbourne on Friday 26th August.

There will be a number of prizes on offer, including major prizes awarded to two teachers who have impressed with their outstanding achievement in inspiring and fostering the participation of girls in mathematics. The top two awardees will each receive $10 000 prize money and an additional $10 000 to support their school mathematics program. Additionally, eight of Australia's leading mathematics teachers will each receive $1 000 and with an additional $1 000 for their school mathematics program.

For more information on eligibility, the nomination process and selection criteria visit www.choosemathsawards.org.au

2D &3D Integration in Fluid Dynamics

Clint Therakam

Introduction

Integration has been a very useful mathematical tool with respect to analysing functions and applications into other areas of mathematics. In the past few centuries, integration has also become an extremely beneficial device especially in the field of engineering. One highly applicable area is fluid dynamics. Integration can be used in places as simple as identifying the cross-sectional area of a pipe and can further extend into 3D modelling of irregular shapes and finding their respective volumes. With the recent advancements in technology; IT and graphics allows engineers to mathematically describe 3D objects in 2D planes. The connection between these two fields can be seen in the application section of this exploration.

Personally, I find this topic quite interesting as integration is a powerful and efficient tool which can find virtually any mathematically describable 2D area and more importantly 3D volumetric relationships. This provides mathematicians and engineers the capability to model prototypes in the digital world and have an enhanced understanding of how such objects can interact together efficiently. In the area of fluid dynamics, this allows engineers to anticipate the behaviour of fluids through mathematical relationships. Furthermore, in the real world this knowledge has been applied to fluid control and even optimising volume capacities of vessels. Overall, there are multiple areas where this mathematics can and has been used to date; and this development in my appreciation for the use of mathematics in the real world, is the key reason I have decided to pursue this exploration topic.

Exploration and Applications

First let us have a look at a simplified version of this situation. What is the volume capacity of a 10m wide pipe with a $1\ \mathrm{m}$ internal radius?

Geometric Method

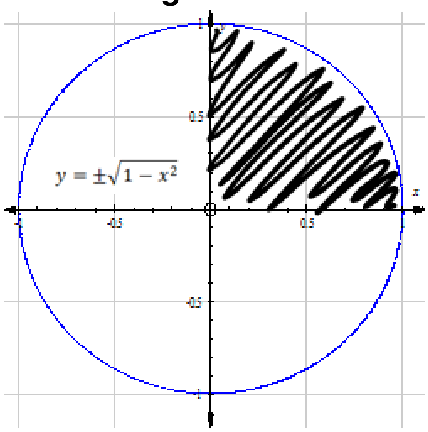
The easiest method to find the volume of the pipe is to use the cross-sectional area multiplied by the length of the pipe i.e. $\pi r^2 \times L = \pi(1^2) \times 10 = 10\pi\ \mathrm{m}^3$.

Integration Method (AnalyzeMath, 2013)

The pipe of $1\ \mathrm{m}$ radius is described by the function (Figure 1):

$$x^2 + y^2 = 1$$
$$y = \pm\sqrt{1 - x^2}$$

Figure 1



The cross-sectional area of the quadrant (the positive region bounded by $x = 0$ and $x = 1$) can be used to find the area of the circle:

$$Area\ of\ circle = 4 \times \int_0^1 \left(\sqrt{1 - x^2}\right) dx$$

$Let, x = \sin k$	**The corresponding domains are**
$\therefore \frac{dx}{dk} = \cos k$	$x \to 1, \quad k \to \frac{\pi}{2}$
$dx = \cos k\, dk$	$x \to 0, \quad k \to 0$

Therefore by substituting x in terms of k:

$$Area\ of\ circle = 4\int_0^{\frac{\pi}{2}} \left(\sqrt{1-\sin^2 k}\right)\cos k\, dk$$

Since $\cos^2 k + \sin^2 k = 1$; $\cos k = +\sqrt{1-\sin^2 k}$ for $0 \le k \le \frac{\pi}{2}$

$$\therefore Area\ of\ circle = 4\int_0^{\frac{\pi}{2}} \cos^2 k\, dk$$

$$\text{Since, } \cos 2k = \cos^2 k - \sin^2 k = 2\cos^2 k - 1\ (Double\ angle\ identity)$$

$$\text{Hence, } \cos^2 k = \frac{\cos 2k + 1}{2} \quad (Allows\ for\ linerization\ of\ integrand)$$

Therefore by substituting $\cos^2 k$ in the equation:

$$\begin{aligned} Area\ of\ circle &= 4\int_0^{\frac{\pi}{2}} \frac{\cos 2k + 1}{2} dk \\ &= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}(\cos 2k + 1)\, dk \\ &= 4 \times \frac{1}{2}\left[\frac{1}{2}\sin 2k + k\right]_0^{\frac{\pi}{2}} \\ &= 2\left[\left(\frac{\pi}{2}\right) - 0\right] \\ &= \pi\ \mathrm{m}^2 \end{aligned}$$

$$\therefore Volume\ of\ pipe = 10\pi\ \mathrm{m}^3\ (same\ as\ geometric\ method)$$

However, the geometric method is not very useful when the pipe is partially full. Whereas, integration can be easily applied to find the volume.

Now let's imagine that only 0.1 m of the pipe is filled from the lower end of the pipe (Figure 2). To assist with integration, the circle formula is horizontally translated by +0.9m (Figure 3):

Figure 2

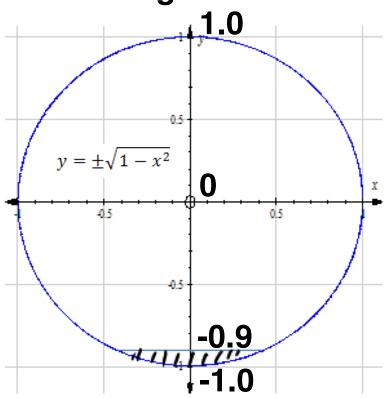
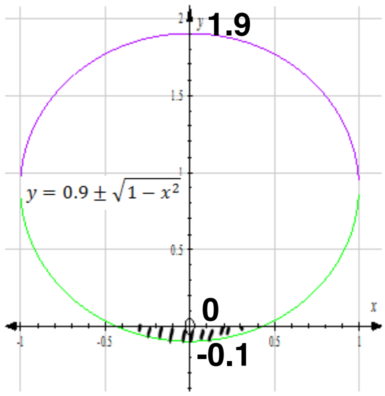


Figure 3



Using algebra and checked with the GDC, the roots of Figure 3 are $x \approx \pm 0.436m(3\ sig.f)$. Since the shaded region is in the bottom half of the circle (outlined in green), the equation of the function integrated is:

$$y = 0.9 - \sqrt{1 - x^2}$$

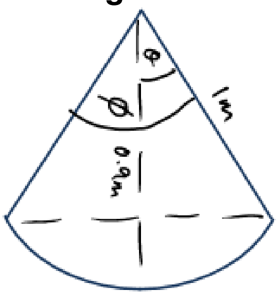
$$\begin{aligned}
Area\ of\ shaded\ region &\approx \int_{-0.436}^{0.436} 0.9\, dx - \int_{-0.436}^{0.436} \sqrt{1 - x^2}\, dx \\
&\approx [0.9x]_{-0.436}^{0.436} - \int_{-0.436}^{0.436} \sqrt{1 - x^2}\, dx \\
&\approx 0.785 - \int_{-0.451}^{0.451} \sqrt{1 - \sin^2 k} \cos k\, dk \text{ (same method as page 1)} \\
&\approx 0.785 - \int_{-0.451}^{0.451} \cos^2 k\, dk
\end{aligned}$$

$$\sin k \approx 0.436$$
$$k \approx \sin^{-1} 0.436$$
$$k \approx 0.451\ radians$$

$$\approx 0.785 - \frac{1}{2}\int_{-0.451}^{0.451} (1 + \cos 2k) dk$$
$$\approx 0.785 - \frac{1}{2}\left[\left(0.451 + \frac{1}{2}\sin(2 \times 0.451)\right) - \left(-0.451 + \frac{1}{2}\sin(-2 \times 0.451)\right)\right]$$
$$\approx 0.785 - 0.843$$
$$\approx 0.058\text{m}^2\ (absolute\ value)$$

PROOF (SECTOR METHOD)

Figure 4



$$\begin{aligned}
Area\ of\ sector &= \frac{1}{2}\emptyset r^2 \\
&\approx \frac{1}{2} \times 0.902 \times 1^2 \\
&\approx 0.451\ m^2
\end{aligned}$$

$$\cos\theta = \frac{0.9}{1}$$
$$\theta \approx 25.8419^o$$
$$\therefore \emptyset \approx 51.7^o$$
$$\approx 0.902\ radians$$

Figure 5

$$\begin{aligned}
Area\ of\ triangle &= \frac{1}{2}ab \sin c \\
&\approx \frac{1}{2} \times 1 \times 1 \times \sin 51.7 \\
&\approx 0.392\ m^2
\end{aligned}$$

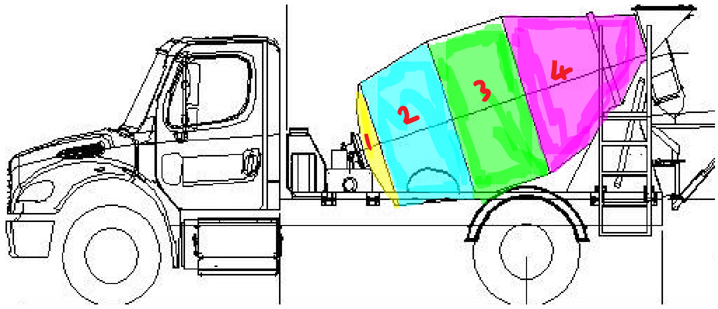
$$\begin{aligned}
\therefore Cross\ sectional\ area &= Sector\ area - Triangle\ area \\
&\approx 0.451 - 0.392 \\
&\approx 0.059\ m^2\ (thus\ proved)
\end{aligned}$$

It is clear that integration is both appropriate and effective in the field of fluid dynamics.

REAL WORLD APPLICATIONS – CEMENT TRUCK

We have seen that in terms of 2D shapes, geometric methods are found to be more straightforward as opposed to calculus. However, in 3D modelling, a field relevant to engineering and fluid dynamics, the effectiveness of integration is far more profound. In other words, through the utilisation of different coordinate systems and triple integrals, the volume of 3D objects such as cones and cylinders can be accurately found. Integration plays a crucial role in finding the volume of vessels of complex shapes and sizes in the real world. One such vessel, integral to the construction industry, is a cement mixer. In order to see if the mathematics works, I had contacted a cement mixer company and received the dimensions of a cement barrel *(Ernest Industries, 2013)*.

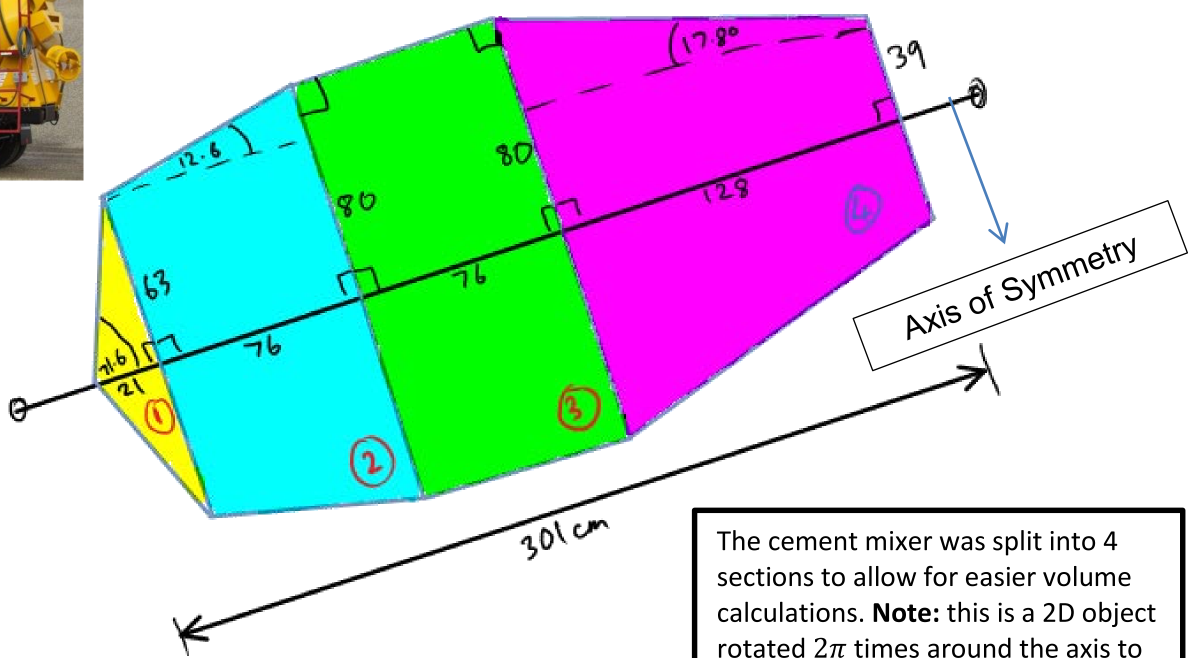
Figure 6 ***(Ernest Industries, 2013)***



By assuming the barrel can be split into 4 regular shapes and through the use of exact ratios and trigonometry; the barrel was modelled to life scale as shown below *(Figure 7).*

CEMENT MIXER DIMENSIONS TO LIFE SCALE (cm)

Figure 7

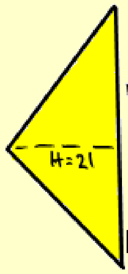


The cement mixer was split into 4 sections to allow for easier volume calculations. **Note:** this is a 2D object rotated 2π times around the axis to create the 3D object.

GEOMETRIC METHOD

Section 1 - Volume of Cone

$$\boldsymbol{V_1} = \frac{\pi}{3} \times r^2 \times H$$
$$= \frac{\pi}{3} \times 63^2 \times 21$$
$$\approx 87{,}283\ cm^3$$



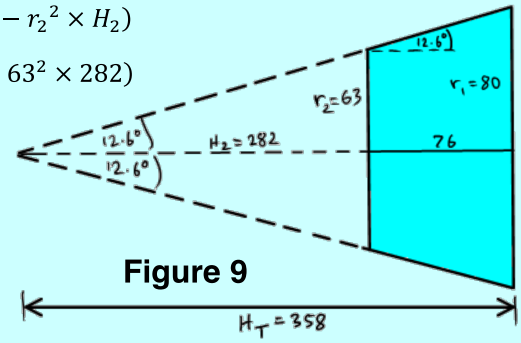
①

Figure 8

Section 2 - Volume of Cone Frustum

The volume of a cone frustum is the difference between the larger (external) hypothetical cone and smaller (internal) hypothetical cone (derived by extrapolation).

$$\boldsymbol{V_2} = \frac{\pi}{3}({r_1}^2 \times H_{Total} - {r_2}^2 \times H_2)$$
$$= \frac{\pi}{3}(80^2 \times 358 - 63^2 \times 282)$$
$$\approx 1{,}227{,}255\ cm^3$$

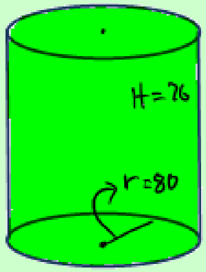


②

Figure 9

Section 3 - Volume of Cylinder

$$\boldsymbol{V_3} = \pi \times r^2 \times H$$
$$= \pi \times 80^2 \times 76$$
$$\approx 1{,}528{,}071\ cm^3$$



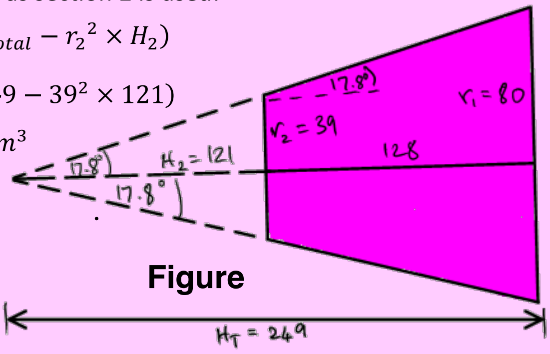
③

Figure

Section 4 - Volume of Cone Frustum

The same method as section 2 is used:

$$\boldsymbol{V_4} = \frac{\pi}{3}({r_1}^2 \times H_{Total} - {r_2}^2 \times H_2)$$
$$= \frac{\pi}{3}(80^2 \times 249 - 39^2 \times 121)$$
$$\approx 1{,}476{,}087\ cm^3$$



④

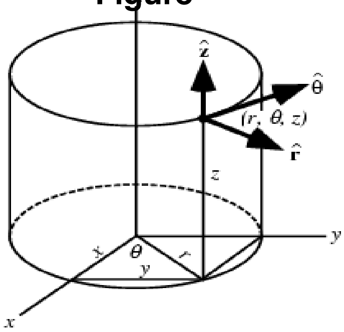
Figure

$$\boldsymbol{Total\ volume\ capacity\ of\ cement\ drum\ \approx\ 4318696\ cm^3 \approx 4.32\ m^3}$$

TRIPLE INTEGRAL METHOD – DIVERGENCE THEOREM

Firstly, it is important to change from use of the Cartesian coordinate system $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ to the 3D cylindrical coordinate system $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{z})$ where $\boldsymbol{r}$ is radius, $\boldsymbol{\theta}$ is the angle made by $\boldsymbol{r}$ and the x-axis and $\boldsymbol{z}$ is the vertical component as used in the Cartesian system *(Figure 12)*. To find $\boldsymbol{r}$ from $(\boldsymbol{x}, \boldsymbol{y})$, the simple Pythagorean law is applied: $\boldsymbol{r} = \sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2}$

Figure



(Wolfram MathWorld)

For the purposes of this investigation, the proved **divergence theorem** is used to find the volume of a 3D shape bounded by a closed surface. It states a volumetric relationship of the cylindrical coordinate system and its respective bounds *(Math24.net, 2013)*:

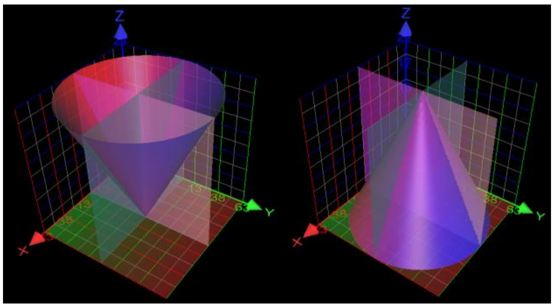
$$\boldsymbol{Volume} = \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2}(\boldsymbol{r})\boldsymbol{dzdrd\theta}$$

Basically this formula uses the triple integral to find the area of all three planes of the cylindrical coordinate system with respect to $\boldsymbol{r}$ and hence this finds the volume. The proof is not important here, we are just using it to explore further into integration.

VOLUME OF SECTION 1

The formula of a cone is in the form: $x^2 + y^2 = z^2\left(\frac{r}{h}\right)^2$

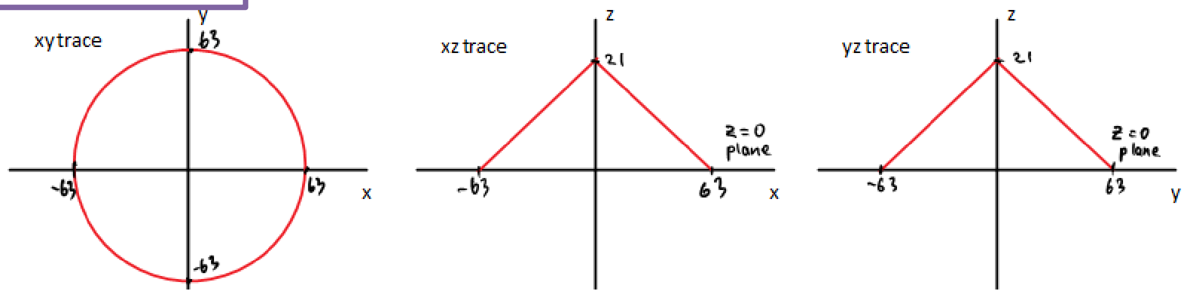
Using Figure 8: $z = \left(\frac{21}{63}\right)\sqrt{x^2+y^2} = \left(\frac{1}{3}\right)\sqrt{x^2+y^2}$



However this cone is positioned upside down with the tip touching the xy plane. Hence, the cone is flipped over with the circular base in line with the XY plane. This creates a closed 3D shape (with the $\boldsymbol{z = 0}$ plane) through which the volume can be found. Hence the equation is translated along the z axis by 21 and a flip along the xy plane:

$\boldsymbol{z = 21 - \frac{1}{3}\sqrt{x^2+y^2}}$

Before using the divergence theorem, the following plane traces are drawn:



The triple integral is used to find the volume which can be split into the three planes as seen in the traces. Hence:

$\boldsymbol{\theta}$ is bounded from 0 to 2π since that is one full revolution forming the circular base. $\boldsymbol{r}$ is bounded from 0 to 63 because that is the length of the radius of the cone. $\boldsymbol{z}$ is bounded from 0 to $\left(21 - \frac{r}{3}\right)$ because previously it was found that $z = 21 - \frac{1}{3}\sqrt{x^2+y^2}$. Hence using the Pythagorean law: $r = \sqrt{x^2+y^2}$, z is brought in terms of r and simultaneously the x and y variables are eliminated.
Each individual integrand (first involving $\boldsymbol{z}$, then $\boldsymbol{r}$, then $\boldsymbol{\theta}$) is solved using normal means till the volume, is found.

$$Volume = \int_0^{2\pi}\int_0^{63}\int_0^{21-\frac{r}{3}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{63} r[z]_0^{21-\frac{r}{3}}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{63}\left(21r - \frac{r^2}{3}\right)dr\,d\theta$$

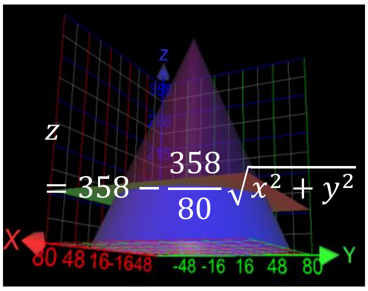
$$= \int_0^{2\pi}\left[21\frac{r^2}{2} - \frac{r^3}{9}\right]_0^{63} d\theta$$

$$= \left[21\frac{63^2}{2} - \frac{63^3}{9}\right]\int_0^{2\pi} d\theta$$

(matches geometry method)

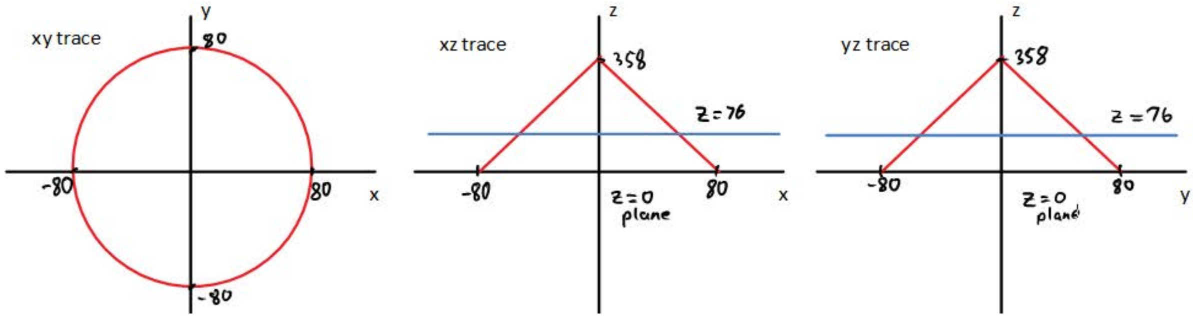
$$\approx 13891.5 \times [\theta]_0^{2\pi} \approx 13891.5 \times 2\pi \approx \boldsymbol{87,283\ cm^3}$$

VOLUME OF SECTION 2



As seen in the geometry method, section 2 of the cement mixer is a frustum. So in order to find the volume of it, the larger cone's volume needs to be found first. Hence the planes $\boldsymbol{z = 0}$ and $\boldsymbol{z = 76}$ are used to show the frustum. Using Figure 9, the form: $x^2 + y^2 = z^2\left(\frac{r}{h}\right)^2$ and the appropriate translations as seen in section 1, the larger section 2 cone can be described by:

$$\boldsymbol{z = 358 - \frac{358}{80}\sqrt{x^2 + y^2}}$$



$$V = \int_0^{2\pi}\int_0^{80}\int_0^{358-\frac{358}{80}r} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{80} r[z]_0^{358-\frac{358}{80}r}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{80} \left(358r - \frac{358}{80}r^2\right) dr\,d\theta$$

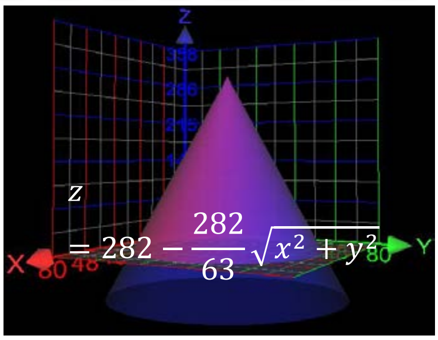
$$= \int_0^{2\pi} \left[179r^2 - \frac{358}{240}r^3\right]_0^{80} d\theta$$

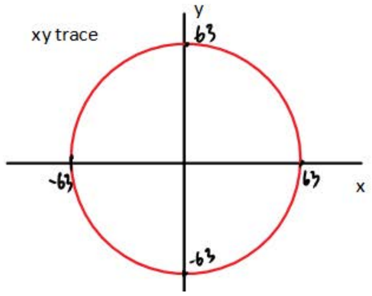
$$= \left[179 \times 80^2 - \frac{358}{240} \times 80^3\right] \int_0^{2\pi} d\theta$$

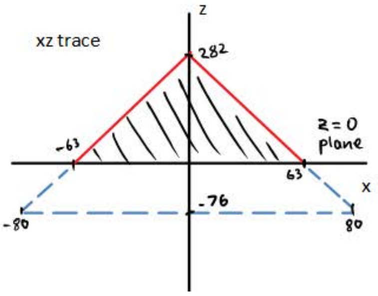
$$= \frac{1145600}{3} \times [\theta]_0^{2\pi} = \frac{1145600}{3} \times 2\pi \approx \boldsymbol{2,399,339\ cm^3}$$

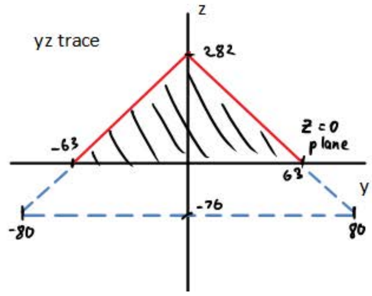
To find the smaller cone's volume, it needs to be treated separately to the larger cone. Hence, by using the dimensions from earlier it can be described by

$$\boldsymbol{z = 282 - \frac{282}{63}\sqrt{x^2 + y^2}}$$









$$V = \int_0^{2\pi}\int_0^{63}\int_0^{282-\frac{282}{63}r} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{63} r[z]_0^{282-\frac{282}{63}r}\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{63}\left(282r - \frac{282}{63}r^2\right)dr\,d\theta$$

$$= \int_0^{2\pi}\left[141r^2 - \frac{282}{189}r^3\right]_0^{63} d\theta$$

$$= \left[141 \times 63^2 - \frac{282}{189}r^3\right]\int_0^{2\pi} d\theta$$

$$= 186543 \times [\theta]_0^{2\pi} = 186543 \times 2\pi \approx \mathbf{1,172,084\ cm^3}$$

Hence, $\boldsymbol{net\ volume\ of\ section\ 2} \approx 2399339 - 1172084 \approx \mathbf{1,227,255\ cm^3}$ (matches geometry method)

VOLUME OF SECTION 3

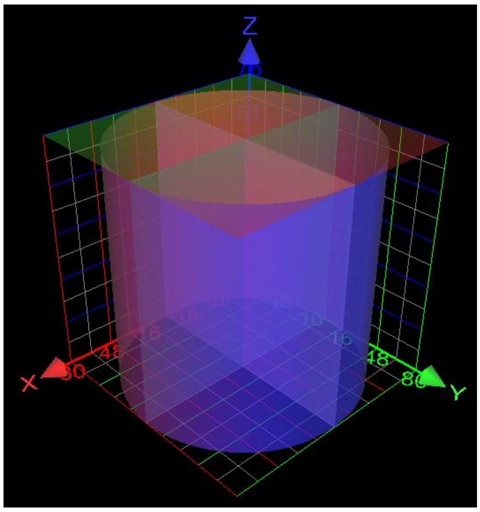
The formula of a cylinder is in the form:

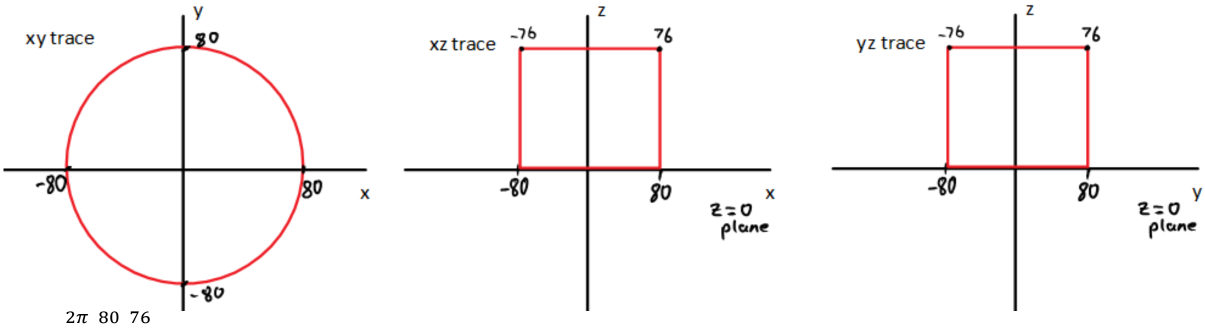
$$x^2 + y^2 = r^2$$
$$\therefore x^2 = 80^2 - y^2$$

Hence using Figure 10, the cylinder can be described by:

$$x = \pm\sqrt{80^2 - y^2}$$

The planes used are $\boldsymbol{z = 0}$ and $\boldsymbol{z = 76}$





$$V = \int_0^{2\pi}\int_0^{80}\int_0^{76} r dz dr d\theta$$

$$= \int_0^{2\pi}\int_0^{80} r[z]_0^{76} dr d\theta$$

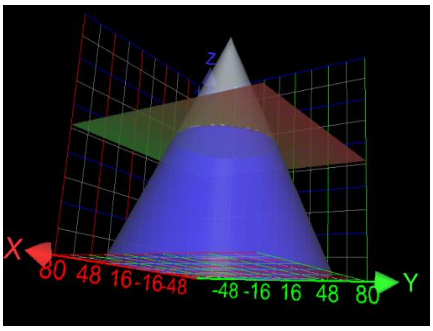
$$= \int_0^{2\pi}\int_0^{80} (76r) dr d\theta$$

$$= \int_0^{2\pi} [38r^2]_0^{80} d\theta$$

$$= [243200]\int_0^{2\pi} d\theta$$

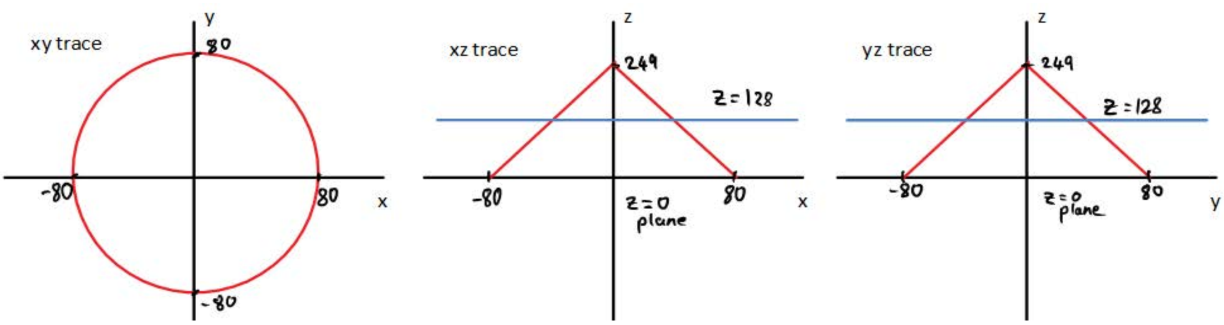
$$= 243200 \times [\theta]_0^{2\pi} = 243200 \times 2\pi \approx \mathbf{158,071\ cm^3}$$

(matches geometry method)



VOLUME OF SECTION 4

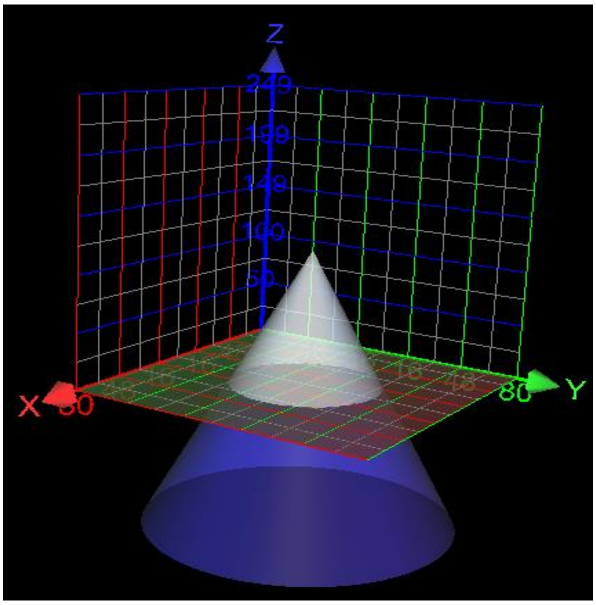
The approach to section 4 is the exact same as section 2 (the smaller cone within the larger cone method). The planes $\boldsymbol{z = 0}$ and $\boldsymbol{z = 128}$ are used. Using Figure 11, the larger cone can be described by $\boldsymbol{z = 249 - \frac{249}{80}\sqrt{x^2 + y^2}}$

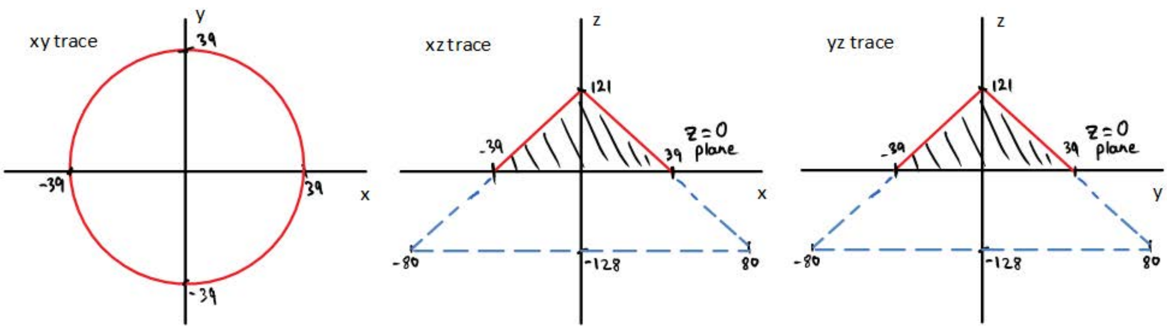


$$V=\int_0^{2\pi}\int_0^{80}\int_0^{249-\frac{249}{80}r} r\,dz\,dr\,d\theta$$
$$=\int_0^{2\pi}\int_0^{80} r[z]_0^{249-\frac{249}{80}r}\,dr\,d\theta$$
$$=\int_0^{2\pi}\int_0^{80}\left(249r-\frac{249}{80}r^2\right)dr\,d\theta$$
$$=\int_0^{2\pi}\left[\frac{249}{2}r^2-\frac{249}{240}r^3\right]_0^{80}d\theta$$
$$=\left[\frac{249}{2}\times 80^2-\frac{249}{240}\times 80^3\right]\int_0^{2\pi}d\theta$$
$$=265600\times[\theta]_0^{2\pi}=265600\times 2\pi\approx \mathbf{1,668,814\ cm^3}$$

The smaller cone can be described by:

$$\boldsymbol{z=121-\frac{121}{39}\sqrt{x^2+y^2}}$$



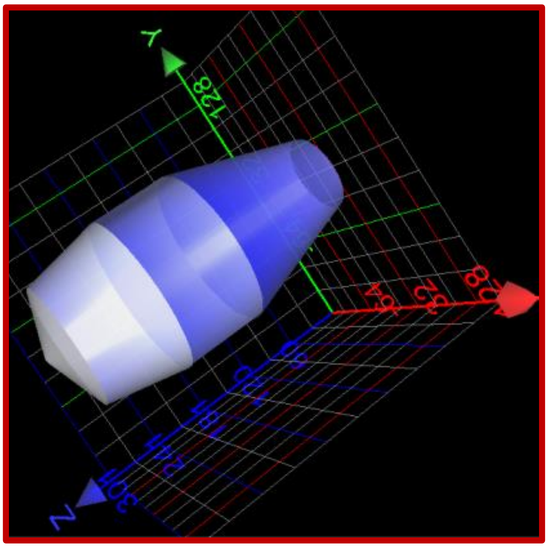


$$V = \int_0^{2\pi}\int_0^{39}\int_0^{121-\frac{121}{39}r} r\,dz\,dr\,d\theta$$
$$= \int_0^{2\pi}\int_0^{39} r[z]_0^{121-\frac{121}{39}r}\,dr\,d\theta$$
$$= \int_0^{2\pi}\int_0^{39}\left(121r - \frac{121}{39}r^2\right)dr\,d\theta$$
$$= \int_0^{2\pi}\left[\frac{121}{2}r^2 - \frac{121}{117}r^3\right]_0^{39} d\theta$$
$$= \left[\frac{121}{2}\times 39^2 - \frac{121}{117}\times 39^3\right]\int_0^{2\pi} d\theta$$
$$= 30673.5 \times [\theta]_0^{2\pi} = 30673.5 \times 2\pi \approx \mathbf{192,727\ cm^3}$$

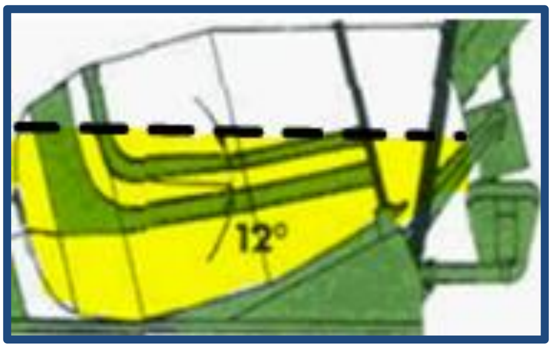
Hence, $\boldsymbol{net\ volume\ of\ section\ 4} = 1668814 - 192727 \approx \mathbf{1,476,087\ cm^3}$ (matches geometry method)

Therefore, by adding all individual components of the mixer:

$$\boldsymbol{Total\ Volume} = Part\ 1 + Part\ 2 + Part\ 3 + Part\ 4$$
$$\approx 87283 + 1227255 + 1528071 + 1476087$$
$$\approx 4{,}318{,}696\ cm^3 \approx \mathbf{4.32\ m^3}$$



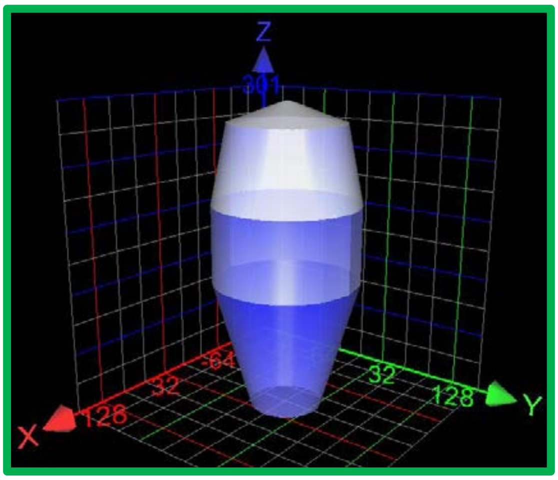
Since both the 'Geometry method' *(see page 4)* and the 'Integration method' arrived at the same volume: 4.32 m^3; it can be concluded that the triple integral divergence theorem is a powerful tool with regards to finding volumes and volumetric relationships of 3D irregular objects in the field of fluid dynamics.



Furthermore, the true effectiveness of using this field of maths in fluid dynamics can be seen when cross-referenced with the specifications by the manufacturer, stating that this cement mixer model is capable of transporting 3.5 $yard^3 \approx 2.68\ m^3$ at a slump *(Ernest Industries, 2013)*. There is clearly a large discrepency between this value and 4.32 m^3. However this can be explained by the 'slump'. When transporting cement, the mixers are required to maintain a certain angle and hence the total volume of mixer isn't filled. According to a universal concrete equipment specialist, *Utranazz*, this angle is approximately 12° and the lower end of max filling coefficient is about 66% *(Ultranazz, 2012)*. Hence: $66\% \times 4.32 \approx 2.85\ m^3$. Including the simplification of shapes and other assumptions, the volume is reasonably close to the specified 2.68 m^3. Once more, this is further evidence of the efficieny and relevance of integration in the real world.

REFLECTION AND CONCLUSION

This exploration has facilitated in displaying the effectiveness and importance of integration in engineering and specifically, the field of fluid mechanics. Firstly, it could be seen as an appropriate tool in order to find cross-sectional areas of simple circular functions and even areas of pipes partially-filled. However, efficiency is vital in the modern society and hence one would question the effectiveness of using complex integration over simple and quick geometry. Afterall, it was clear that equations of circles are derived through means of integration and calculus anyways.



However in the 3D world (real world) where fluid dynamics play a far more vital role, geometry is far less effective when in comes to irregular shapes. This is primarily because finding the volume of cones and cylinders can be done through the shape's perpendicular height and radius. The process itslef shows nothing about the properties of the objects in the 3D world. However, the process of solving volume problems using triple integration, provides more information about the object by way of a mathematical equation, graph traces, 3D modelling properties and interaction. Modelling such objects in life scale is an extremely useful tool for mathematicians and engineers. This aspect of integration is enhanced through the developments in technology. It can be seen how the two fields interact effectively as well as work to provide a reasonable model of the real world. Furthermore, such applications of 3D modelling enhance the value of graphics and IT, which in turn provides job opportunities and ultimately meets social and moral obligations.

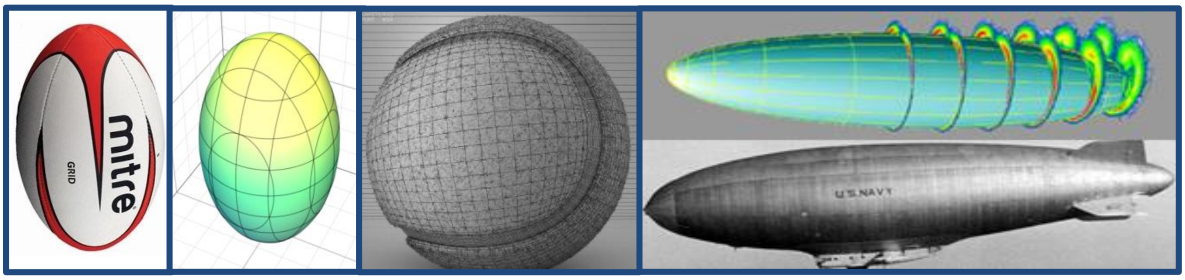
More than often not, several intense branches in mathematics are heavily reliant on mathematical theory and have little use in the real world. However, when such areas of mathematics do branch into the real world (applied maths), the true importance is evident. Mathematics not only becomes a useful tool, it becomes the core aspect of analysing such scenarios. An excellent example of this progess and relationship between mathematics and 3D modelling is the cement mixer exmaple used. I for one was amazed to find that mathematics plays a crucial role in such modelling. In fact I now understand that commonly used design programs such as Auto CAD requres mathematics behind its core functioning. Once again I can see how IT and mathematics have influenced each other. When considering the historical aspect this becomes even more significant. Integration has existed for many centuries as of yet, however noticible developments in IT have only occurred in the past few decades.

Such developments have brought rise to moral, social and ethical advantages. The true beauty of mathematics is that ideas can be universally communicated; hence surpassing language and ethical barriers while allowing cross-cultural work. To me, this understanding has provided an appreciation for mathematics not just on paper but in real life.

With regard to possible extensions for the exploration; the nature of the topic itself lends well into different areas of fluid and aerodynamics. To begin with, the volume of slightly more complex shapes can be found. This could be from modelling water tanks or silo tanks for example. Allternatively, the area of plumbing and drainage can be considered. In the example of the cement mixer, all segments were rotated the full 2π times.

Although, in areas such as drainage, fluids do not always fill the entire vessel which means that mathematics regarding limits and bounds will prove useful. Another interesting area of further extension is to look at different coordinate systems. Spherical coordinates for example would prove useful in finding the volume of 3D spheres and prolate spheroids as seen in the pictures below:

This could therefore branch into blimps, air balloons and even tennis balls & footballs. With such 3D objects, optimising aerodynamics and reducing drag becomes important. Such extensions could prove useful in increasing fuel efficiency; a real world application.



Overall, it is clear that mathematics, especially in the area of calculus, is an important tool in the real world. Through this exploration I have not only been able to appreciate calculus as being an efficient analytical tool, I now appreciate the relevance of such mathematics in the real world.

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2nd National Conference

Transforming Mathematics Education for Indigenous Learners

ATSIMA

Value us. Value our education. Value our future.

Register here: http://bit.ly/25siFFA

SUNDAY 30 OCTOBER - WEDNESDAY 2 NOVEMBER, 2016

At the University of Wollongong and Sandon Point Tent Embassy (for Opening & Closing Ceremonies)

About this conference
A collaboration of leaders, educators and stakeholders from Community, education, research with the Aboriginal Communities of the Dharawal and Yuin peoples of the Illawarra and South East Coast.

Keynote speakers
Value our education Emeritus Professor Alan J. Bishop
Value our future Ken Markwell, Indigenous Sector Practice Director, EY.
Go to: http://atsimanational.ning.com/speakers

Call for presenters
Submissions are open for Community, education and business presentations. Go to: atsimanational.ning.com/submissons.

Location
Sandon Point Tent Embassy & University of Wollongong's SMART Infrastructure Facility

Sponsorship
ATSIMA invites organisations and individuals to sponsor this important event. For more information, go to: atsimanational.ning.com/sponsorship or email us: enquiries@atsima.org.

Conference website: atsimanational.ning.com/events

Want to register: Go to http://bit.ly/25siFFA

About ATSIMA
The Aboriginal and Torres Strait Islander Mathematics Alliance is an Aboriginal led, non-profit, member-based group representing Communities, education and business.

Contact us
Dr Chris Matthews, ATSIMA Chair
chrismatthews@atsima.org
enquiries@atsima.org
atsimanational.ning.com

QAMT Dates
Professional Development
Planning for 2016

Thursday, 28th July
Australian Mathematics Competition
Australian Mathematics Trust
Details www.amt.canberra.edu.au

QAMT Year 7/8 Quiz – Sponsored by QT Mutual Bank
A quiz style competition with 3 members to a team suggested dates are

Round 1:	July
Round 2:	Early August
State Grand Final:	Tuesday, 6th September
Entry fee:	$25 per team
	$10 administration fee for non-member schools
State Co-ordinator:	Peter Cooper, spcooper@uq.net.au

Saturday, 20th August
August Forum and AGM

A one-day event for all maths educators from prep-tertiary.
For Prep to Tertiary teachers. $85 members; $55 students & presenters; non-members $150. QAMT School members can send up to 4 teachers at member prices.

Details at http://www.qamt.org/qamt/august-forum-qamt-agm-1

Senior Mathematics Teaching
Helping Children Learn: Curriculum

Conference:	5th December
Venue:	The University of Queensland
Cost:	$140

MaSER

TEMPEST

(Towards Education Mathematics Professionals Encompassing Science and Technology)

TEMPEST is a national project focussing on the professional learning available to mathematics teachers. It aims to identify, evaluate and provide access to quality professional learning through *Dimensions*, an online portal hosted by the Australian Association of Mathematics Teachers (AAMT).

How will TEMPEST achieve this?
We will work with teachers, schools, associations and those involved in mathematics education/professional learning to identify effective professional learning and trial and evaluate selected activities.

What can you and/or your school do?
We need evidence of impact of quality professional learning on mathematics teachers' capabilities and engagement, and on student interest and learning. We seek your participation to trial quality professional learning and use selected evaluation tools.

Who will you work with?
There are five Implementation Officers working throughout Australia. Their job is to work with you to identify and facilitate professional learning suitable for your needs.

Desley Pidgeon
TEMPEST Implementation Officer for Queensland

Desley has been actively involved in education over the past twenty years, predominantly teaching at secondary level in the areas of mathematics, agriculture and business. She has also delivered programs in the VET/TAFE sector to both school-based learners and adults. Desley has a particular interest in collaborative teaching pedagogy to support student learning. Desley is based at CQ University in Rockhampton, Queensland, and also works as a casual lecturer in an enabling mathematics program at the School of Education.

d.pidgeon@cqu.edu.au (07) 4923 2771 ext 52771 0413 923 540 re(Solve) MATHS BY INQUIRY

reSolve: Maths by Inquiry
Promoting a spirit of inquiry in school mathematics

reSolve: Maths by Inquiry **is a national project that provides Australian schools with resources to help students learn mathematics in an innovative and engaging way. It is led by a team of expert teachers and academics from around the country.**

The *reSolve* vision

At the centre of ***reSolve*** is the ***reSolve: Maths by Inquiry*** Protocol. This sets out a vision for teaching and learning mathematics and underpins all aspects of the project. The protocol is organised around three focal points:

reSolve **mathematics is purposeful.**

reSolve **mathematics is challenging yet accessible.**

reSolve **mathematics promotes a supportive knowledge-building culture.**

The nuts and bolts

reSolve provides principals, teachers, schools and students with resources for professional learning and classroom teaching. It is based on relevant real-world examples to help students deal with complex situations using a variety of mathematical methods.

Its professional resources promote individual teacher learning and whole-school change, plus highlight all aspects of the ***reSolve: Maths by Inquiry*** Protocol. The resources support the Australian Institute for Teaching and School Leadership (AITSL) Australian Professional Standards for Teachers.

Its classroom resources exemplify the ***reSolve: Maths by Inquiry*** Protocol. These support the Australian Curriculum: Mathematics and include learning experiences for every year level from Foundation to Year 10, with a particular focus on understanding, reasoning and problem solving. They provide examples of how mathematics is used in everyday life and mathematical investigations and proof, while some capitalise on emerging technologies and mathematically able software. The classroom resources range from single lessons to substantial units of work.

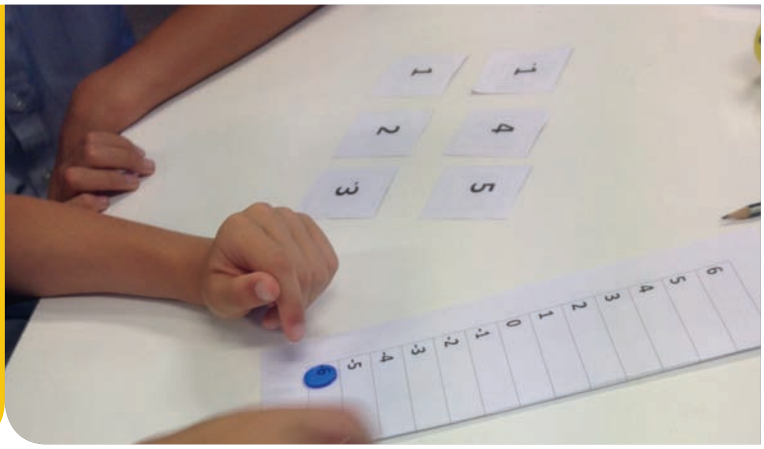
Who's behind *reSolve*?

reSolve is a key programme under the Australian Government's Restoring the focus on STEM in schools initiative. It is managed by the Australian Academy of Science in collaboration with the Australian Association of Mathematics Teachers.
The programme runs from November 2015 to June 2018, with resources becoming available from November 2016.

We're thrilled with the opportunity to continue partnering with the *reSolve* team and to trial new resources as they become available.

Sam Hardwicke
Year 5 and 6 Team Leader at Turner School

re(Solve) MATHS BY INQUIRY



reSolve will support collaborative actions in the National STEM School Education Strategy 2016-2026, endorsed by Australian Education Ministers in December 2015. The strategy aims to make sure all students finish school with strong mathematics knowledge and that they are inspired and equipped to take on more challenging STEM subjects at senior school, including intermediate and advanced mathematics.

reSolve: Maths by Inquiry is supported by the Australian Government Department of Education and Training.

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Executive Director
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Matt Skoss
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mskoss@aamt.edu.au
0418 624 631

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Curious to know more?

Contact the reSolve team, go to
www.science.org.au/resolve
or scan our QR code.

Australian Academy of Science

Ian Potter House, 9 Gordon Street, Canberra ACT 2601
+61 2 6201 9490
www.science.org.au/resolve

I liked the lesson because it was different to all the other ones. It was "funner". And the game was useful to me and helped me.

Year 6 Student
Amaroo School

Australian Academy of Science

aamt
Australian Association of Mathematics Teachers

QUEENSLAND STEM EDUCATION NETWORK

SCIENCE | TECHNOLOGY | ENGINEERING | MATHEMATICS

What is STEM Education?

STEM stands for Science, Technology, Engineering and Mathematics, and the Queensland STEM Education Network is a tertiary consortium established in 2015 which aims to improve and advance Science, Technology, Engineering and Mathematics capacity in Queensland. Griffith University is leading the way in partnership with the Queensland University of Technology, James Cook University and the University of the Sunshine Coast. This year, the network expanded via inclusion of the University of Southern Queensland and CQ University representation to enable state-wide collaboration and sharing of best practice.

In July 2013, Professor Ian Chub released a position paper on the development of a national strategy to guide STEM in Australia, highlighting four essential elements:

Education: formal and informal.

Knowledge: ensuring continuous flow of new ideas and their dissemination.

Innovation: using knowledge to produce high value goods and services.

Influence: utilising networks, collaboration and alliances to ensure Australia's international place.

What are the goals of QSEN?

1. To raise awareness, interest and achievement in science and mathematics among Queensland junior secondary students leading to increased STEM enrolments at senior secondary and tertiary levels.

2. To engage students, parents, teachers, guidance officers and the broader community to demystify and raise awareness of the importance of STEM education and STEM related careers.

3. To de-construct known identity/cultural factors which underpin STEM-aversion in junior secondary students, especially in low socio-economic status and Indigenous communities.

How does QSEN plan to achieve these goals?
QSEN will build on existing STEM outreach programs to allow state-wide sharing of best practice and the co-development of new student engagement initiatives tailored to meet local needs. QSEN also plans to focus on the reengagement of students in the classroom through the co-development of innovative in-class curriculum resources complemented by the provision of engaging, informative, out-of-class STEM experiences which take advantage of the unique expertise, resources and infrastructure available at each partner university. Additionally, targeted workshops are being designed to engage with students in low SES and Indigenous communities, as well as to "influence the influencers" of student choices in order to demystify STEM-related careers and identify and remove myths, misconceptions and road-blocks to engagement in STEM studies.

What is the value of STEM qualified individuals? The value of STEM qualified individuals lies with the skill development that they bring to their jobs including active learning, design thinking, interpersonal skills, system analysis and evaluation, and strategic thinking, to name a few. Deloitte conducted a survey in 2014 of employers about the value of STEM qualified individuals in their organisations. The survey concluded that:

- Employers of Science qualified individuals were closely aligned to the overall trend of skill importance, attributing marginally more importance for complex problem solving and creative problem solving.

- Employers of Technology qualified individuals placed more importance on programming.

- Employers of engineering qualified individuals placed more importance on design thinking.

- Employers of Mathematics qualified individuals placed more importance on critical thinking.

For further details contact:

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Manager, Queensland STEM Education Network

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www.queenslandstem.edu.au

Teaching Ideas

Activities with Asteroids

Stephen Broderick
St Ursula's College, Toowoomba

Activities with Asteroids

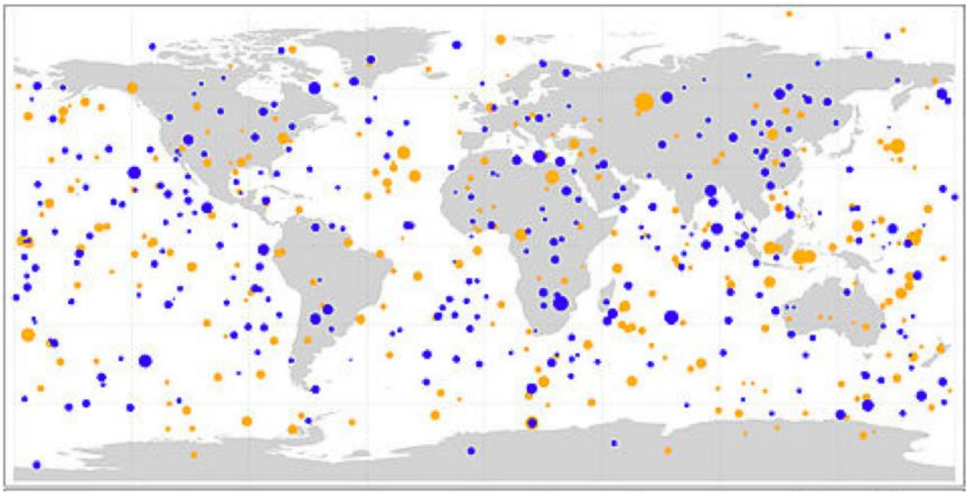
On April 13, 2029, a 325 metre wide asteroid called Apophis will be visible in broad daylight as it passes the Earth below the orbits of geosynchronous communication satellites. It will be approximately 31 000 kilometres from the Earth after initial calculations indicated that it could collide with the Earth. Luckily there are a number of contingency plans to deal with rogue asteroids, provided they are spotted well in advance of any possible collision.

1. Probability of an asteroid impact

Asteroids with a one kilometre diameter strike the Earth on average every 500 000 years. An event like this would be felt over the entire surface of the Earth. Such an impact would result in mass extinctions across the planet. Larger asteroids 5 km in diameter impact on average every 20 million years, while the last known impact of an object of 10km or more in diameter was at the Cretaceous–Paleogene extinction event 66 million years ago. This event wiped out the dinosaurs.

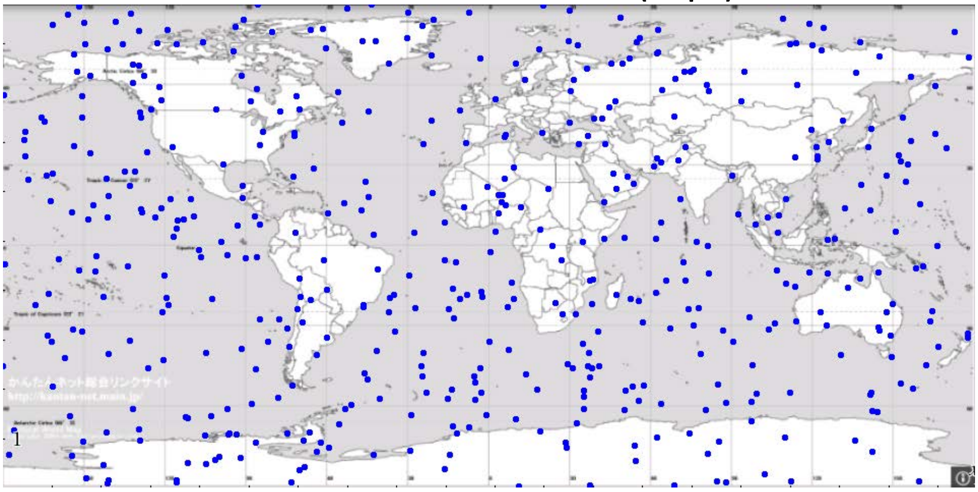
The first map (map 1) below shows the frequency and location of asteroids between 1 and 20 metres in diameter which disintegrate in the Earth's atmosphere. These asteroids are also known as bolides. The second map (map 2) is a Monte Carlo simulation of the bolides across the planet. Note how closely the simulation matches the actual data for Australia.

Bolide Events 1994-2013 (Map 1)



Frequency of small asteroids (1- 20 metres diameter) impacting Earth's atmosphere

Monte Carlo Simulation of Bolide Events (Map 2)

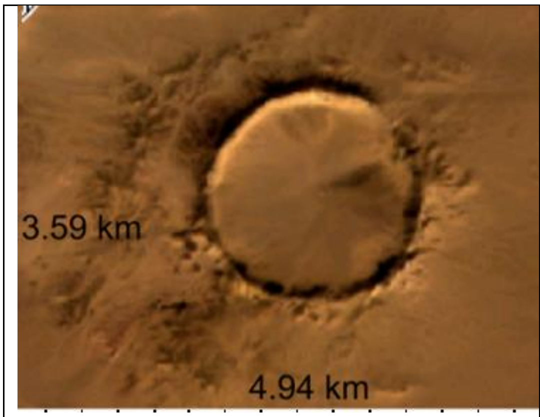


2. Using probabilities to estimate the area of a meteorite crater

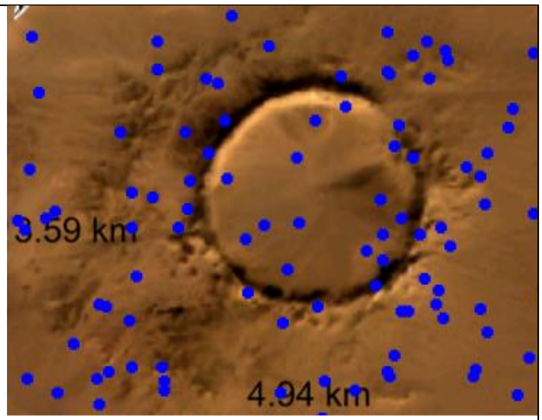
The Tenoumer impact crater in Mauratnia is almost a perfect circle. The rectangular image below has dimensions 3.59 km by 4.94 km with a total area of 17.73 km^2.

The Monte Carlo method uses probability to estimate areas of a selected region. This technique can be used to estimate the area of the Tenoumer crater.

One hundred Cartesian co-ordinates are randomly generated in the rectangular region and 17 points are inside the crater. The crater thus represents 17% of the total rectangular area and equals 3.014 km^2.



rectangular area = 17.73 km^2



crater area $\approx \frac{17}{100} \times 17.73 = 3.014$ km^2

Since the area of the impact crater is 3.014 km^2 and from the area of a circle formulae, the radius is 0.9794 km. Therefore, the diameter is 1.96 km. The actual diameter is listed as 1.9 km. The Monte Carlo method is quite useful for determining the areas of irregular craters.

3. Asteroid diameter V Crater diameter

Asteroids with a diameter of 4 metres impact Earth (as meteorites) approximately once per year. Larger meteorites and the frequency of their occurrence are represented in the table below. For example, (from the first row of the table) an asteroid with a diameter of 100 metres will impact the Earth approximately every 5 200 years and create a crater with an approximate diameter of 1.2 kilometres.

There are a number of contingency plans for deflecting a potential asteroid impact, providing there is enough warning time.

One such plan is painting one hemisphere of the asteroid with a dark material to absorb more sunlight, taking advantage of the Yarkovsky effect. An asteroids surface heats up and as it rotates away from the Sun, this heat is emitted back into space as thermal infrared radiation. The rate of absorption and emission is unbalanced, and the infrared emissions carry away momentum from the asteroid, leading to the sunlight gently pushing the asteroid in one direction. Painting one hemisphere dark results in a stronger push.

DE-STAR (Directed Energy system for Targeting of Asteroids and exploRations) involves converting solar energy using photovoltaic cells into a laser beam. The laser beam then heats up the surface of the asteroid to around 2 700°C, which would cause a jet of vaporized material to be ejected, pushing the asteroid in the opposite direction. The deflection would be successful if there were at least 4 years warning time.

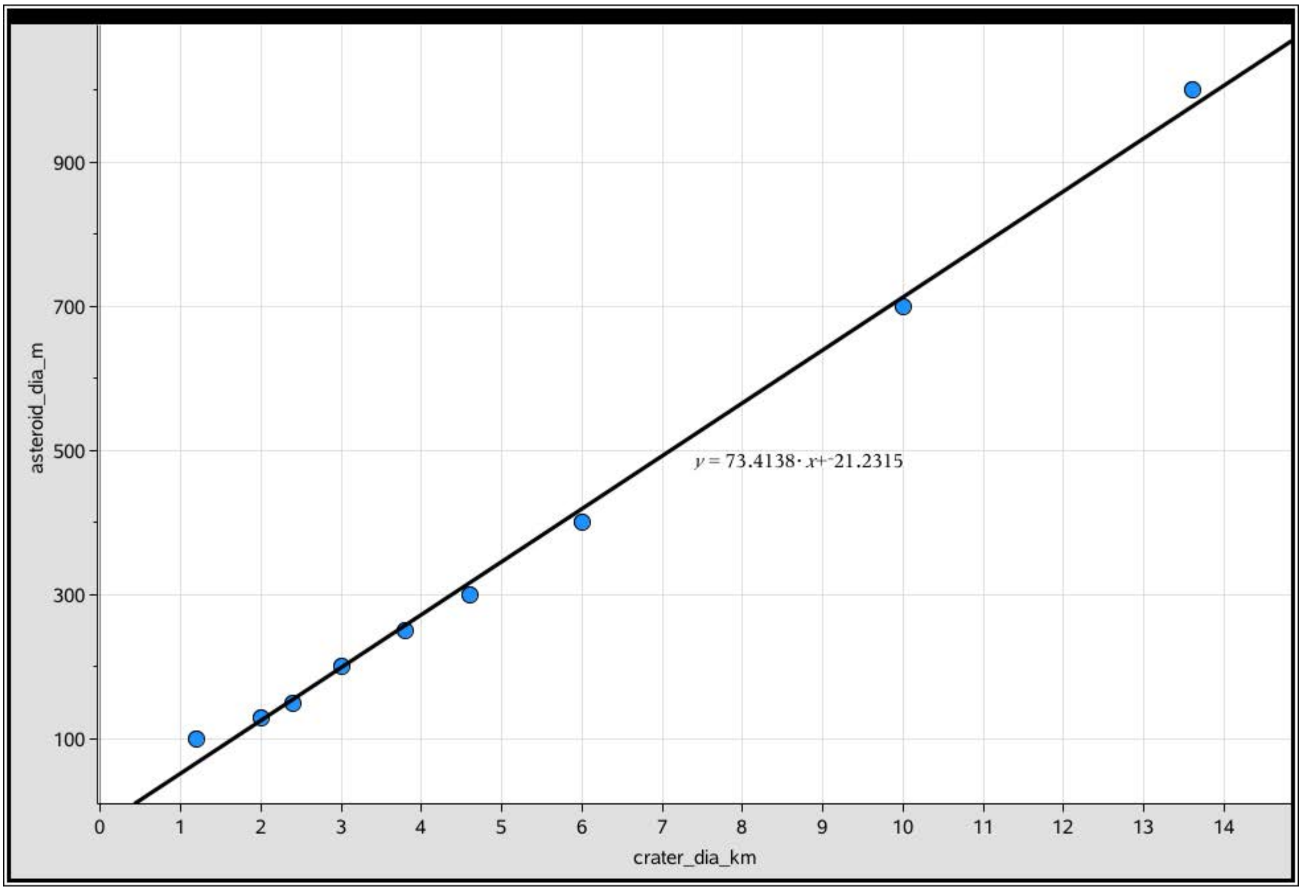
Another method of deflection is known as a gravity tractor. A spacecraft hovers close to an asteroid and the gravitational attraction between the two is sufficient to pull the asteroid off course. This technique would take many years to sufficiently deflect an asteroid.

Asteroid diameter (metres)	Impact energy (Megatons of TNT)	Crater diameter (kilometres)	Frequency of occurrence In years
100	38	1.2	5 200
130	64.8	2	11 000
150	71.5	2.4	16 000
200	261	3	36 000
250	598	3.8	59 000
300	1 110	4.6	73 000
400	2 800	6	100 000
700	15 700	10	190 000
1 000	46 300	13.6	440 000

The Barrington crater in Arizona is just over 1 kilometre in diameter and approximately 50 000 years old. It is 170 metres deep. How big was the asteroid that created the crater?

*This data is based on an asteroid density of 2 600 kg/m^3, velocity of 17 km/second and impact angle of 45°.

By plotting Asteroid diameter (metres) V Crater diameter (kilometres), the size of the asteroid that created the Barrington crater can be approximated from the linear relationship below.



Based on the information in the graph, the diameter of the asteroid before impact is approximately 50 metres.

Using the model Y = 73.4138 x + -21.2315 gives a more accurate value of 52.18 metres.

What about the amount of energy released during the Barrington impact?

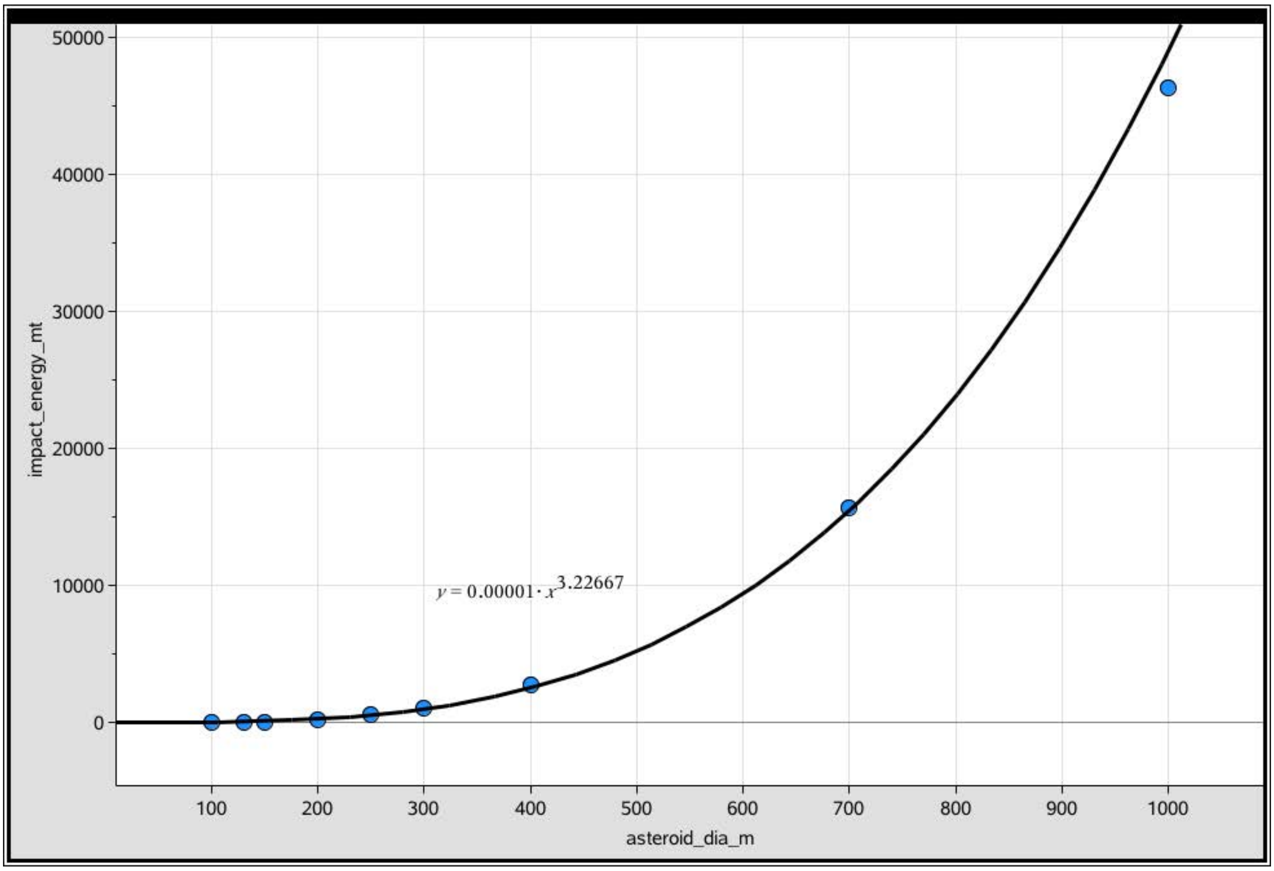
By graphing either Asteroid diameter V Impact energy or Crater diameter V Impact energy, it is evident that the relationship is exponential.

The exponential nature arises from the kinetic energy of the asteroid at impact.

Since kinetic energy = $\frac{1}{2}mv^2$ and the velocity is constant for this data set (v = 17 km/s), then the kinetic energy is proportional to the mass or volume.

Assuming the asteroids are spherical, then volume of a sphere can be used to model the data.
V = $\frac{4}{3}\pi r^3$.

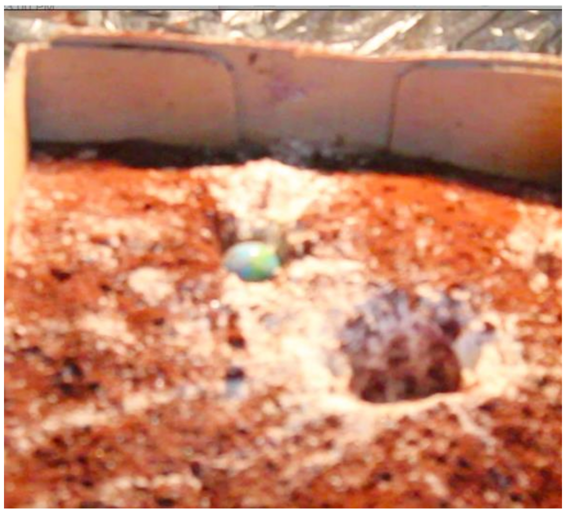
This means the kinetic energy is proportional to r^3. For example, doubling the radius increases the kinetic energy by a factor of 8.



Using the model $y = 0.00001 \times x^{3.22667}$ with a diameter of 52.18 metres produces a total energy of 3.48 Mt of TNT. (3.48 million tons of TNT) for the Barrington asteroid impact. This is equivalent to 232 Hiroshima atomic bombs.

4. Modeling impact craters with marbles and flour

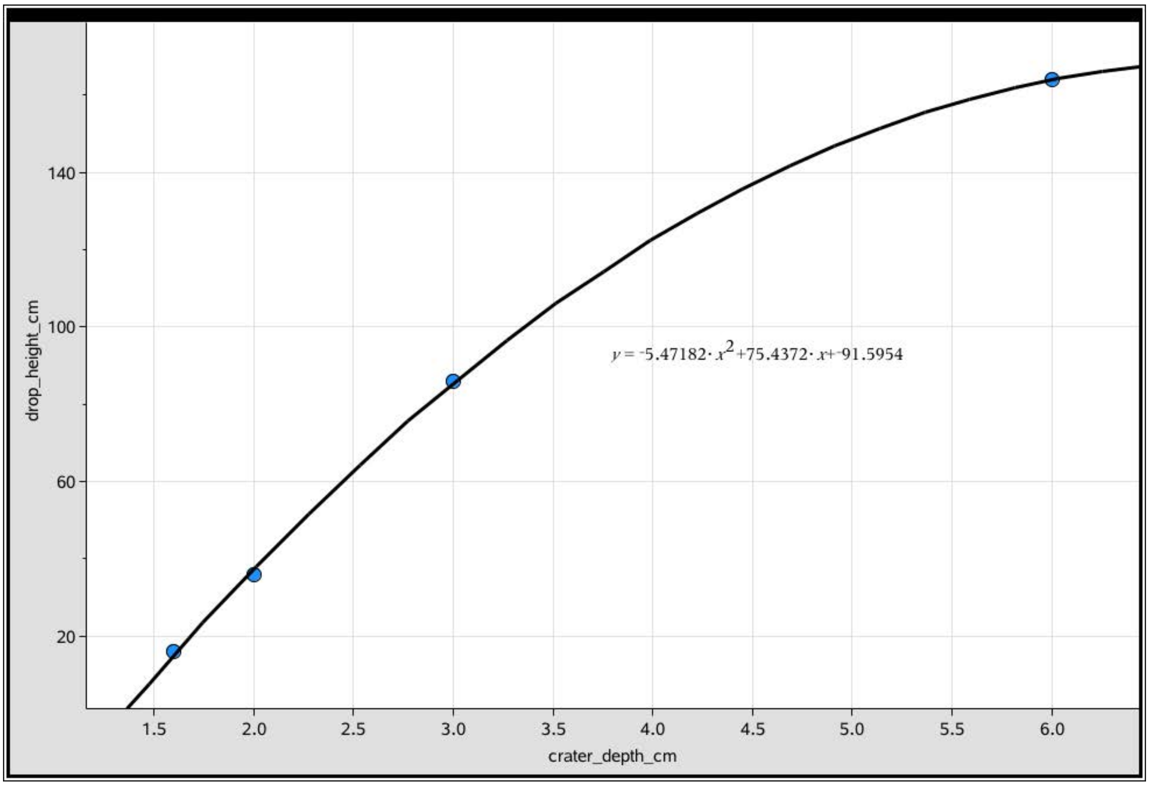
The following activity uses flour, coco powder and marbles to produce meteorite impact craters. Fast frame cameras can also obtain frame-by-frame images of the shape and distribution of the ejecta mid-flight.



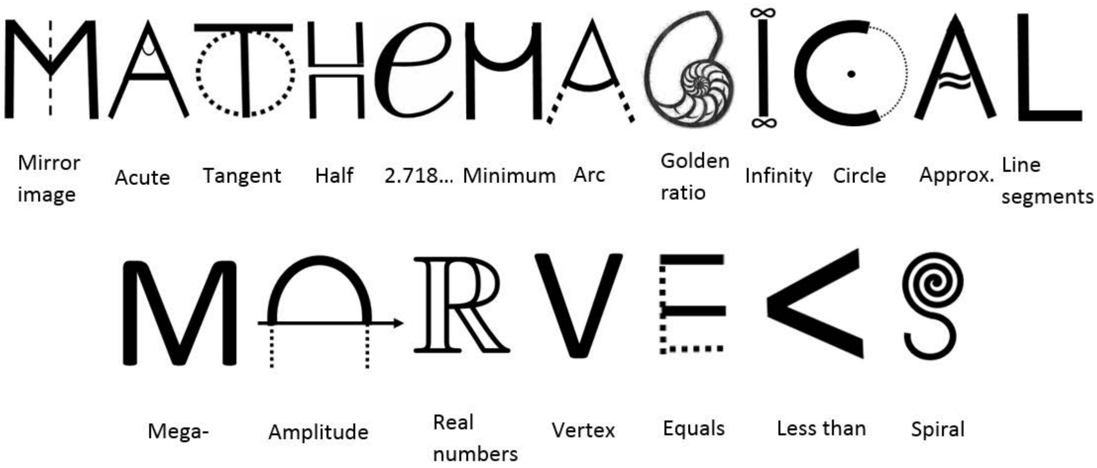
To obtain data, a marble was dropped from four different heights and the depth of the crater was recorded.

The data is represented in the table below and a quadratic equation was used to model the data.

Marble drop height (cm)	Crater depth (cm)
16	1.6
36	2
86	3
164	6



References
'Astronmoy Now' Averting Asteroid Armageddon, July 2015
Wikipedia 'Impact Event' https://en.wikipedia.org/wiki/Impact_event



Andrew Wrigley
Somerset College

Methods of Multiplication

"It is unworthy of excellent men to lose hours like slaves in the labour of calculation which could be relegated to anyone else if machines were used".

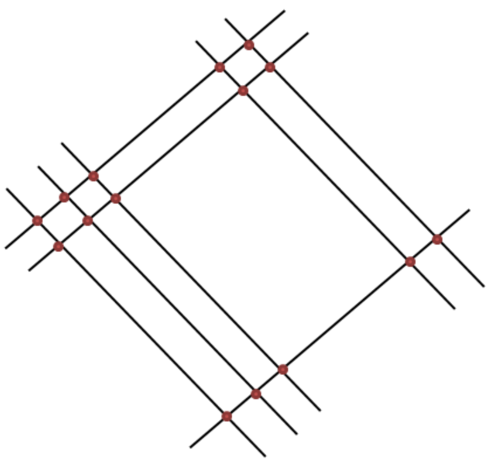
Gottfried Wilhelm Leibniz

The first hint of evidence of man's ability to multiply numbers is found on the Ishango bone, a baboon's femur containing tally marks and dating back 20 000 years.

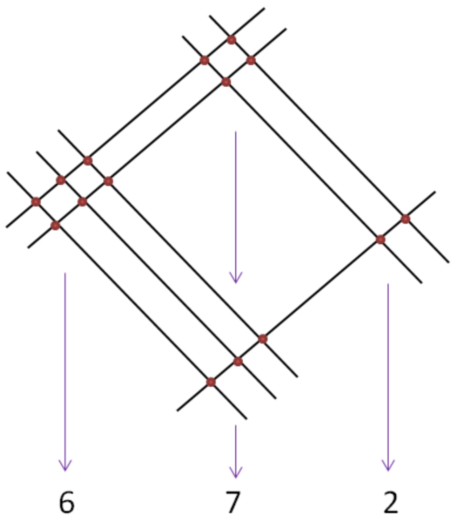
Throughout history, cultures have developed their own methods of multiplication, many of which provide us with a good teaching tool today. They can certainly bring variety to a process that can need livening up and are a good way of avoiding the inevitable use of the calculator while providing some insight into the process. It would be a useful exercise for students to consider why these methods work.

The Mayan method

I will start with what is often referred to as Mayan multiplication, even though there is no evidence that the Maya ever did multiplication. It uses lines to represent numbers, e.g., 21 × 32 (below).



Count the line intersections and then add them vertically,



This gives the required result that 21 x 32 = 672. Sometimes you have 'carry' digits.

The Ancient Egyptian method

One of the oldest written methods, it is simple and involves a doubling method that links it to binary numbers. The method would be used by professionals like astronomers, architects and, of course, mathematicians.

They would multiply 21 x 32 as follows; the larger number (32) is in the right hand column and doubled. The left hand column starts at 1 and keeps doubling up to the largest number less than 21.

1	32
~~2~~	~~64~~
4	128
~~8~~	~~256~~
16	512

Select numbers in the left hand column add to 21 i.e. 16 + 4 + 1 and cross out the rest.
Add the remaining numbers in the right hand column are added together. i.e. 32 + 128 + 512 = 672

The Russian Peasant method

This is similar to the Egyptian method but a little easier to follow.

The two numbers are placed in two columns as before. The first column is halved (and rounded down) while the second column is doubled.

21	32
~~10~~	~~64~~
5	128
~~2~~	~~256~~
1	512

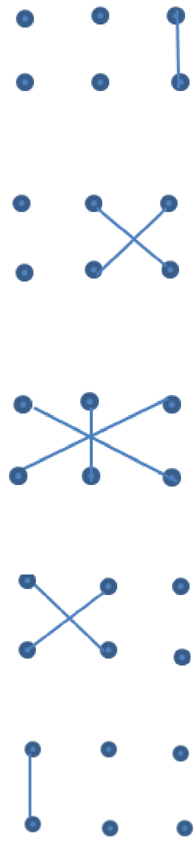
This time, even numbers in the first column eliminate the whole row and the remaining numbers in the second column are then added together. 32 + 128 + 512 = 672.

The Vedic Method

This technique requires the student to know their tables and store numbers in their head before writing down the answer in one line. The key is to see the pattern of digit multiplication.

Example   310 x 212

If we consider each digit as a point on the right hand side then the order of multiplying is shown in five steps.



Step 1 is 0 x 2 =0

Step2 is 1 x 2 + 0 x 1 = 2

Step3 is 3 x 2 + 0 x 2 + 1 x 1 = 7 etc. etc.

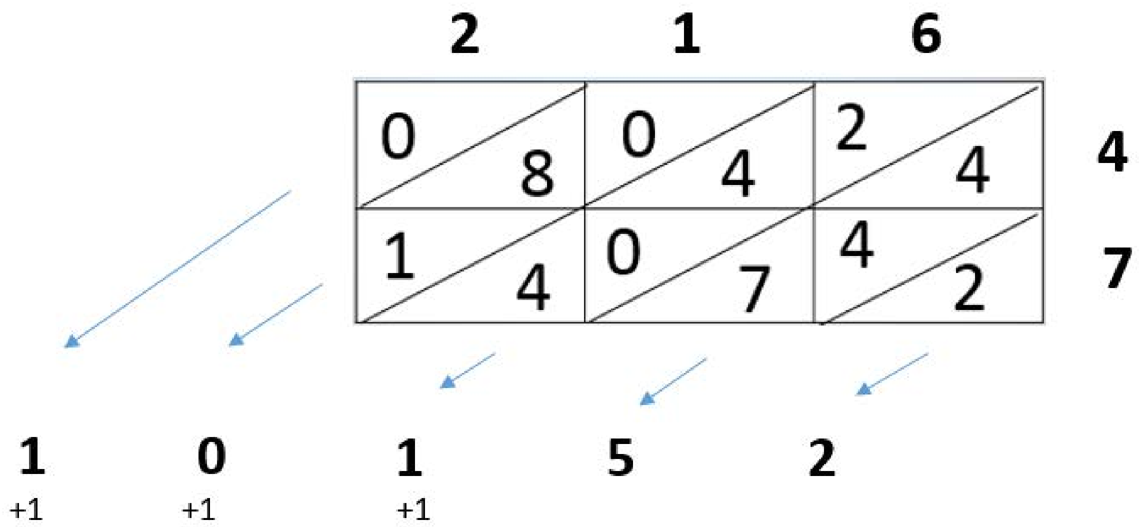
$$\begin{array}{r} 310 \\ \underline{\times\ 212} \\ 65720 \end{array}$$

Sometimes carry digits are needed.

The Lattice (Gelosia) method

This method originated in India and appeared in Europe in the 14th century.

A grid is drawn and the two numbers to be multiplied (216 x 47) are placed at the top and to the side. Each grid is filled with the product of two numbers and then the digits are added diagonally (with carries) to get the answer 10 152.



John Napier probably took this idea to make his calculating device now known as Napier's bones. It was Napier, of course who was credited with the invention of logarithms for multiplying and dividing numbers. Sadly these days log tables are no longer included in Maths textbooks. Even the humble slide rule has become extinct. This was an ingenious calculating device, nicknamed the '*slipstick*', which used logarithmic scales. It became a fashion accessory of all engineers in the 50s and 60s but died a sudden death in the mid 70's with the advent of pocket calculators. It should be noted here, however, that in 1969 Buzz Aldrin used a $10 slide rule on the Apollo 11 mission to navigate all the way to the moon and back.

Visualising Subtraction

Brenda Kettle
Principal Project Officer
QCAA

There is now compelling brain evidence to support the use of visualisation in developing deep mathematical understanding. The following describes a sequence of visual activities designed to build students' conceptual understanding of subtraction using the same question: 58 – 35 = ?

It is important that for each strategy, students are able to explain the steps they took to arrive at an answer.

The activities described provide a firm foundation for students to mentally visualise subtraction situations. In addition to being a precursor to students performing calculations mentally, these visual strategies are also a tool for understanding more complex subtraction situations.

Activity 1
Subtraction with the Maths Mat
A *maths mat* can be made from shade cloth and gaffer tape (tape with a high strength cloth backing). Use laminated cards to jointly construct the hundreds board within the taped squares.

Process: Have a student stand on the starting number 58 and ask them to demonstrate the subtraction by stepping out how they would take away 35.

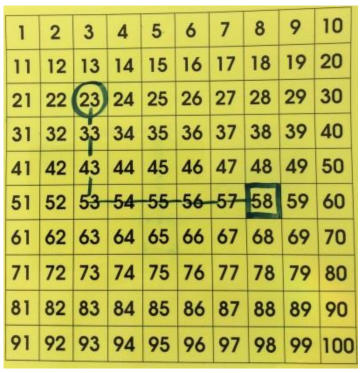
Option 1	Option 2
Begin with taking away the tens by taking three steps up the mat – 48, 38, 28.	Begin with taking away the ones by taking five steps to the left – 57, 56, 55, 54, 53.
Continue with taking away the ones by taking five steps to the left – 27, 26, 25, 24, 23.	Continue with taking away the tens by taking three steps up the mat – 43, 33, 23.
	

It is important that as students work through the problem they explain why they are taking steps up the mat (taking away 10 each time) or across the mat (taking away 1 each time).

Activity 2
Subtraction with the Hundreds Board

Students use the knowledge created in the whole body kinaesthetic modelling on the maths mat (Activity 1) and apply it to subtraction with individual A4 sized 100's boards.

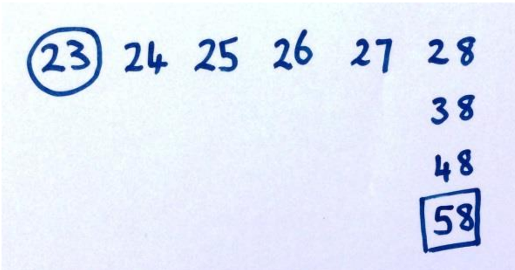
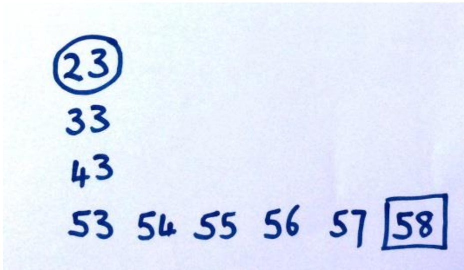
Process: Have students outline the starting number 58 and ask them to show how they would take away 35. Steps are recorded as if they were walking on the maths mat.

Option 1	Option 2
	

Activity 3
Subtraction Using Hundreds Board Jottings

Students visualise the subtraction process created on the hundreds board and use informal jottings to assist them in solving subtraction problems.

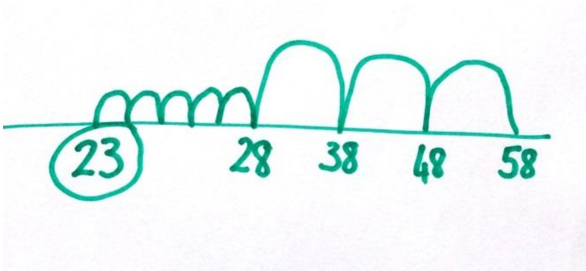
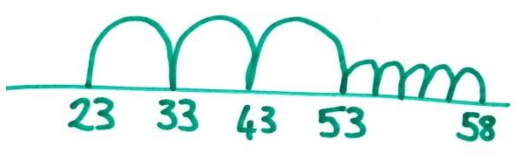
Process: Have students write the number 58 and then write the steps taken to work back towards 23 as if they were walking on the maths mat or using the hundreds board.

Option 1	Option 2
	

Activity 4
Subtraction on an Open Number Line

Students visualise the subtraction process and use jottings on an open number line to assist them in solving subtraction problems.

Process: Students construct a number line to demonstrate the sequence used to subtract 35. Encouraging students to use different sized "jumps" will enhance the capacity to visualise the difference between steps representing tens and those representing ones.

Option 1	Option 2
	

2D & 3D Modelling with Mathematica

Miles Ford
St Johns Anglican College

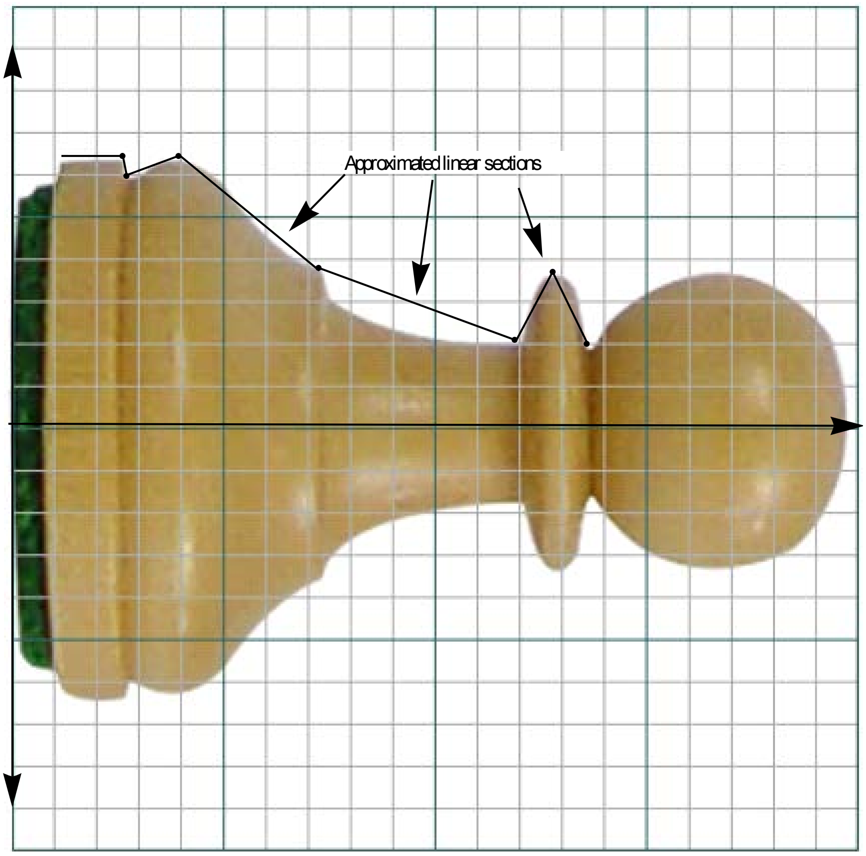
This activity is designed for use with an 11 Mathematics B class prior to them having been introduced to a wide array of function types. If this is being done with a class further into the Mathematics B content then the limitation of using linear and circular functions is not relevant.

Modelling a Chess Piece

Access the internet to get an image of a chess pawn (or use the one included in this activity).

Sketch the pawn onto a piece of graph paper using predominantly circular curves and straight lines.

Now add an x and y axis to your sketch. These can go anywhere, but setting them as shown in the diagram should help simplify identifying the points.



Work only with the image in the first quadrant (above the x-axis). Mark off the start and end points of each linear section and identify the coordinates of the points.

Create a variable for each set of points:

```
In[1]:= points1 = {{1, 6.3}, {2.8, 6.3}};
        points2 = {{2.8, 6.3}, {2.9, 6}};
        points3 = {{2.9, 6}, {4, 6.3}};
        points4 = {{4, 6.3}, {7.3, 3.8}};
        points5 = {{7.3, 3.8}, {12, 2}};
        points6 = {{12, 2}, {12.9, 3.8}};
        points7 = {{12.9, 3.8}, {13.6, 2}};
```

The semicolon at the end of each line suppresses the output as all we're doing here is assigning values to the variables

Then, determine the equation of each line. For the first segment we can see the equation is $y = 6.3$ with a domain of $(1, 2.8)$. For the rest of the segments use *Mathematica*:

```
In[8]:= model2 = LinearModelFit[points2, x, x]["BestFit"]
        model3 = LinearModelFit[points3, x, x]["BestFit"]
        model4 = LinearModelFit[points4, x, x]["BestFit"]
        model5 = LinearModelFit[points5, x, x]["BestFit"]
        model6 = LinearModelFit[points6, x, x]["BestFit"]
        model7 = LinearModelFit[points7, x, x]["BestFit"]
```

Out[8]= $14.7 - 3.\, x$

Out[9]= $0.272727\, x + 5.20909$

Out[10]= $9.3303 - 0.757576\, x$

Out[11]= $6.59574 - 0.382979\, x$

Out[12]= $2.\, x - 22.$

Out[13]= $36.9714 - 2.57143\, x$

We could use a semicolon again as we don't really need to see the equations of the functions at this point if we wished

For the semicircle section (the head of the pawn) visually identify the centre (h,k) and radius (r):

```
In[14]:= r = 3.5; h = 16.5; k = 0;
         circle = k + Sqrt[r^2 - (x - h)^2]
```

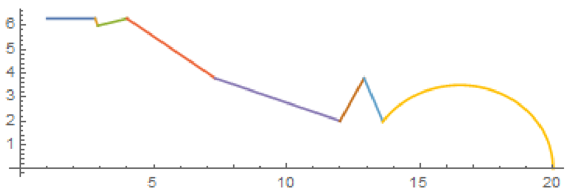
$$\text{circle} = k + \sqrt{r^2 - (x - h)^2}$$

Out[15]= $\sqrt{12.25 - (x - 16.5)^2}$

Plot the functions with their domains (as specified by the x-values of each set of points). To restrict a function to the given domain use && - the logical AND construct:

```
In[16]:= Plot[{6.3 && 1 < x < 2.8, model2 && 2.8 < x < 2.9, model3 && 2.9 < x < 4, model4 && 4 < x < 7.3,
   model5 && 7.3 < x < 12, model6 && 12 < x < 12.9, model7 && 12.9 < x < 13.6, circle && 13.6 < x < 20},
  {x, 0, 20}, AspectRatio -> 6/20]
```

Out[16]=



The AspectRatio is set purely for display purposes and is determined from the overall domain and range indicated on the axes.

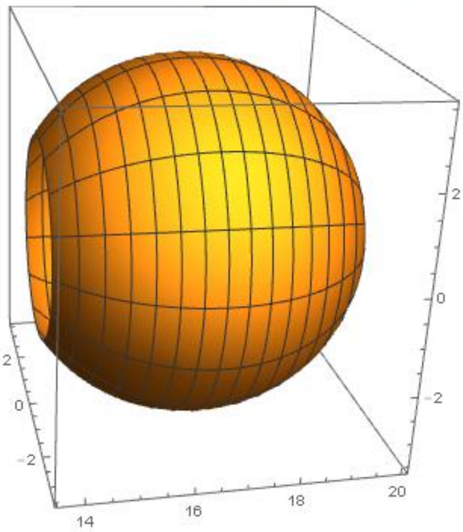
You can then repeat this process for the lower half to produce a 2D profile of the pawn.

3D Potting

Instead of Plotting the functions use RevolutionPlot3D[] on them instead. Here's a simple example using the semicircular pawn head:

```
In[17]:= RevolutionPlot3D[circle, {x, 13.6, 20}, RevolutionAxis → {1, 0, 0}]
```

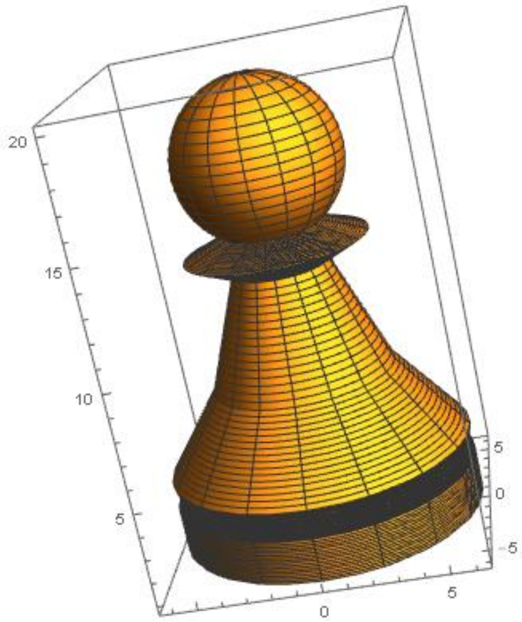
Out[17]=



To display all the functions together on one display use the command Show[] with option PlotRange -> All. Repeat the RevolutionPlot3D[] command for the rest of the functions to produce a 3D model of the pawn:

```
In[18]:= Show[{
    RevolutionPlot3D[6.3, {x, 1, 2.8}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model2, {x, 2.8, 2.9}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model3, {x, 2.9, 4}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model4, {x, 4, 7.3}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model5, {x, 7.3, 12}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model6, {x, 12, 12.9}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[model7, {x, 12.9, 13.6}, RevolutionAxis → {1, 0, 0}, PlotRange → All],
    RevolutionPlot3D[circle, {x, 13.6, 20}, RevolutionAxis → {1, 0, 0}, PlotRange → All]
   }]
```

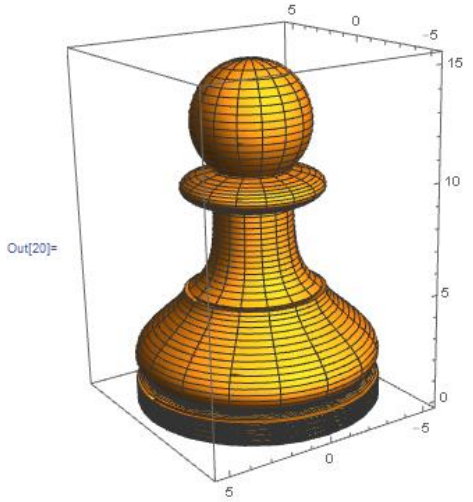
Out[18]=



Going Further

The code shown here produces a more accurate model of the pawn and can be printed from a 3D printer. It utilises a broader variety of functions than the original project, such as exponential and quartic functions. It used a function called NonLinearModelFit[] to determine the functions for the individual sections. The full mathematical functions have been included in the code so that it can be copied straight into *Mathematica* and will execute successfully.

```
In[19]:= a = 0.3;
  pawn = Show[
    RevolutionPlot3D[5.3, {x, 0, 1}, RevolutionAxis → {1, 0, 0}, PlotStyle → Thickness[a], PlotRange → All],
    RevolutionPlot3D[-3.125 x^3 + 15.625 x^2 - 25 x + 17.8, {x, 1, 1.6}, PlotStyle → Thickness[a],
     RevolutionAxis → {1, 0, 0}], RevolutionPlot3D[0.177537 x^3 - 1.94344 x^2 + 5.92484 x - 0.223016,
     {x, 1.6, 5}, PlotStyle → Thickness[a], RevolutionAxis → {1, 0, 0}],
    RevolutionPlot3D[443.224 × 0.3110906170364932^x + 1.592712882247984, {x, 5, 9}, PlotStyle → Thickness[a],
     RevolutionAxis → {1, 0, 0}],
    RevolutionPlot3D[-1.5341489341673953 x^4 + 58.388338364256796 x^3 - 836.0336265536673 x^2 +
      5338.495429731742 x - 12825.727068101038, {x, 9, 10.35}, PlotStyle → Thickness[a],
     RevolutionAxis → {1, 0, 0}], RevolutionPlot3D[Sqrt[2.5^2 - (x - 12.55)^2], {x, 10.3, 15.05},
     PlotStyle → Thickness[a], RevolutionAxis → {1, 0, 0}]]
```



To 3D print the design use the Export[] function and export the file as a .stl file.

```
Export["pawn.stl", %]
```

Using '%' means export the last executed code, so do this as the very next action after having executed the model producing code.

Mathematica® is a registered trademark of Wolfram Research, Inc.

Face book competition: Consider a mathematician that has featured heavily in your teaching or influenced your views about mathematics and the teaching of mathematics.

My favourite mathematician is Benoit B. Mandelbrot because he somehow made geometry interesting with his development of fractals.

Brad Patten
Winner of the 'The Man Who Knew Infinity' Movie Tickets

In **THE MAN WHO KNEW INFINITY,** Dev Patel stars alongside Jeremy Irons and explores the brilliance of a man many believed could decipher the very fabric of the universe - possibly existence itself. Driven by his destiny for a greater calling, Ramanujan's (Patel) life was turned upside down when Cambridge professor, G.H. Hardy (Irons) discovered his talents and plucked him from obscurity in his homeland of India. The pair would go on to become unlikely friends and make up one of history's most bewildering and productive collaborations. As mathematicians they worked on the most complex problems known to man and much of Ramanujan's work is still relevant in maths and science today.

Gauss – A Fundamentally Complex Life

Johann Carl Friedrich Gauss (30 April 1777 – 23 February 1855) was a German mathematician who contributed significantly to many fields, including number theory, algebra, statistics, analysis, differential geometry, geodesy, geophysics, mechanics, electrostatics, astronomy, matrix theory, and optics. Often referred to as the, "the foremost of mathematicians" and "greatest mathematician since antiquity", Gauss had an exceptional influence in many fields of mathematics and science and is ranked as one of history's most influential mathematicians.

Students frequently get told the story of Gauss as a child prodigy and his method of adding the numbers from 1 to 100 when he was eight. Although this story may be contested his later achievements as a mathematician are unquestionable. This title refers to work in his 1799 doctorate *A new proof of the theorem that every integral rational algebraic function of one variable can be resolved into real factors of the first or second degree*. In this work Gauss proved the fundamental theorem of algebra which states that every non-constant single-variable polynomial with complex coefficients has at least one complex root. His initial proof was called into question for lacking rigour but it is accepted that a subsequent version in 1849 was generally rigorous. His attempts clarified the concept of complex numbers considerably along the way.

This contains many powerful messages for students of high level mathematics subjects. Gauss spent fifty years and had many attempts to produce a proof acceptable to his peers. When mathematical problem solving is required then perseverance is essential.

Jim Lowe
Winner the Men of Vision DVD pack which includes the following DVDs: I'm Not There; Love & Mercy; Nowhere Boy; Slumdog Millionaire; The Motorcycle Diaries and The World's Fastest Indian

Answers to Student Problems

Volume 41, Number 1

1. $\frac{(a^x)^2}{a^{-2}} = a^{4x} \times a$

$\frac{a^{2x}}{a^{-2}} = a^{4x+1}$

$a^{2x+2} = a^{4x+1}$

$2x + 2 = 4x + 1$ Equate Indices

$x = \frac{1}{2}$

2. $\frac{11+n}{23+n} = \frac{1}{5}$

$5(11 + n) = 1(23 + n)$

$55 + 5n = 23 + n$

$4n = -32$

$n = -8$

3. $\sqrt{b} = 1\frac{1}{2}$

$\sqrt{b} = \frac{3}{2}$

$b = \frac{9}{4}$

$b^3 = \left(\frac{9}{4}\right)^3$

$b^3 = \frac{729}{64}$

$b^3 = 11\frac{25}{64}$

4. Let first integer = n

$$\frac{n+n+1+n+2+n+3+n+4+n+5+n+6+n+7+n+8+n+9+n+10+n+11}{4}$$

$$= \frac{12n+66}{4}$$

$$= \frac{12n}{4} + \frac{66}{4}$$

$$= 3n + 16 + \frac{2}{4}$$

The remainder is 2.

5. $(a+b)^2 = (a-b)^2 + a^2$ Using Pythagoras' Theorem

$$a^2 + 2ab + b^2 = a^2 - 2ab + b^2 + a^2$$

$$2ab = -2ab + a^2$$

$$4ab = a^2$$

$$4 = \frac{a^2}{ab}$$

$$\frac{4}{1} = \frac{a}{b}$$

$a:b$ is $4:1$

Entries
Solutions for the student Problems were submitted by All Saints Anglican School, Centenary SHS, Kingaroy SHS, King's Christian College, Moreton Bay College, St Teresa's Catholic College Noosaville, St Ursula's College Toowoomba

Winners
Congratulations are extended to Maeve Cairns of All Saints Anglican School and Willow Wilkes of Moreton Bay College.

Prizes are provided by our generous sponsor, The University of Queensland.

Submitting Solutions
Students are invited to submit solutions to the Student Problems.

Please photocopy the problem page and clearly print your name, your school, and your year level.

Write your solutions (with full working) for each question.

Send your solutions to:

"QAMT Student Problems"
C/- Rodney Anderson
Moreton Bay College
PO Box 84
WYNNUM QLD 4178

Closing date is Friday, 12th August

Queensland STEM Education Network Reference Group

Student Problems

Name: .... School: .... Year: ....

Write your solutions (with working) next to each question or on a separate sheet of paper or by filling in the appropriate boxes.

A pdf copy of the student problems is on www.qamt.org/resources.

Question 1. Given

$a = b$	
$a^2 = ab$	multiply both sides by a
$a^2 - b^2 = ab - b^2$	subtract both sides by b^2
$(a-b)(a+b) = a(a-b)$	factorise
$a + b = a$	cancel
$2a = a$	$b = a$
$2 = 1$	

How? Algebraically it is correct.

Question 2. Algebraically determine the maximum number of gold spheres of radius p cm that can be made from a solid cube of gold with a side length of 5p centimetres. (The gold is melted to form the spheres.)

Question 3. What is the maximum area that can be enclosed by ten (non-bending) 1 metre fence sections?

Question 4. A rectangular box has faces with areas of 48 cm^2, 60 cm^2 and 80 cm^2. Algebraically determine the side lengths and the volume of the box.

Question 5. Show full algebraic reasoning to determine the value(s) of n such that

$$8^{2n} = 4^{n^2}$$

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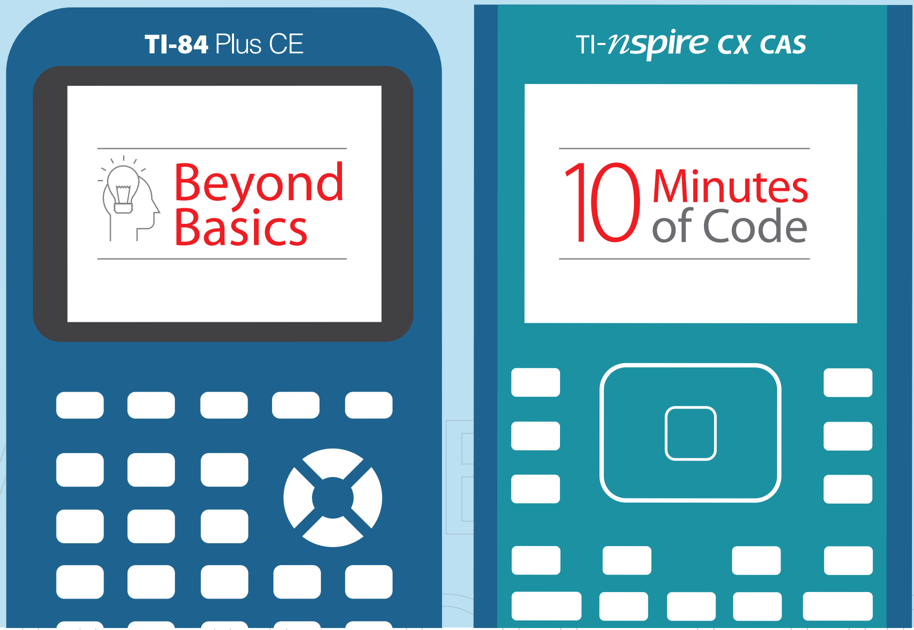
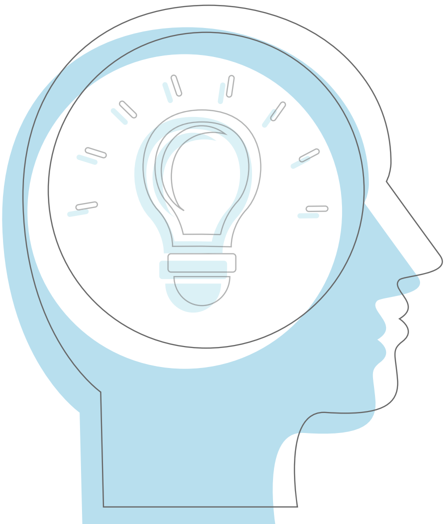


Encourage students to explore basic programming using the TI-Nspire™ or TI-84 Plus technology they use every day.



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Engage students in short activities that build understanding of maths concepts, programming logic and coding skills.

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