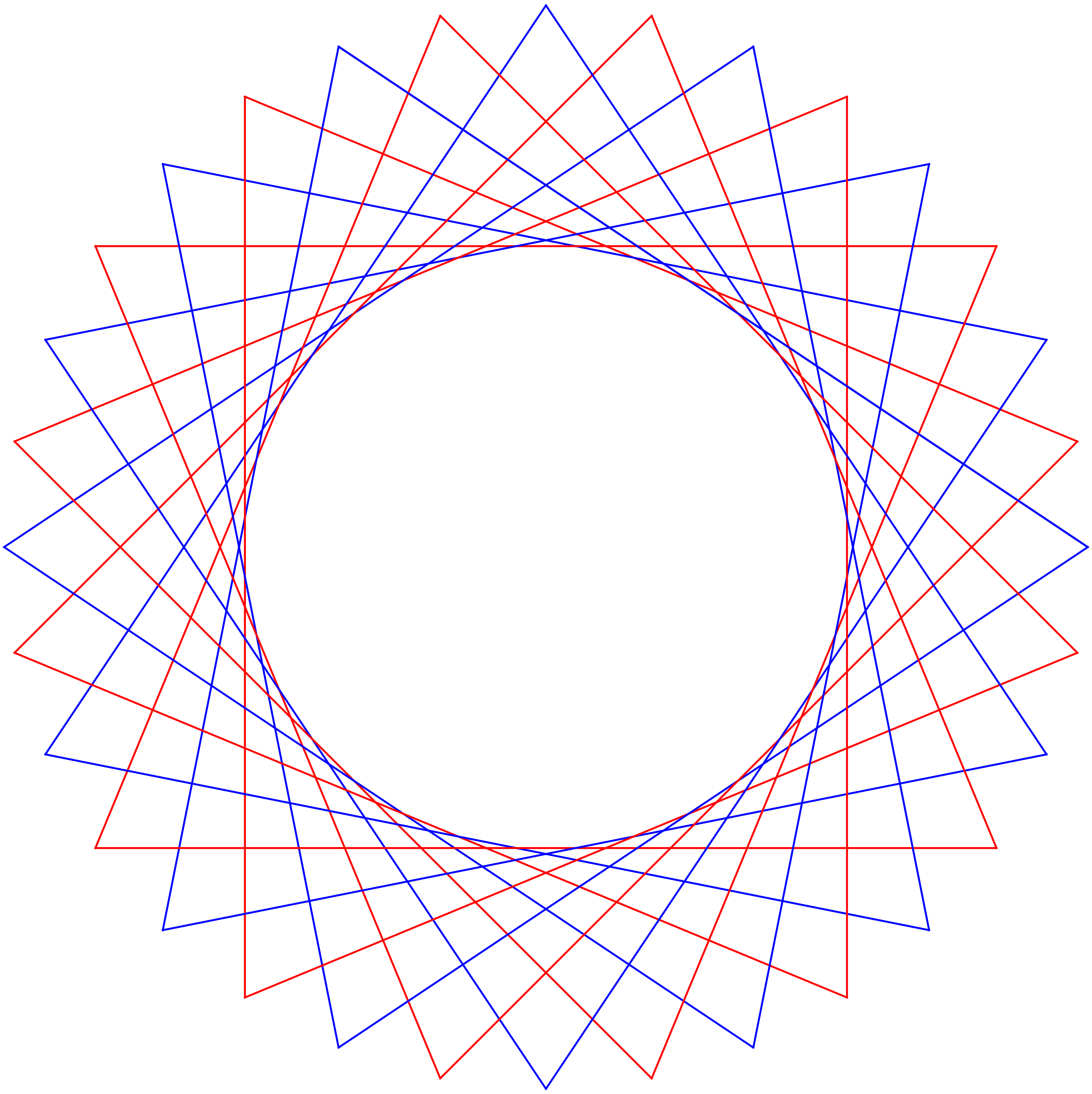
TEACHING MATHEMATICS

Volume 42 Number 3 August 2017



The Journal of the Queensland Association of Mathematics Teachers Inc

August 2017 Volume 42 No. 3

TEACHING MATHEMATICS

Regulars

Features

Teaching Ideas

Front Cover

The cover shows a simple 'string graph'. See the article on Page 24 for instructions and more examples.

QAMT Executive & Committee

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Teaching Mathematics is the official journal of the Queensland Association of Mathematics Teachers Inc. The journal is published four times a year (March, June, September and November) and is provided as a right to all members of the association.

The aims of the journal reflect, put into effect, and communicate the objectives of the association:

- to promote interest in mathematical education at all levels;
- to encourage and promote research in the teaching of mathematics;
- to speak and act on matters related to mathematical education.

Advertising material published in and accompanying this journal does not imply endorsement by the association.

Educators, administrators, parents and students are invited and encouraged to contribute articles which relate to the teaching of mathematics at any level. These may be of any length from interesting 2-line snippets, through short letters concerned with topical issues to longer in-depth articles. Critical comments and advice about the future directions of the journal are always welcome.

Materials should be sent to the Editor, Rodney Anderson. The preferred way is by email. Contact details are as follows:

Rodney Anderson
Moreton Bay College

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Microsoft Word is the preferred format. All receipts will be acknowledged - if you haven't heard within a week, e-mail Rodney to check. Copy dates are: mid-February; mid-May; mid-August; mid-October.

The views expressed in articles contained in Teaching Mathematics belong to the respective authors and do not necessarily correspond to the views and opinions of the Queensland Association of Mathematics Teachers.

If you have any questions regarding Teaching Mathematics, contact Rodney Anderson. Publications sub-committee members are listed below. Feel free to contact any of these concerning other publication matters.

Rodney Anderson (Convenor)	Moreton Bay College	07 3390 8555
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Books, software etc. for review should be sent to Rodney Anderson; information to go in the newsletter should be sent to Gaynor Johnson at the QAMT Office. Newsletter copy dates are the beginning of each term.

Contact the QAMT office for advertising enquiries.

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QAMT also now offers a colour advertising service for the covers.

From the President

Greg Bland

I would like to start by thanking those of you who were able to make it to Toowoomba for the QAMT Annual Conference this year. The Executive Committee received some very positive and constructive feedback from conference delegates, and one of the most pleasing aspects from my perspective was the quality and variety of individual break-out presentations that we had in the program. We had a significant number of first-time presenters who did a fantastic job, as well as the evergreen regulars who have shared so much of their wisdom and enthusiasm with us over the years.

As well as receiving much-appreciated feedback from those who attended the conference, I would be equally keen to hear from those of you who were not able to make it or chose not to attend for whatever reasons. Despite the quality presentations, it was a little disheartening to have fewer than 100 people attend the conference – we would sincerely like to aim for at least double that number in what should be our flagship professional development event of the calendar year.

Clearly, when we move the conference out of the capital, it becomes more difficult for us to anticipate numbers as there are additional complications in terms of flights and transport, etc. (even though Toowoomba is only 90 minutes' drive from Brisbane). The PD Subcommittee would be interested to hear your thoughts about our current policy of alternating between Brisbane and another regional setting for our conference, and we would also appreciate any thoughts you might have about the timing of the event. I'd also like to hear your thoughts on whether you'd be more inclined to attend a more residential-style conference in a non-school venue (our conference at Twin Waters comes to mind).

Throughout my time with QAMT, we have experimented with different dates, different venues, and also other arrangements such as having a joint conference with the Science Teachers Association of Queensland (STAQ). I'd be very keen to hear from you if you have any thoughts or observations about what we could do to make our annual conference a more appealing event (and don't worry about being blunt!). My email address is g.bland@twgs.qld.edu.au.

I hope that this busy term is treating you well, and I look forward to catching up with you soon.

Greg Bland
President, QAMT

From the Editor

Rodney Anderson

A reminder that we encourage contributions from members for the Journal, since after all, it is your Journal. This is a chance to share your ideas and practices with other members. We also welcome suggestions for particular topics that you would like to read about.

E-mail suggestions and submissions to andersonr@mbc.qld.edu.au

Incentive to contribute articles/teaching ideas to the journal

QAMT MEMBERSHIP DRAW

1 For every article/teaching idea contributed, the author will receive a ticket in a Membership Draw.

2 If you are contributing an article/teaching idea for the first time, the author will receive two tickets in a Membership Draw.

Student Teacher Bursary

Lisa Gisik

I feel privileged to have been given the opportunity for a bursary to attend the QAMT conference this year. The positives were the warm welcome the Pre-service teachers all received, we had the opportunity to meet so many senior mathematics teachers from around Queensland and the exposure and quality of all the guest speakers was fantastic.

The networking opportunities the conference gave me was also highly beneficial, as I learnt so much about different mathematics teachers experiences in various schools and was given an insight into how to I can become a better mathematics teacher. I learnt that there are many ways to engage students to enjoy mathematics and a wide variety of teaching strategies to be used to make 'Maths fun'.

Overall, I thoroughly enjoyed participating in the 2017 QAMT and look forward to future conferences.

Coco Bu

Positives

There are so many positives in this year's conference. The large number of quality presentations is by far the most beneficial aspect for me from a learner's point of view. The presentations also closely reflected the theme of the conference—A Game changer. It inspires me of how I can use games, and interactive and hands-on activities in class to raise my students' interests and participation. The other big positive for me is the networking opportunities the conference provided, which is valuable for my professional development.

Networking

The conference provided fun and relaxing networking opportunities through star-gazing, welcome drinks, and dinner gathering. I found it easy and inspiring talking to experienced and passionate teachers through those social events. I keep in touch with several teachers via email and LinkedIn, some of whom I am going to visit in the short future and some of whom might potentially become my supervisor in the coming practicum. Through talking to the professionals, I not only learned their teaching strategies, curriculum changes but also gained confidence in myself as I believe by being in such a supportive community, one day, I might become one of the confident and inspiring teachers as well.

What you learnt?

I learned so much. One thing that stands out from all of the PD workshops is the use of technology, especially calculators. I wasn't a great fan of calculators as I think students depend on them too much. But after hearing some of the presenters on how they made use of the calculators, I started to realise maybe they can be used much more or in a much better way than just computation. It also made me realise that as a teacher, I should have the fluency in using the calculators and be able to design when and how to use them so that students can benefit from them the most.

General comments

It was a shame that I couldn't go to more workshops than I did. I missed some of the interesting ones as the time clashed. However, having such a large collection of presentations is also the biggest positive and the greatest success of the conference. Overall, a great success and an invaluable experience for me as a pre-service teacher! I thoroughly enjoyed it!

On the next two pages are photographs taken at the 2017 Annual Conference.


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National Mathematics Summer School

Rachel Hauenschild

In January, I was lucky to be selected as one of 12 Queensland students to attend the National Mathematics Summer School at ANU. It was an incredible experience, spending two weeks learning maths and meeting like-minded people from all over Australia.

At NMSS, we studied three courses: number theory, projective geometry and languages and automata. The lecturers were all engaging and passionate, and the tutorials gave us the opportunity to ask questions and really understand the topics. Evening private study gave us even more time to talk the tutors and lecturers, which made the learning very personalised and rewarding.

We also had lots of opportunities to explore Canberra on the weekend and free afternoons. At the college, there were countless board games and puzzles, and always friendly people to talk to. The two weeks culminated in a formal dinner, concert and party to farewell the amazing people we had shared the experience with.

I would like to thank all the staff and students for a truly incredible experience. In two short weeks, I was able to learn so much maths and meet amazing people who I look forward to staying in touch with everyone in the future. The NMSS was a wonderful experience that I would highly recommend to anyone with an interest in maths and desire to meet like-minded people.

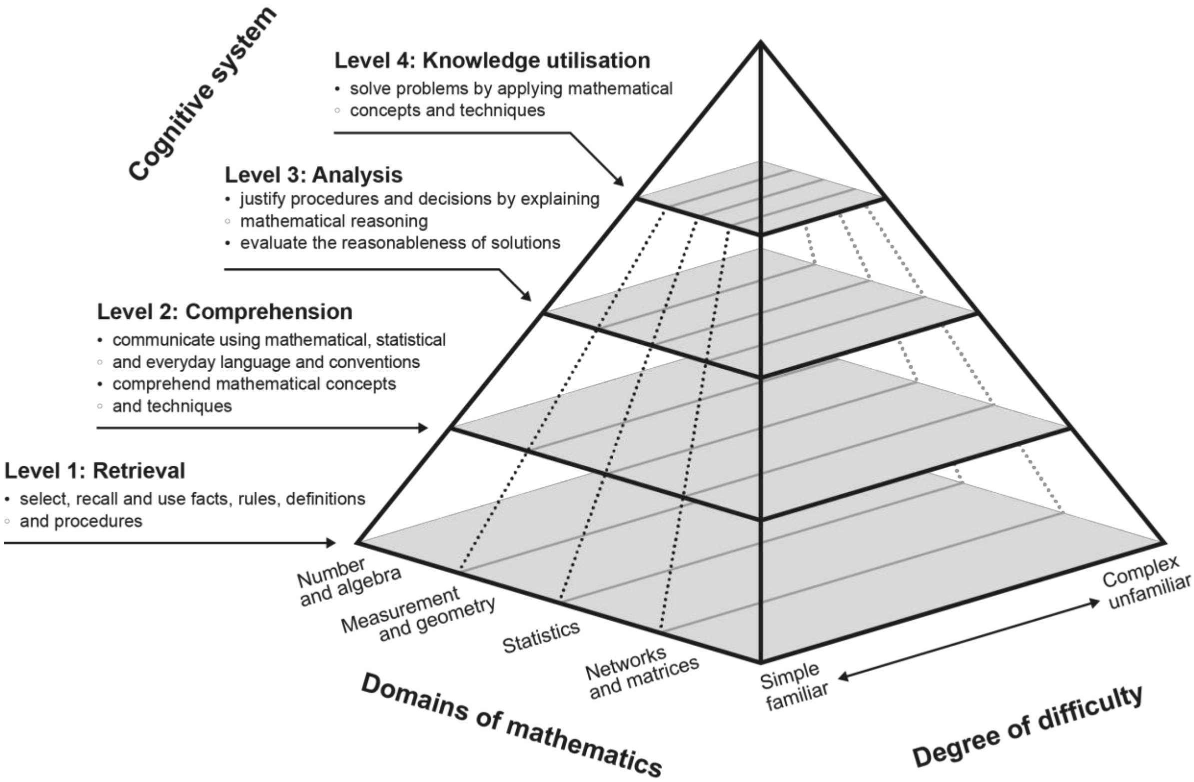
The Pedagogical and Conceptual Frameworks in the new Queensland Senior Mathematics Syllabuses

The Queensland Curriculum and Assessment Authority (QCAA) is redeveloping senior syllabuses to support the introduction of the new QCE system from 2019. Teaching, learning and assessment in the new senior mathematics syllabuses is underpinned researched-based pedagogical and conceptual frameworks. The first framework, *The relationship between foundational knowledge and problem-solving*, outlines the approach to developing mathematics problems and examination design in the syllabuses. The second framework, *Problem-solving and mathematical modelling*, outlines a four-stage iterative process that students will undertake when solving real-world mathematics problems and provides the structure for the Problem-solving and modelling task (PSMT) in the syllabuses.

The following is based on an extract of these frameworks from General Mathematics 2017 v1.1.

The relationship between foundational knowledge and problem-solving
To succeed in mathematics assessment, students must understand the subject matter (organised in domains of mathematics), draw on a range of cognitive skills, and apply these to problems of varying degrees of difficulty, from simple and routine, through to unfamiliar situations, complex contexts, and multi-step solutions (Grønmo et al. 2015). The relationship between the domains of mathematics in General Mathematics, level of cognitive skill required (syllabus objective) and degree of difficulty is represented in three dimensions for mathematics problems in the following diagram.

Figure 1: Assessment pyramid



Adapted from Verhage & de Lange (1997) and Marzano & Kendall (2007).

Principles of developing mathematics problems

This representation, known as the 'assessment pyramid', shows the relative distribution of thinking and range of difficulty of mathematics problems. It places an emphasis on building up from the basics. Success in mathematics is built on knowledge of basic facts and proficiency with foundational processes (Norton & O'Connor 2016). With a solid foundation, students can then be asked to apply higher level cognitive processes in more complex and unfamiliar situations that require the application of a wider range of concepts and skills.

The cognitive system

To solve a full range of mathematics problems, students are required to engage the cognitive system at all four levels of processing knowledge: retrieval, comprehension, analysis and knowledge utilisation (Marzano & Kendall 2007). The syllabus objectives are represented in the pyramid model through their alignment to these levels.

The degree of difficulty

The difficulty of a problem is defined by its complexity and a student's familiarity with it, not the level of cognitive process required to solve it. The complexity of a particular type of problem doesn't change, but familiarity does. With practice, students become more familiar with a process and can execute it more quickly and easily (Marzano & Kendall 2007).

Using a full range of questions

The pyramid model shows that Level 1 problems can be hard and relatively complex, even though they are based on 'retrieval' and therefore might seem easy and straightforward (Shafer & Foster 1997). Problems in the higher levels are not necessarily more difficult than those in Level 1. There are some students who find Level 1 more challenging and have more success in solving problems in Levels 2, 3 and 4 (Webb 2009).

The distance along the domains of mathematics dimension and the degree of difficulty dimension decreases for higher levels. Level 1 problems can more easily be based on distinct subject matter and the difference between easy and hard can be great. Problems that require students to use more levels of cognition tend to also involve making connections with subject matter within and across the domains of mathematics. They are often placed in contexts that require strategic mathematical decisions and making representations according to situation and purpose. At higher levels the difference between easy and hard is smaller (Shafer & Foster 1997; Webb 2009). Students should master basic facts and processes through practising simple familiar problems before moving on to those that are more complex and unfamiliar, at any level.[1]

The assessment pyramid helps visualise what is necessary for a complete assessment program. Problems in a complete mathematics program need to assess a student's growth and achievement in all domains of mathematics and across the full range of objectives. Over time, through a teaching and learning period, students will be exposed to problems that 'fill the pyramid'. Each assessment instrument will reflect this for the relevant subject matter, providing students with the opportunity to demonstrate what they know and can do at all levels of thinking and varying degrees of difficulty (Shafer & Foster 1997).

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.

Problem-solving
Problem-solving is required when a task or goal has limiting conditions placed upon it or an obstacle blocking the path to a solution (Marzano & Kendall 2007). It involves

- knowledge of the relevant details
- using generalisations and principles to identify, define and interpret the problem
- mental computation and estimation
- critical, creative and lateral thinking
- creating or choosing a strategy
- making decisions
- testing, monitoring and evaluating solutions.

Problem-solving requires students to explain their mathematical thinking and develop strong conceptual foundations. They must do more than follow set procedures and mimic examples without understanding. Through problem-solving, students will make connections between mathematics topics, across the curriculum and with the real world, and see the value and usefulness of mathematics. Problems may be real-world or abstract, and presented to students as issues, statements or questions that may require them to use primary or secondary data.

Mathematical modelling
Mathematical modelling begins from an assumption that mathematics is everywhere in the world around us — a challenge is to identify where it is present, access it and apply it productively. Models are developed in order to better understand real-world phenomena, to make predictions and answer questions. A mathematical model depicts a situation by expressing relationships using mathematical concepts and language. It refers to the set of simplifying assumptions (such as the relevant variables or the shape of something); the set of assumed relationships between variables; and the resulting representation (such as a formula) that can be used to generate an answer (Stacey 2015).

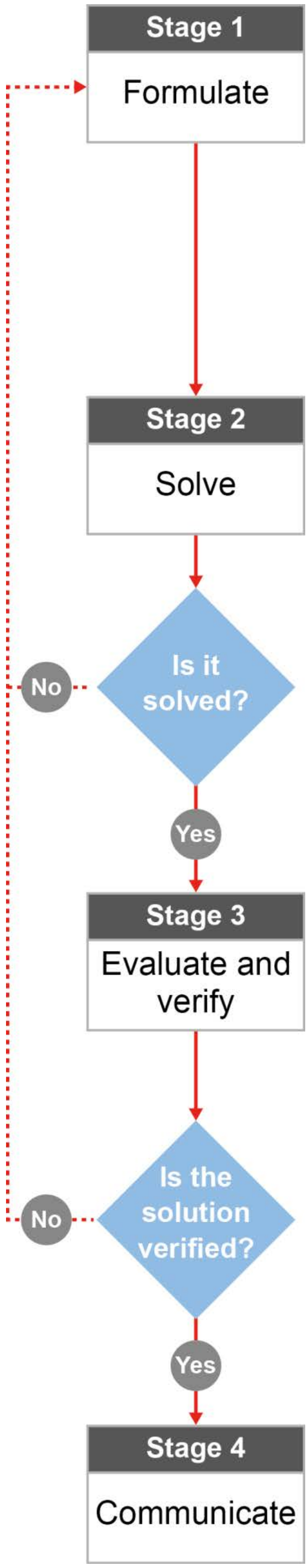
Mathematical modelling involves:

- formulating a mathematical representation of a problem derived from within a real-world context
- using mathematics concepts and techniques to obtain results
- interpreting the results by referring back to the original problem context
- revising the model (where necessary) (Geiger, Faragher & Goos 2010).

Through developing and applying mathematical models, students cumulatively become real-world problem-solvers. Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.

The following section outlines an approach to problem-solving and mathematical modelling. Problems must be real-world, and can be presented to students as issues, statements or questions that may require them to use primary or secondary data.

Figure 2: An approach to problem-solving and mathematical modelling



Once students understand what the problem is asking, they must design a plan to solve the problem. Students translate the problem into a mathematically purposeful representation by first determining the applicable mathematical and/or statistical principles, concepts, techniques and technology that are required to make progress with the problem. Appropriate assumptions, variables and observations are identified and documented, based on the logic of a proposed solution and/or model.

In mathematical modelling, formulating a model involves the process of mathematisation — moving from the real world to the mathematical world.

Students select and apply mathematical and/or statistical procedures, concepts and techniques previously learnt to solve the mathematical problem to be addressed through their model. Possible approaches are wide-ranging and include synthesising and refining existing models, and generating and testing hypotheses with primary or secondary data and information, as well as using standard mathematical techniques. This process may require returning to the initial observations and assumptions, and reconsidering and modifying them to ensure the problem has been solved or can actually be solved.

Solutions can be found using algebraic, graphic, arithmetic and/or numeric methods, with and/or without technology.

Once a possible solution has been achieved, students need to consider the reasonableness of the solution and/or the utility of the model in terms of the problem. They evaluate their results and make a judgment about the solution/s to the problem in relation to the original issue, statement or question.

This involves exploring the strengths and limitations of the solution and/or model. Where necessary, this will require going back through the process to further refine the solution and/or model. In mathematical modelling, students must check that the output of their model provides a valid solution to the real-world problem it has been designed to address.

This stage emphasises the importance of methodological rigour and the fact that problem-solving and mathematical modelling is not usually linear and involves an iterative process.

The development of solutions and models to abstract and real-world problems must be capable of being evaluated and used by others and so need to be communicated clearly and fully. Students communicate findings systematically and concisely using mathematical, statistical and everyday language. They draw conclusions, discussing the key results and the strengths and limitations of the solution and/or model. Students could offer further explanation, justification, and/or recommendations, framed in the context of the initial problem.

Approaches to problem-solving and mathematical modelling in the classroom

When teaching problem-solving and mathematical modelling, teachers should consider teaching for and learning through problem-solving and mathematical modelling. When teaching for, students are taught the specific mathematical rules, definitions, procedures, problem-solving strategies and critical elements of the model, that are needed to solve a given problem. When learning through, students are presented with problems to solve, but must apply the knowledge and skills they have previously been taught to solve it. By solving these problems, students are able to develop new mathematical understanding and skills. This requires an explicit and connected approach to teaching problem-solving and mathematical modelling that necessitates fluency of critical facts and processes at each step.

The following describes three different approaches to teaching problem-solving and mathematical modelling along the continua between *teaching for* and *learning through*:

Approach	Description	***Teaching for*** **or** ***learning through***
Dependent	The teacher explicitly demonstrates and teaches the concepts and techniques required to solve the problem, and/or develop a mathematical model. This usually involves students solving (stage 2), and evaluating and verifying (stage 3).	*Teaching for*
Guided	The teacher influences the choice of concepts and techniques, and/or model that students use to solve the problem. Guidance is provided and all stages of the approach are used.	Moving towards *learning through*
Independent	The teacher cedes control and students work independently, choosing their own solution and/or model, and working at their own level of mathematics. The independent approach is the most challenging.	*Learning through*

These approaches are not mutually exclusive. An independent approach (learning through) might be undertaken as an extension of a dependent or guided activity that students have previously undertaken (teaching for). Students need to have attained the relevant foundational understanding and skills before working independently during a problem-solving and modelling task. This capacity needs to be built over time through the course of study with teachers closely monitoring student progress.

[1] Complex unfamiliar questions that require more levels of cognitive skills should not be equated with elaborate problem-solving tasks and modelling questions only. A single-answer, conventional question, such as: ‘Find the equation of the line passing through the points (2,1) and (1,3)’ can be adapted to a more open-ended question, such as: ‘Write the equations of at least five lines passing through the point (2,1)’ (Goos 2014). This revised question targets the identical subject matter but provides the possibility of easily identifying diverse student understanding and skills by moving it towards complex unfamiliar questions and assessing more cognitive skills. For further examples, see White et al. (2000).

[2] A wide variety of frameworks for problem-solving and modelling exist in mathematics education literature. The approach outlined here aligns with and is informed by other approaches, such as Polya in *How to Solve It: A new aspect of mathematical method* (1957), the Australian Curriculum (2015) *Statistical investigation process*, the OECD/PISA Mathematics framework (OECD 2015, 2003) and ‘A framework for success in implementing mathematical modelling in the secondary classroom’ (Stillman et al. 2007). For further reading see Blum et al. (2007); Kaiser et al. (2011); and Stillman et al. (2013).

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NZAMT New Zealand Association of Mathematics Teachers

BACK TO THE FUTURE 2017

15th Biennial Conference

proudly hosted by the Canterbury Mathematical Association www.nzamt2017.com

Diarise these dates

The Canterbury Mathematical Association has the pleasure of hosting the biennial NZAMT conference in Christchurch from Tuesday 3rd October to Friday 6th October 2017. We invite you to join us and enrich the conference experience to help make it the best NZAMT event yet.

Our theme for the conference is Back to the Future and will focus on Blast from the Past, Present Practice and Preparing for the future. We plan our conference to be simultaneously future focused and mindful of past wisdom. The conference theme embraces moving mathematics and statistics education forward with all its possibilities of incorporating innovation, expanding knowledge and technology, while at the same time, looking back, learning from and honouring what has gone before.

Our conference will offer a thought provoking, exciting and fun programme to leave you rejuvenated and full of inspiration for your return to the classroom. Please check out our conference website http://www.nzamt2017.com/.

Important dates for your diary:

- Conference: 3 October – 6 October 2017
- Abstract submission: open now, closes 31 May 2017 (for further details see our Call for Abstracts flyer attached, and on the conference website)
- Conference registration: opens 10 February 2017

We look forward to hearing from you.

NZAMT2017 organising committee
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Teaching Ideas

Square of a Sum – Going to the Pictures

Robert Nelder

We normally use any of several ways to try to teach students that in general

$$(a+b)^2 \neq a^2 + b^2$$

These would include algebraic expansion, area diagrams, numerical verification with calculator memories, etc.

Many students forget about the "correction term" *2ab.* They believe that this extra term wrecks the innocent simplicity of a beautiful formula. *"But it works if the 'plus' changes to 'multiply', Sir! No correction term is needed there."*

What about this next idea, as yet another attempt to convince students about the rule? (I wish I'd thought of this variation during my career).

Seven boys and three girls go together to the pictures. That means the movies. Admission costs $7 each and each person buys a $3 chocolate on the way in.

We have (7 + 3) people each spending (7 + 3) dollars.

So the total amount spent should be (7 + 3)(7 + 3) or $(7 + 3)^2$ dollars. $100.

But if you say that this is the same as 7×7 + 3×3, i.e. $(7 + 3)^2 = 7^2 + 3^2$, then look what's happened

- The boys have their tickets, but no chocolates, and
- The girls have their chocolates, but no tickets.

To fix this, we need to pay the extra amounts of

7×3 so that the boys will have chocolates, and

3×7 so that the girls will have their tickets.

Therefore (7 + 3)(7 + 3) or simply $(7 + 3)^2$ must be the same as $7^2 + 3^2 + 2×7×3$.

Dimension 5 – Habits of Mind in Action

Benjamin Morris
Christopher Blood
Brisbane Boys' College

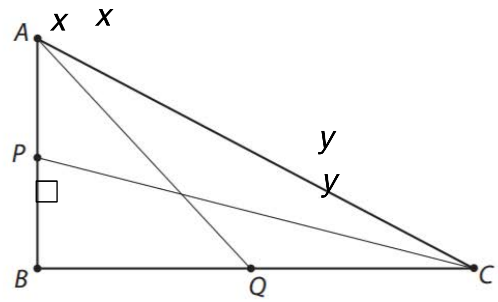
In the Dimensions of Learning teaching and learning model, Dimension 5 is often referred to as the Habits of Mind area of learning, where students develop thinking skills that enhance the connection of learning with the long term memory.

Benjamin Morris is a final year student. He remembered the question in this article from the 2016 QAMT Problem Solving Competition. He had not obtained a satisfactory solution though had not given up. This persistence enabled him to retain his familiarity of the problem until new information acquired in year 12 Mathematics C enabled him to complete an articulate solution. The problem and the solution are provided below and can serve as a model for other students to see how a student can enjoy mathematics to the level displayed by this student.

Question 5

Triangle ABC is right angled at B, as shown below. P is a point on AB with $\angle BCP = \angle PCA$. Q is a point on BC with $\angle BAQ = \angle QAC$. If $AQ = 9$ and $CP = 8\sqrt{2}$, what is the length AC?

(2016 QAMT Problem Solving Competition)



Let $\angle BAQ = \angle QAC = x$ and $\angle BCP = \angle PCA = y$ and $\angle ABC = \frac{\pi}{2}$.

Thus $\frac{\pi}{2} + 2x + 2y = \pi \;\rightarrow\; x + y = \frac{\pi}{4} \;\rightarrow\; y = \frac{\pi}{4} - x$ (A)

In ΔABQ, $AB = AQ\cos x \;\rightarrow\; AB = 9\cos x$ (B)

In ΔCBP, $CB = CP\cos y \;\rightarrow\; CB = 8\sqrt{2}\cos\left(\frac{\pi}{4} - x\right)$ (from A)

From addition formula,

$$CB = 8\sqrt{2}\left(\cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x\right) = 8\sqrt{2}\left(\frac{\cos x + \sin x}{\sqrt{2}}\right) = 8\cos x + 8\sin x \qquad \text{(C)}$$

In ΔABC, $\tan 2x = \dfrac{CB}{AB}$

From double angle formula and equations B and C,

$$\frac{2\tan x}{1 - \tan^2 x} = \frac{8\cos x + 8\sin x}{9\cos x} \;\rightarrow\; 18\sin x = 8\left(1 - \tan^2 x\right)\left(\cos x + \sin x\right)$$

Upon simplification,

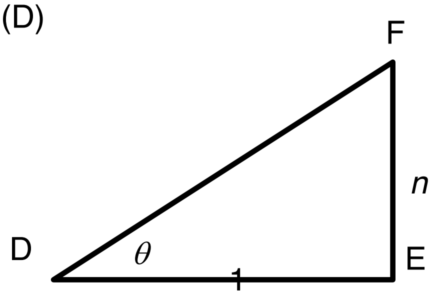
$$4\tan^3 x + 4\tan^2 x + 5\tan x - 4 = 0 \quad \rightarrow \quad 4z^3 + 4z^2 + 5z - 4 = 0 \qquad (z = \tan x)$$

$$(2z-1)(2z^2+3z+4) = 0 \qquad \text{(from polynomial division)}$$

$$z = \frac{1}{2} \text{ or } 2z^2 + 3z + 4 = 0$$

However, $2z^2 + 3z + 4 = 0$ has no real roots as the discriminant $b^2 - 4ac < 0$.

Therefore, $z = \tan x = \frac{1}{2} \quad \rightarrow \quad x = \tan^{-1}\left(\frac{1}{2}\right)$ (D)



In order to proceed consider the right triangle, ΔDEF with $\angle EDF = \theta,\ DE = 1,\ FE = n.$

From Pythagoras, $DF = \sqrt{1+n^2}$

$$\tan\theta = n \ \rightarrow \ \theta = \tan^{-1} n$$

$$\cos\theta = \frac{1}{\sqrt{1+n^2}} \quad \rightarrow \quad \boxed{\cos\left(\tan^{-1} n\right) = \frac{1}{\sqrt{1+n^2}} = \frac{\sqrt{1+n^2}}{1+n^2}}$$

$$\sin\theta = \frac{n}{\sqrt{1+n^2}} \quad \rightarrow \quad \boxed{\sin\left(\tan^{-1} n\right) = \frac{n}{\sqrt{1+n^2}} = \frac{n\sqrt{1+n^2}}{1+n^2}}$$

In

$$\Delta ABC, \qquad AB = 9\cos\left(\tan^{-1}\frac{1}{2}\right) = \frac{18\sqrt{5}}{5} \qquad \text{(from D)}$$

$$CB = 8\cos\left(\tan^{-1}\frac{1}{2}\right) + 8\sin\left(\tan^{-1}\frac{1}{2}\right) = \frac{24\sqrt{5}}{5}$$

So,

$$AC^2 = AB^2 + CB^2$$

$$= \left(\frac{18\sqrt{5}}{5}\right)^2 + \left(\frac{24\sqrt{5}}{5}\right)^2 = 180$$

$$AC = 6\sqrt{5}$$

The length of the side AC is $6\sqrt{5}$ units.

Drones or Remotely Piloted Aircraft

The following teaching resources may be useful if you are planning to use drones at school for teaching and learning. Australian legislation must be adhered to
https://www.legislation.gov.au/Details/F2016C00889

Schools will need to plan full risk assessments and further considerations are at the discretion of the leadership team. Students need to be taught rules and responsibilities for safe and ethical use prior to engaging with the drones. These two introductory lessons may be a useful start to your program. They are shared from an iTunesU course (Coding Drones in Primary Education) created by Dan Martinez and Beth Claydon from St Hilda's School, Gold Coast. www.sthildas.qld.edu.au . Further teacher resources can be found after the lesson outlines.

Lesson outlines:
Lesson 1
In this lesson, students will realise the safety issues relating to flying drones in a school environment (hall/undercover area)

Materials needed
- iPad
- Flight Safety Lesson Set up Document http://bit.ly/2oZQSfi

Lesson outline:
Drones are a fun and exciting technology. However, they can also be dangerous and cause injury if not used safely.

The following suggestions are designed to provide a safe and structured approach to your lessons.

You may wish to adapt or modify these safety suggestions to best suit your environment. The duty of care for the safety of students is always the responsibility of the teacher. It is paramount and essential that students listen carefully and follow instructions.

These are strictly recommendations and do not dismiss teachers of their duty of care during the lesson, nor from preparation of a risk assessment.

We recommend you review the flight safety lesson set up document prior to the lesson.
https://drive.google.com/file/d/0B16uXkNkvazjR0wwXzJnMWY2TmM/view

Flight Zone
1. Establish a clear Flight Zone. A large open and clear space works best. A school hall, gymna sium etc

2. Make it clear and explicit that the flight zone is out of bounds for students.

3. Students are not to enter the flight zone to collect their drones until instructed by the teacher.

Control Zone

1. Establish a clear Control Zone

2. Establish clear roles for students working in pairs. “Controller” - controls the Drone “Spotter/Drone Collector” - collects the drone when given the “All Clear”

3. Explain that no student is allowed to enter the flight zone until the entire class has been given the “All Clear” by the teacher.

"All Clear" Instruction

1. Once all drones are grounded, have each Controller place their iPad on the floor. This is to stop any accidental propeller ignitions.

2. Once all Controller iPads are on the ground, and the Flight Zone is safe, call “All Clear”

3. Then the Spotters/Collectors retrieve their drone.

Task:

Using information from the video clip (Drone rules in Australia) and the document in the ‘Materials needed’ list, create a visual representation of each step as outlined within the lesson setup document, via an application of your choice. You may wish to complete this task as a collage, comic within Book Creator, iMovie etc. https://www.youtube.com/watch?v=72TNbtQCB-A&feature=youtu.be&t=3s

Lesson Resources:

Drone rules in Australia https://youtu.be/72TNbtQCB-A

Lesson 2

In this lesson, students will

• View and communicate ideas and information surrounding the ethical and social protocols relating to commercial drones

• Plan and share ethical solutions for commercial drone use

Materials Needed:

• iPad and headphones

• Applications: Popplet, PicCollage or presentation tool of choice

Lesson Outline:

1. DISCUSS and DEFINE the following terms: ethics, ethical decisions, morals, protocols, implications, deployment, recreational, commercial, military arenas, remotely piloted aircraft, unmanned aircraft and CASA (Civil Aviation Safety Authority)

2. In pairs or small teams, VIEW, READ and DISCUSS the resources from the list below that outline why and how commercial drones are used.

3. In either Popplet, PicCollage or presentation tool of choice, CREATE a presentation that illustrates your understanding of appropriate use of drones - What are the Pros and Cons of Using Drones?

4. You must include a relevant title and associated images and text. Use the following questions as prompts to help you share ideas and concepts in your presentation. You do not have to directly answer each question; they are simply to get your ideas flowing.

- What are the pros and cons of using drones?
- How can they be used appropriately? Or inappropriately?
- What are some of the safety issues surrounding the use of drones?
- How might drones be used in the future?
- What are the CASA regulations?
- Why is it important to think about the ethical use of drones?
- Why do rules and regulations need to be in place?
- What can go wrong when using drones?
- How can drones benefit our lives?
- Why do you need to learn about using drones?

5. PRINT, SHARE and DISPLAY your presentation with others.

Lesson Resources:
Drone Debate – BTN
http://www.abc.net.au/btn/story/s3840212.htm

Why children should have a drone
http://www.news.com.au/technology/why-children-should-be-given-a-drone-for-christmas/news-story/44750687e3aa4af59c269434cf0ab112

Use of drones by Australia Post
http://mobile.abc.net.au/news/2016-04-15/australia-post-to-trial-drone-parcel-delivery-of-online-shopping/7331170

Drones to the rescue
http://www.kidsdiscover.com/teacherresources/drones-uavs-rescue/

Drones can create issues
http://abc7news.com/technology/toy-drone-creates-new-issues-for-firefighters/472520/

Additional Teacher Resources:
A free online interactive learning module:
http://services.casa.gov.au/elearning/casa_101/

A free online drones course for adult learners:
https://alison.com/courses/introduction-to-drones

Factsheet for flying drones:
https://www.casa.gov.au/file/176971/download?token=yQxTYAcn

A short safety poster/brochure:
https://www.casa.gov.au/file/172036/download?token=CMcZWoBV

INDEPENDENT SCHOOLS QUEENSLAND 07 32281515 office@isq.qld.edu.au

Strings Graphs - Problems worth Exploring

Peter Fox
Texas Instruments Australia

Great mathematics problems engage students in thinking; provide connections between multiple areas of mathematical studies and contain multiple entry and exit points. Most textbook questions focus on skill and drill; they typically contain a single process or procedure; occur in clusters of similar problems and often rely on repetition and rote. While skill and drill type questions serve a purpose they should not constitute the majority of our student's mathematical diet. Some neurological evidence supports the use of repetition; it can assist memory and the subsequent efficiency to recall certain facts and procedures. Repetition however does not generate understanding and can have a negative impact on mental engagement. Finding the right balance of rehearsal, problem solving and investigations is just one of the many challenges faced by teachers of mathematics. The Strings problem explored in this article is lovely example of a problem worth exploring. It contains multiple entry and exit points from Year 9 through to Specialist Mathematics in Year 12. There are opportunities to include a reasonable amount of rehearsal throughout the problem combined with the appropriate use of technology to support and feedback. In this article the TI-Nspire CX (non-CAS) has been used.

The problem starts by exploring a family of linear function that produce an aesthetically pleasing pattern. The visual appeal has been incorporated into a number of popular architectural structures such as the "Chords bridge" in Jerusalem (Figure 1) and the Margaret Hunt Hill Bridge in Dallas (Figure 5). We start by looking at how the problem fits within the Year 9 curriculum.

Figure 1 – Chords Bridge, Jerasulem **(Wikipedia)**

Year 9

Start by graphing the family of ten straight lines that pass through the points: (0, 10) & (1, 0); (0, 9) & (2, 0); (0, 8) & (3, 0) … (0, 1) & (10, 0). In each case students can use rise over run to calculate the gradient and the y intercept for the standard form of a straight line.

Equation 1:

$$\frac{Rise}{Run}=\frac{-10}{1}$$ Y Intercept = 10

$$y=\frac{-10}{1}x+10$$

Equation 2:

$$\frac{Rise}{Run}=\frac{-9}{2}$$ Y Intercept = 9

$$y=\frac{-9}{2}x+9$$

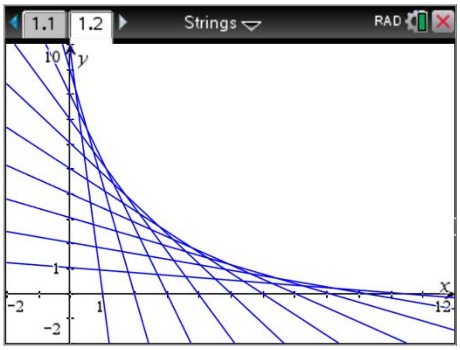


Figure 2 - String Graphs: Eqns (1,2 ..10)

The graphs for equations 1 to 10 are shown in figure 2. Students may choose to assign domain constraints to the equations so that they terminate on the y and x axis respectively.

The calculation of the gradient presents a powerful opportunity to illustrate the importance of using exact values in computations. Compare the two sets of gradient calculations:

$\{\frac{-10}{1}, \frac{-9}{2}, \frac{-8}{3}, \frac{-7}{4}, \ldots \frac{-1}{10}\}$

{-10, -4.5, -2.66, -1.75 … -0.1}

If students use exact values for the gradient they see a pattern emerge, students that use the approximated values rarely identify the pattern. This helps reinforce the necessity to work with exact values on other occasions such as surds, trigonometric functions and exponentials.

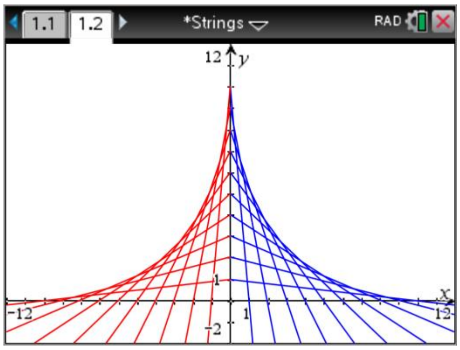


Figure 3 - String Graphs Equations (11, 12 ... 20)

A second set of equations can be generated to help complete the imagery associated with the Chords Bridge in Jerusalem. In Figure 3 equations 11 through to 20 consist of lines joining points: (-10, 0) & (0, 1); (-9, 0) & (0, 2) … (-1, 0) & (10, 0), all with domains restricted such that x < 0.

There are many ways this activity can be further investigated at this level, where the first component serves as the scaffolding on which students can use as the basis for further investigations: Extend the existing pattern beyond (0, 1) and (10, 0) to include: (0, -1) & (11, 0), (0, -2) & (12, 0) …. The resulting pattern surprises some students as the curve starts to turn around. Some students predict that it will ultimately turn around on itself; however this is not the case as will be seen later.

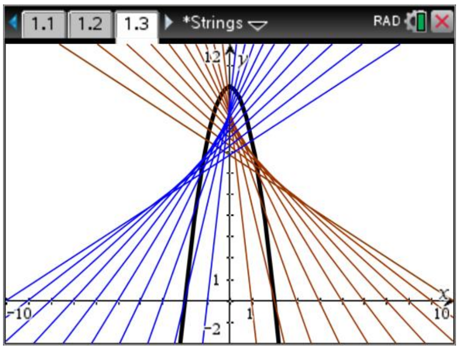


Figure 4 – Stitching lines to a curve

Determine the equations connecting: (-10, 10) & (1, 1); (-9, 9) & (2, 2) … (-1, 1) & (10, 10). This equates to stitching points along the lines $y = -x$ and $y = x$. This produces a very recognisable curve in the envelope created by the lines and supports the notion for the previous point that the curve does not return onto itself. This example also moves away from gradient and y intercept to equation of a line given two points. This investigation can be further extended to stitching points along the lines *y = mx* and *y = -mx* and seeing how the gradient (m) effects the shape of the resulting curve. The equation to a straight line given two points can be calculated using the gradient and translational form of a straight line: $y = m(x - h) + k$. This form allows students to identify patterns particularly if exact values are used.


Figure 5 – Margaret Hunt Hill Bridge (Wikipedia)

For a more challenging task consider stitching points to a curve rather than straight lines. The Margaret Hunt Hill bridge is a lovely example of stitching a family of straight lines to a parabola. Mathematically the complexity is only one step up on the previous extension; in this case students are given one point and have to compute the second.

All of the above activities provide a reasonable amount of rehearsal and a natural progression with regards to the determination of a linear equation and subsequent graphs. Students may also produce physical constructions using nails and strings.

This activity is a great opportunity to introduce graphing calculators. From a practical perspective students gain appropriate experience entering and editing graphs and moving between calculations and graphs. From a pedagogical perspective the graphs provide visual feedback with respect to the accuracy of the equations determined without looking to BoB (Back of Book). Unlike BoB, the visual feedback provided by the calculator's graph allows students to consider any errors and the cause of such errors. They can determine whether it is the calculation of the gradient, x or y axis intercept or one of the points that has been incorrectly determined and therefore focus their attention on the relevant computations. There are also more efficient means of entering the equations by using parameters.

Year 10 & 11

The original problem can be extended by considering the points of intersection of consecutive straight line equations. The pairs of simultaneous equations are relatively simple to solve:

Equation 1: $y=-10x+10$ Equation 2: $y=\frac{-9}{10}x+9$

Solution: Point of intersection: *x* - coordinate

$$-10x+10=\frac{-9}{2}x+9$$

$$1=\frac{11}{2}x$$

$$x=\frac{2}{11}$$

Point of intersection: *y* - coordinate

$$y=-10\left(\frac{2}{11}\right)+10$$

$$y=\frac{90}{11}$$

The table below shows consecutive points of intersection for some of the pairs of equations. The table also includes difference values to help identify the patterns in the points of intersection.

Equations	Point (t)	x-Coordinate	1st Order Difference	2nd Order Difference	y-Coordinate	1st Order Difference	2nd Order Difference
Eqns: 1 & 2	1	$\frac{2}{11}$	$\frac{4}{11}$	$\frac{2}{11}$	$\frac{90}{11}$	$-\frac{18}{11}$	$\frac{2}{11}$
Eqns: 2 & 3	2	$\frac{6}{11}$	$\frac{6}{11}$	$\frac{2}{11}$	$\frac{72}{11}$	$-\frac{16}{11}$	$\frac{2}{11}$
Eqns: 3 & 4	3	$\frac{12}{11}$	$\frac{10}{11}$	$\frac{2}{11}$	$\frac{56}{11}$	$-\frac{14}{11}$	$\frac{2}{11}$
Eqns: 4 & 5	4	$\frac{20}{11}$	$\frac{14}{11}$		$\frac{42}{11}$	$-\frac{12}{11}$	
Eqns: 5 & 6	5	$\frac{34}{11}$			$\frac{30}{11}$		

In both cases a constant difference is observed in the second order signifying that the points of intersection for the x and y coordinates can both be generated by quadratic equations. Students can focus on the numerator to make the equations easier to identify: {2, 6, 12, 20, 34 …} and {90, 72, 56, 42, 30 …}.

There are numerous techniques for determining the quadratic equations. With (t) representing the equation number the coordinates for the consecutive points of intersection can be determined by the equations:

$$x = \frac{t(t+1)}{11} \quad \& \quad y = \frac{(t-10)(t-11)}{11}$$

These equations are referred to as parametric equations and serve as a great introduction for advanced year 10 students or those studying Specialist Mathematics in year 11. The original family of straight lines can also be generated using a parameter:

$$f(x,t) = \frac{t-11}{t}x + 11 - t$$

Examples: $f(x,1) = -\frac{10}{1}x + 10$, $f(x,2) = -\frac{9}{2}x + 9$ …

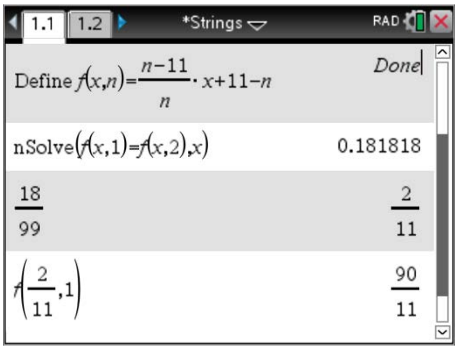


Figure 6 – Defining an equation

If students determine the general equation then it is much easier to graph the family of curves and also to find consecutive points of intersection. The entire family of straight lines is graphed in a single line in figure 7. More lines can be graphed by having:

t = {11, 12, 13 … 20}

Similarly:

t = {-10, -9, -8 …}

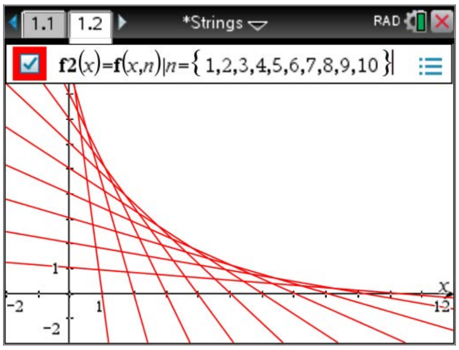


Figure 7 – Family of lines from one equation.

The advantage of parametric equations is that they also allow students to generalise the solution generating more equations and plotting more points to create the envelope defined by the lines and the consecutive points of intersection.

An earlier extension question pertaining to stitching the lines y = x and y = - x now becomes highly relevant. The envelope formed for this task is a simple parabola. Students can investigate what happens to the parabolic envelope when stitching y = mx and y = -mx. Figure 8 shows the parametric curve for the original points of intersection, this line looks more like a parabola. Thinking carefully, some students may realise that this is simply a 45° rotation of a previous extension stitching the lines:

y = x and y = -x.

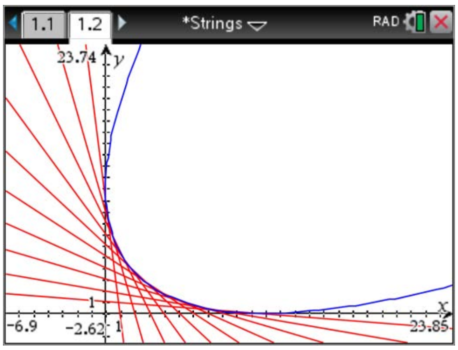


Figure 8 – Parametric Description

Of course as a rotation, all of these solution points and equations could be obtained using matrices, but that's a whole new article!

Year 12

To date the task has essentially focused on some relatively straight forward algebra. In the absence of rotational matrices, algebra and calculus can be used to show that the envelope formed is parabolic.

Consider the optical properties of a parabolic curve:

A beam of light that travels parallel to the principle axis will be reflected through the focal point.

A simple case of this is where the light strikes the surface of the parabola at a 45° angle. The symmetry of the envelope means the principle axis is the line y = x, so a light ray that strikes the parabola where the gradient is zero (or infinite) will make a 45° angle.

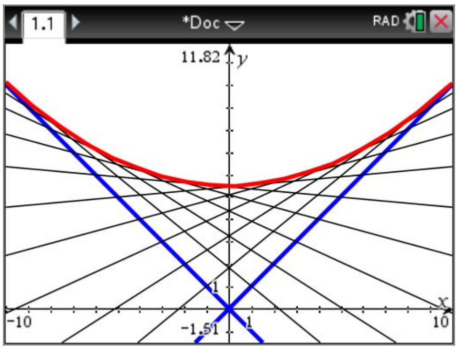


Figure 9 – Stitching y=mx and y=-mx

Let's start with determining the gradient of the curve at any point. Using the parametric equations this amounts to a simple application of the chain rule:

$$x = \frac{t(t+1)}{11} \qquad y = \frac{(t-10)(t-11)}{11}$$

$$\frac{dx}{dt} = \frac{2t+1}{11} \qquad \frac{dy}{dt} = \frac{2t-21}{11}$$

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{2t-21}{11}\cdot\frac{11}{2t+1} = \frac{2t-21}{2t+1}$$

The derivative can be used to determine where the gradient of the curve is zero (t = 10.5) or infinite (t = -0.5). These points can then be used to determine the equation for the reflected ray in each case. The point where the two reflected rays meet will be the focal point. A simple diagram will also reveal that these are one in the same lines. This produces the coordinates of the focal point as:

Focal Point: $\left(\frac{483}{44}, \frac{483}{44}\right)$

Similarly the turning point can be determined by considering a gradient of -1.

Turning Point: $(5,5)$

Using information regarding the distance between the focal point and the turning point (or any point on the parabola), the equation to the directrix can be determined.

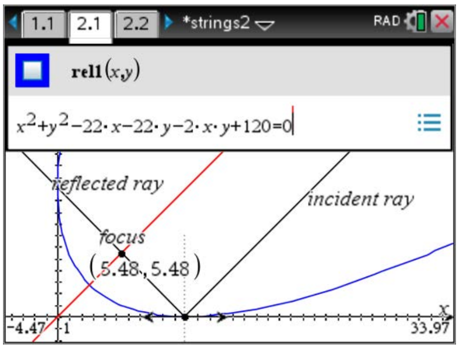


Figure 10 - Conic Section

Directrix: $y = -x - \frac{9}{200}$

The parametric equations can also be converted into a single relation by eliminating the parameter *t*. The results in the equation

$$x^2 + y^2 - 2xy - 22x - 22y + 120 = 0$$

The gradient of this relation can also be determined using implicit differentiation techniques.

$$\frac{dy}{dx} = \frac{y - x + 11}{y - x - 11}$$

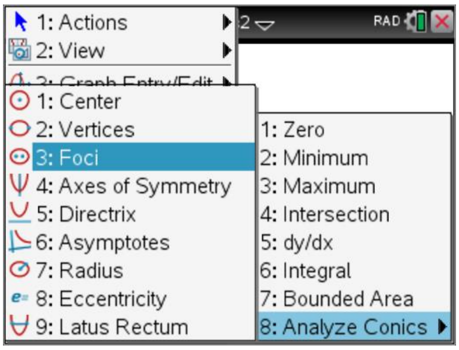


Figure 11 - Analysing Conic Sections

On the TI-Nspire CX it is possible to analyse conic sections and immediately return the coordinates of the foci, the equation to the directrix or axis of symmetry.

So a problem that started as simply determining the equations to a family of straight lines can be extended to implicit differentiation and an analysis of conic sections and includes many options to investigate in between. If you're still hungry for more, try exploring what happens if successive straight lines are moved closer together. The first set of points remain unchanged: (0, 10) & (1, 0), but the second set of points become (0, 9.5) & (1.5, 0) and the third set become: (0, 9) & (2, 0) and the fourth: (8.5, 2.5) and so forth. Successive points of intersection move, so too the envelope formed by the lines. As the points and lines get closer together a curve defines this limiting situation. What is the equation for the curve in the limiting situation?

Alternative Assessments
Two examples of investigation based assessments

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With some clear evidence in the literature regarding the purpose of alternative forms of mathematical assessment and its role in the teaching and learning cycle, I attempted to follow various examples of utilising an investigation style approach to assessment to try to capture a bigger picture of the student, their mathematical thinking and processes as well as their outcomes. What I have not included here are the approaches and time I take across a year to TRY to develop a culture of questioning, justifying, examining and explaining mathematical concepts and importantly their own metacognitive thinking. I wish to profess at this stage that I am sure there are flaws in my approach to achieve the above and also feel there is probably a lot more that can be done to improve these assessment tasks and marking criteria, but the intention to seek alternate ways to assess and gather further insight into the mathematical thinking of my students I believe is quite evident. I also recognise that this approach may not be a possibility for all due to the requirement of some technology (it is undoubtedly easier to do these assessments with a 1 to 1 iPad program, but can be achieved with fewer devices or other recording devices). While I utilise 6 different investigation, in combination with testing formats across the year, I will present 2 such approaches that I have used. These tasks were developed for Year 6.

Example 1: Active Angles investigation
The first assessment is in finding angles. This task is designed to assess the content descriptor of *Investigate, with and without digital technologies, angles on a straight line, angles at a point and vertically opposite angles. Use results to find unknown angles (ACMMG141)*, which links to the achievement standard through *They solve problems using the properties of angles* (Australian Curriculum v8). This content descriptor has changed slightly in the latest version of the Australia curriculum, where previously there was more focus on solving problems related to angles. As such the task outline involved searching for what I termed 'active angles'.

Through our classwork prior to the task we explored angles and where we could find them in our world. This was then combined with naming angle types, how to measure them with and without digital technologies and how you could use one angle to find out missing angles. The assessment task that was developed was to film themselves measuring at least two 'active angles' they could find in their lives. As there is no specific angle that needed to measured, this meant that the task was ceiling-less so that a student could demonstrate their knowledge of a Year 6 curriculum at all levels. Prior to any filming or measuring, students were to provide the teacher with a plan of what they were going to measure, why they were going to measure it and their thoughts behind their selections.

Students sought out a wide variety of angles in and outside of the classroom. Some measured angles of shots on a football (soccer) goal, while others measured the angle created in the minimum elevation required to kick a rugby conversion. Further still, the slope of a walkway or railing, the angle of how much a piece of furniture had moved from its original place, the angle of how much a piece of displayed work was off-centre (apparently my classroom displays need work and my boys wanted to highlight that I think!) and even the angle required to manipulate a piece of furniture to be able to get it through the door were all examples of what the students selected to measure and describe. As you can see the options available are quite vast and can demonstrate a differing level of understanding of angles and their properties.

As part of the task the students were recorded explaining what their angle was (i.e. the properties), how they were to measure it (using digital and non-digital technologies), what further information they could gain from their angle (i.e. vertically opposite angles, angles on a straight line etc) and through each stage they were describing and explaining their mathematical thinking. It is in this area that I was able to extract some very interesting information about their understanding. In addition to the task, there was a 5 question paper component. Some students were able to get most of the questions right until they hit the problem solving questions, but were able to provide interesting insight into their filmed angle investigation. In one case it became clear that the maths metalanguage was what he found most difficult, but through filming his thoughts he was able to articulate this clearly in his own terms when working through the task. This told me that in regards to the aspect of 'solving problems relating to angles' he was able to achieve at a year level standard. He was able to clearly identify 2 active angles and how to measure them to find unknown angles and put his angle understanding to practical use, but not with the appropriate mathematical metalanguage. This contrasts what he produced in the paper based problem solving questions as an understanding of the angles and how to find missing angles was not evident.

Below is an example of the task outline that is presented to the students. They are also provided with a task checklist as part of the task sheet and a marking guide as well. After the first angle has been planned, filmed, explained and reviewed by the teacher, further teaching points are extracted before the students present two more angles.

Task Outline:
You are to demonstrate and explain your understanding of how angles work and how to measure them based on 'active angles'. This means you will be going outside (or inside if it suits your active angle) to show how angles are used in many different ways in your lives. The options are endless! All you have to do is find an example of where you use angles in life (e.g. shooting at a goal, giving directions) and film yourself using the angle, measuring the angle and explaining any details about your angle.

Example 2: It your own game of chance!
The second example looks at an investigation related to the chance and data content descriptors of *Interpret secondary data presented in digital media and elsewhere (ACMSP148), Describe probabilities using fractions, decimals and percentages (ACMSP144), Conduct chance experiments with both small and large numbers of trials using appropriate digital technologies (ACMSP145), Compare observed frequencies across experiments with expected frequencies (ACMSP146) and Interpret and compare a range of data displays, including side-by-side column graphs for two categorical variables (ACMSP147).* These content descriptors are described in the achievement standard as *Students compare observed and expected frequencies. They interpret and compare a variety of data displays including those displays for two categorical variables. They interpret secondary data displayed in the media* and *Students describe probabilities using simple fractions, decimals and percentages*.

The task was a slightly larger investigation than the previous example as it involved a few more steps and considerations. The first step of the task was to look at secondary data and how it can be misleading. In order to show this we created misleading and accurate graphs based on real statistics such as from the State of Origin. The students then created a misrepresented graph and an accurate graph to show how easily a bias can be presented in data. Discussions around data collection, analysis and presentation followed through a class developed experiment. Students were then to design their own group chance experiment. The chance experiment had to have a small and then larger number of trials. The students were to work out an expected probability of an outcome. At our school we have five houses, so many groups chose to do an experiment relating to their houses using things like a fidget spinner with an arrow on it landing on a person in a particular house, or blindfolding and spinning a person in the middle to throw a ball to the other participants on the outside and recording the house of the person who caught it. Using the number of students from each house in their group they were able to calculate the expected frequency across the predetermined number of trials and represented as a decimal, fraction and a percentage. Data collection was to be recorded in a frequency table.

Once the trials and collection was complete students were to record the observed frequency as a fraction, decimal and percentage. In order to delve into the mathematical thinking, a written response was given comparing the expected frequency with the observed frequency and any potential discrepancies were discussed. Also trends in the data was discussed and potential reasons analysed. Finally a data display was required and students were to justify their choice in creating the data display that they did.

Further considerations for this task include the creation of a second misrepresented data display and also a links to life section where students will consider their trial and information that they have found to identify how this understanding has a purpose in their everyday lives or future. Below is the task outline provided to students. Again, a task sheet with a checklist of required elements is provided along with a marking guide.

Task Outline

Design your own chance experiment where you can look at expected frequencies compared with observed frequencies similar to the class example. Record your expected frequency before you begin your trial. Complete your trial and record your results in a table, as a graph and your observed frequencies as a fraction, decimal and percentage.

Each of these tasks was given 5 marks per section similar to a 5 point scale. This means that the year level expectation would be a score of 3 for each section and allows for students to achieve without a ceiling and show achievement significantly above expectations without using a percentage approach to grading. For this to be effective, time also has to be spent investigating expectations to achieve with the task (i.e. a success criteria) and explaining how the marking will work (this may also be reiterated to parents who may question grades). Furthermore for these tasks to have success, there is a need to work with the students on how to show their mathematical thinking whether it be through verbal or written statements. This can be time consuming, but definitely has a purpose. I take the approach of asking my students from the start of the year to 'examine and explain' not only each response to a question, but also their process to get their answer. In addition to this I ask my students to engage in an error analysis, which can be done in two ways – either provide an incorrect answer to a question for the students to correct through justification or questions that they have made an error to go back and identify where their error was specifically and what they need to do to rectify the error. This has a dual purpose of looking for consistent misconceptions or errors and also to ensure that students (and in my case boys, who stereotypically are very keen to move on once completing a question) take time to engage in some metacognitive mathematical thinking.

These tasks are in no way ground-breaking, but they are designed to consider the literature surrounding mathematics anxiety and beliefs and the use of games and investigations to create opportunities to develop and capture mathematical thinking that may be missed in paper tests. Again, I reiterate that I still think that there is a place for testing, but a varied approach to assessment allows for a greater range of thinking, understanding and skills. As suggested by Stoehr (2017), 'one size does not fit all' when it comes to mathematics. There are so many possibilities for developing investigations and ideas that are already published. I believe it is a matter of having the confidence as a teacher to try different approaches. I again do not profess that these examples are perfect and fit for direct copy, but hope that it may give some ideas and thoughts for primary maths teachers to consider trying something a touch different. I presented one example which used videoing to capture thought processes and mathematical understanding and a second that required writing out thoughts, but these aspect can be adjusted to suit the needs of your class and your students. The important point that I wish to convey is to try different ways to capture mathematical thinking and to see if we can change the 'image problem' and as suggested by Darragh (2014) "demystify mathematics so that performing 'good at mathematics' performances is not seen as unattainable" (p. 99) to those who may lack the confidence or belief in their ability to achieve their potential.

References

Darragh, L. (2015). Recognising 'good at mathematics': using a performance lens for identity. *Mathematics Education Research Journal 27*(1), 83-102. doi: 0.1007/s13394-014-0120-0

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A Math Made in Heaven
Mathematics and Money

MoneySmart Teaching Team
Australian Securities & Investments Commission

ASIC's MoneySmart Teaching program builds the consumer and financial literacy capabilities of young Australians and helps educators (and the broader school community) teach young people about money. Using money and finance as a context for teaching mathematics engages students in learning mathematical concepts, enabling them to apply their skills to solve practical problems outside the classroom, contributing to life-long learning.

ASIC's MoneySmart Teaching Program aims to develop and provide young people with the knowledge, skills and behaviours to:

- **Manage money day to day:** Kids need to develop healthy money habits from an early age (including the difference between needs and wants) and learn to create a budget that works for them.
- **Plan for the future:** Setting goals, learning the value of saving, not spending more than you earn.
- **Make informed choices:** How to identify trusted sources of information, understanding consumer rights and responsibilities, understanding the importance of asking questions about financial products and services.

The program builds teacher capacity through providing personal learning and professional development opportunities to teachers alongside Australian Curriculum aligned resources across key learning areas for Foundation to Year 12 students. The MoneySmart website provides freely available teaching resources aligned to the Australian Curriculum (v8.3) to support teachers to incorporate financial literacy into their classroom teaching across all learning areas. Specific Mathematics focused units of work are available for Foundation to Year 10. This provides students with the opportunity to gain money management skills early in their lives and develop their financial knowledge, skills and behaviours through to their final years of school.

Financial literacy is an established part of the Australian Curriculum, particularly in Mathematics. The money and financial mathematics content strand appears across the Year 1 to Year 10 Mathematics curriculum. The general capability numeracy continuum taught across all learning areas also states that students should identify situations where money is used and apply their knowledge of the value of money to purchasing, budgeting and justifying the use of money.

Worksheets from two secondary units of work are featured that highlight how money and finances can be used as contexts for teaching mathematics concepts:

- How can we reduce our spending? (Year 7)
- Smart consumers 4 a smart future: Solar Sums (Year 9)

In the Year 7 Mathematics unit, 'How can we reduce our spending?' students investigate and compare prices as consumers through activities such as grocery shopping and selecting a mobile phone plan. In activities 1 and 2, students investigate the unit pricing of grocery items using a variety of mathematical strategies. The attached worksheet provides alternative strategies which can be used to complement tasks within these activities. Other activities within the unit include students performing calculations related to mobile phone costs and applying their learning to find and recommend ways that their family can reduce their spending.

The Year 9 Mathematics unit, 'Solar sums' is a problem-based investigation that challenges students to apply mathematical concepts to solve a common household issue. Using a hypothetical scenario, students investigate how consumers can reduce electricity consumption to improve cost efficiency. They compare the value of goods and services to inform consumer decisions and investigate the efficiency of solar panels, which is explored further in the attached additional strategies.

ASIC's MoneySmart also provides more general resources which have recently been mapped to the Australian Curriculum and are used extensively in secondary schools especially in upper secondary Mathematics classes. These include the Rookie series and ASIC's Be MoneySmart resource. The Rookie series features first time events in a young person's life such as moving out of home, buying a car or getting a job. ASIC's Be MoneySmart is a five module video based resource with a student workbook covering topics such as saving, budgeting and spending, debt management and taxation. In addition, the MoneySmart website has a number of tools and resources to support teaching activities including calculators, budget planners and financial infographics.

For teachers, the program's professional development plays a key role in building personal and professional confidence and capacity to teach students about money and finances. Teachers who feel more confident in teaching financial literacy are more likely to integrate this into their regular teaching practice. All MoneySmart Teaching professional development aligns with the Australian Professional Standards for Teachers. The online modules include:

- MoneySmart maths for primary teachers' module provides four hours of accredited professional learning. The module explores how money and financial concepts can be used as a tool to engage students in Mathematics, making the subject meaningful through real life learning contexts.

- Introduction to Consumer and Financial Literacy Education provides two hours of accredited professional learning and outlines why financial capability is important and its value for teachers. The MoneySmart Teaching tools, resources and Australian Curriculum links are also explored.

MoneySmart Teaching workshops are available across Queensland in 2017 including remote and regional areas to support teachers in State, Catholic, and Independent schools in the delivery of consumer and financial literacy education. A 60-90 minute professional learning session can be delivered after school or on a student-free day. If you are interested in attending a MoneySmart Teaching workshop or your school would like to host a professional learning session please contact Senior Project Officer Deborah Al Hinai from the Queensland Department of Education and Training on (07) 3513 5965 or email: Deborah.alhinai@det.qld.gov.au.

Mathematics teachers play a crucial role in teaching young people about money and finances through classroom teaching and learning, utilising its rich content for teaching numeracy skills and wider mathematical concepts and content. The MoneySmart Teaching program supports teachers of Mathematics through resources and professional development to implement financial education as part of their regular teaching practice. Enabling young people to develop their financial capability through teaching mathematical concepts using money and finances as the context for learning engages students in authentic learning experiences and provides them with the opportunity to develop the knowledge, skills, attitudes and behaviours needed to meet the challenges of the twenty-first century.

MoneySmart Teaching
https://www.moneysmart.gov.au/teaching

Foundation to Year 10
https://www.moneysmart.gov.au/teaching/teaching-resources/teaching-resources-for-primary-schools

https://www.moneysmart.gov.au/teaching/teaching-resources/teaching-resources-for-secondary-schools

How can we reduce our spending? (Year 7)
https://www.moneysmart.gov.au/media/558603/mst_secondary_maths7_unit.pdf

Smart consumers 4 a smart future: Solar Sums (Year 9)
https://s3-ap-southeast-2.amazonaws.com/mst-resources/smart-consumers-4-a-smart-future/Maths_Yr9/index.htm

'How can we reduce our spending?'
https://www.moneysmart.gov.au/media/558603/mst_secondary_maths7_unit.pdf

'Solar sums'
https://s3-ap-southeast-2.amazonaws.com/mst-resources/smart-consumers-4-a-smart-future/Maths_Yr9/index.htm

Rookie series
https://www.moneysmart.gov.au/teaching/teaching-resources/moneysmart-rookie-for-educators

ASIC's Be MoneySmart
https://www.moneysmart.gov.au/teaching/teaching-resources/teaching-resources-for-vet

Tools and resources
https://www.moneysmart.gov.au/tools-and-resources

Professional development
https://www.moneysmart.gov.au/teaching/professional-development

MoneySmart maths for primary teachers' module
https://www.moneysmart.gov.au/teaching/professional-development/workshops-and-online-learning#maths

Introduction to Consumer and Financial Literacy Education
https://www.moneysmart.gov.au/teaching/professional-development/workshops-and-online-learning#om1

The Bargain Hunter Shopping Challenge

An engaging and competitive activity for year 7 and 8 students.

Overview

The lesson is designed to replicate real-world shopping ventures through the interpretation of numbers, discounts and multi-buy product savings.

Targeted at students in years 7 and 8, 'bargain hunters' are equipped with a shopping list and a budget. They must navigate the 'store' and identify all the items on the list. The catch is that there are multiple brands with different pricing structures and incentives on display. The challenge is to minimise expenditure through the use of these discounts and special offers. Throughout the game, 'shoppers' will have to organise the data and formulate their own optimisation strategies to beat the budget.

Background

Shopping for groceries is an everyday activity that we really take for granted. Yet big brand supermarkets have complicated this experience by bombarding shoppers with a range of discount offers, specials, reward programs and multi-buy options. Perhaps, now more than ever, we need to be savvy shoppers to interpret these incentives and keep within the budget. And although we do not need to be mathematicians to buy our groceries, there is value in developing strategies to effectively engage in this activity.

So I was challenged to consider how these everyday skills could be applied in a classroom context.

This has culminated in what I have called the 'Bargain Hunter Shopping Challenge' (BHSC).

It has been designed to cater to the financial mathematics components of the ACARA 8.3 national curriculum for the year 7 and 8 cohorts, while offering a fun and simple approach for both teachers and students.

Objectives

The BHSC is an active learning task which requires students to engage with their surrounds, retrieve and catalogue information, and use higher order critical thinking skills.

In doing the activity, students will further develop:

- Numeracy skills as this everyday example demonstrates the role and purposeful use of mathematics in a context that is both fun and challenging.
- Critical thinking skills to organise and evaluate data, investigate possibilities, consider alternatives and solve problems.
- Literacy skills through:
    - the reading and interpretation of 3D models, signage and associated mathematical texts.
    - discussing, producing and explaining solutions.

The Task

Given a ten (10) item shopping list, students must purchase one of each item from the store.
The goal is to minimise spending.

To achieve this, students will need to carefully read the advertised specials and combine items with similar pricing structures.

Students must collect the data and develop their own strategies to optimise savings.

I would recommend that students should work in groups of 2-3 so that the team can experiment with possible solutions and then present their ‘best buy’ to the class.

The teacher may choose to provide hints along the way to further challenge students and/or help those who are struggling to get started.

Delivery
To simulate the shopping experience, illustrations of all the store items are mounted on cardboard props displayed around the classroom. Consequently, students have to navigate the room and collect their own data before attempting the problem. All the information they need is on display, including pricing and discount offers where they apply.

The task complexity is achieved by the introduction of competitive brands for some items. Students need to document options carefully and investigate various combinations in order to purchase all ten items on the shopping list while undercutting the $32.50 budget limit.

Here is a summary of the items and pricing options

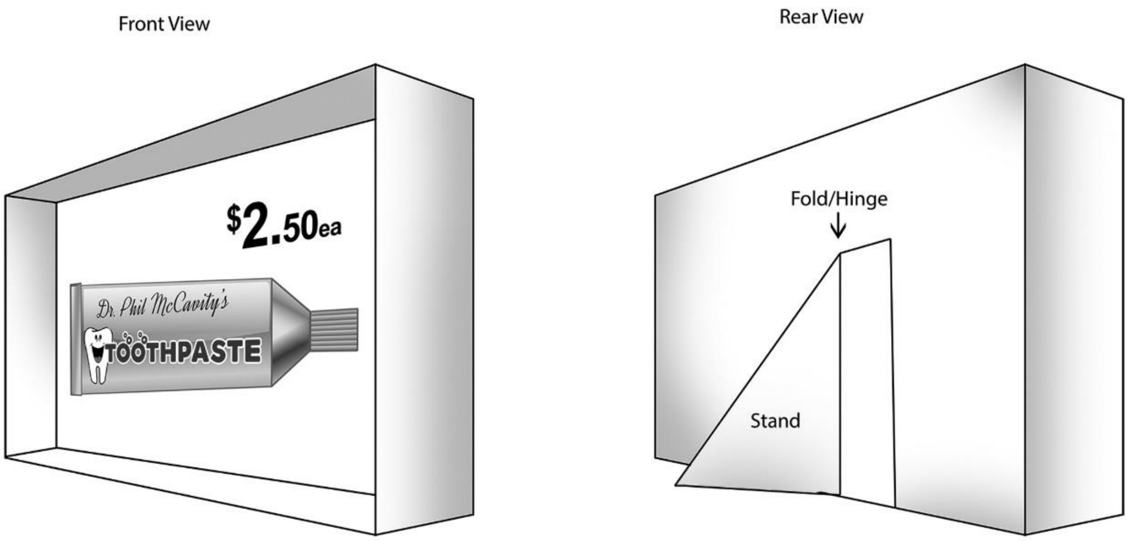
Shopping Items	Brand/s	Regular Price		Discount Tag
Pkt Museli Bars	Yum Yum (5 pack)	$	3.50	**Blue**
Sugar	Queensland	$	2.80	**Green**
Salt	Sea Salt	$	0.80	
	Rock Salt	$	0.90	**Green**
Toothpaste	Dr Phil McCavity's	$	2.50	
Flashlight	Blue	$	8.00	**Blue**
	Green	$	8.00	**Green**
Batteries	Red	$	6.00	30% off
	White	$	6.00	$2 off
Duct/Packaging Tape	Super Strong	$	7.40	**Blue**
Can Opener	Yellow	$	8.00	**Yellow**
	Blue	$	8.00	**Blue**
Bottled Water	Filtered	$	1.90	**Green**
	Spring	$	1.90	**Yellow**
Dishwashing Liquid	Lime	$	2.30	**Green**
	Strawberry	$	2.30	$1 off

Guide to Specials & Discounts

Yellow Tag	60% off purchase of any 2 different yellow tag items.
Blue Tag	Buy 2 different blue tag items and get a different blue tag item of equal or lesser value for free.
Green Tag	Buy 1 green tag item, get a different green tag item of equal or lesser value 1/2 off.
Other	This week's special deal. Prices as marked.

The basic activity would require a 70-minute lesson for junior secondary students to complete. However, the task could be easily extended to include greater student involvement (e.g. making the store props) or reformatted as a mathematics assignment.

Below is a sample design for the shelf prop which has been constructed from an A4 paper box lid. Colour prints of each item and the relevant discount tags are glued inside.



Suggested Strategies & Solutions
As there is an element of competitiveness to the task, students will often start out with a trial and error method, thinking this will lead to a quick resolution. However, the real problem-solving skills lie in the student's ability to

- work in teams
- accurately record information
- organise the data
- interpret offers and calculate discounts
- identify items that do not have an alternative brand (so they must buy the item as is)
- group items with common discount offers
- recognise that they must buy sufficient Yellow, Blue and Green tag items to qualify for these discounts in each category
- recognise that they will need to apply all colour coded discounts in their solution
- experiment with different combinations
- accurately calculate savings and the total buy price.
- explain their method and solution to others.

Only by the elevated use of these skills, and some patience, will the best results be found.

Surprisingly, there are three (3) solutions to which I have awarded a 1-star, 2-star and 3-star rating depending on the savings achieved.

Item List 1-Star	Item List 2-Star	Item List 3-Star
Yum Yum Bars Rock Salt Queensland Sugar Dr Phil McCavity's Toothpaste White Batteries Blue Flashlight Super Strong Duct Tape Yellow Can Opener Spring Water Lime Dishwashing Liquid	Yum Yum Bars Sea Salt Queensland Sugar Dr Phil McCavity's Toothpaste White Batteries Blue Flashlight Super Strong Duct Tape Yellow Can Opener Spring Water Lime Dishwashing Liquid	Yum Yum Bars Rock Salt Queensland Sugar Dr Phil McCavity's Toothpaste White Batteries Blue Flashlight Super Strong Duct Tape Yellow Can Opener Spring Water Strawberry Dishwashing Liquid
Total Cost: $31.41	Total Cost: $30.61	Total Cost: $30.41

Resourcing
The complete kit of teaching resources for this activity include:

- student worksheet
- session notes
- classroom presentation (MS PowerPoint)
- worked solutions (MS Excel)
- image files for shopping items and discount tags (JPEGs)
- illustrations of prop construction.

Anyone wishing to obtain a copy of these resources for educational purposes only (not for resale), should contact me (eleanorknie@gmail.com).

APPENDIX

ACARA Mappings
Year 7
Achievement Standard—Compare the cost of items to make financial decisions

- Find percentages of quantities and express one quantity as a percentage of another, with and without digital technologies (ACMNA158)
- Investigate and calculate 'best buys', with and without digital technologies (ACMNA174)

Year 8
Achievement Standard—Students solve everyday problems involving rates, ratios and percentages

- Solve problems involving the use of percentages, including percentage increases and decreases, with and without digital technologies (ACMNA187)
- Solve a range of problems involving rates and ratios, with and without digital technologies (ACMNA188)
- Solve problems involving profit and loss, with and without digital technologies (ACMNA189)

Attributions

Concept:	Nancy Drew, The Trail of the Twister, *Her Interactive*
Graphic Design, Illustrations & Props:	Paul Knie
Course Design & Content:	Eleanor Knie

"How can I use Problem Solving in my classroom?"

Geoff Todman
Ex-Lecturer in "Mathematics Education" at Churchlands Teachers College of Advanced Learning, Edith Cowan University, WA. 1977.

Preface: This article was presented as a workshop for Teachers at the recent QAMT Conference in Toowoomba, June, 2017. Theme was "Game Changers".

Use games as a model to solve problems.

"Big Brick Puzzle"

Maths Strand Geometry
Topic Space and Position
Strategy Work backwards / Trial and Error / Looking for patterns
Proficiency Problem Solving

Task

Change the position of the 6 coloured blocks, so that a red block is on top AND no block of a colour touches another of the same colour.

Answer

		Red
	Blue	Yellow
Yellow	Red	Blue

Follow Up: Discover the other possible combination. (Red must still be on top)

Background

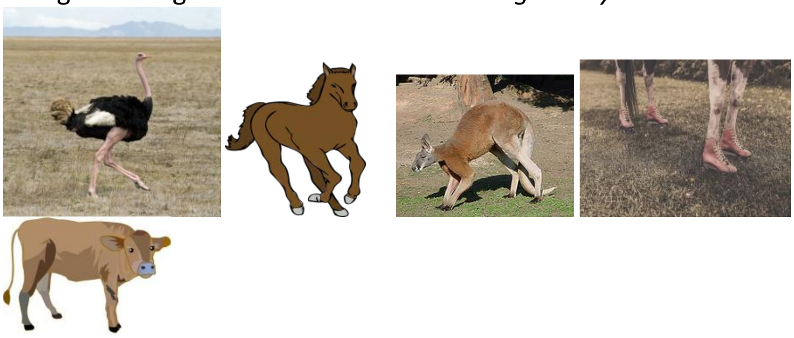
As a Primary/ High School teacher in Perth, in the 70's - 80's - 90's, I found that the norm for learning maths and then finding the answers was to learn or memorise the formulas, then apply them to the problem to find out the answer. In Maths I learnt the formulas, then applied them in Physics to calculate the answer.

Problem solving is found not just in Physics, but in all subjects. Environmental, Economical, Health, Engineering, Transport, Art and Music, etc.

"Can you see a pattern in the musical scale?"

Today, we encourage students to **"Think Mathematically"**

Task
“The young farmer’s daughter looks into the stock yards. She counts 20 legs on the ground.” *What animals might they be?*

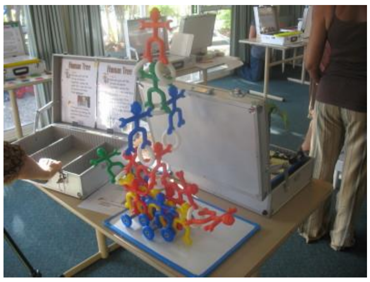


I realised that what I knew as "Problem Solving" was being presented to students as "exercises" or word sentences, not really understanding how to solve a real life problem that hadn't been encountered before.

"How can you tie a knot in a skipping rope, without letting go of the hands?"

Act it out using practical materials. A problem is an unfamiliar situation that requires previously applied known skills to discover the answer (of which there may be several).

"Forty tennis players enter a knockout singles competition. How many games must be played to determine a winner?"



Use practical materials to model.

Maths teaching of this subject has to change from being presented as a series of structured steps to one which allowed the student to work creatively, flexibly and logically. Lateral thinking is very important.

Open Ended
"*How many different ways can you make up 100 cents?"*

Strategy: *Make a table or chart.*

"Can you think "Outside the Box?"

"Can you find the diagonal of the box, without lifting the lid?"

Now we find quoted in the latest Qld and N.S.W. Syllabus:
"*Problem Solving is the most important aspects of school mathematics".*

"There needs to be a de-emphasis of the narrow basic-skills approach"

Quote from United Kingdom maths syllabus - "The whole maths curriculum should be organised around problem solving."

Introduction
It is my objective in this article to make teachers aware of the ways that there are strategies behind Problem Solving that can be presented to students in order for them to select one or more of them to solve a problem.

Why Teach Problem Solving?
Employers these days require that their employees be flexible thinkers and good problem solvers in the workplace.
"What do you need to do to a client's web page so that they achieve #1 Google rating?"

The Strategies of Problem Solving.
1. After reading and discussing, Locate Key Words.
2. Look for a Pattern
3. Guess and Check (Trial and Error)
4. Simplify the problem
5. Draw a diagram or graph, table or chart
6. Act it out
7. Make a model
8. Work backwards
9. Adopt a different point of view (Lateral Thinking)
10. Logical Reasoning.

For example

Noodle Game of NIM.
All Stages
Strategy: Work Backwards.

Start with 11 noodles in a bin or bucket. One has a flashing light.

1. Take turns to remove either one (1) or two (2) noodles from one bin to the other until only one noodle (the one with the flashing light) is left.
2. Whoever ends up with this one, gets arrested and goes to goal and misses a turn.

Press the button to make sound and light to flash.

Being Creative:

Edward de Bono in "Children Solve Problems" found that children from a very early age can solve problems.

Creativity is used a lot here. Not afraid of a right or wrong answer, they were more open to be creative.

"How do you stop a cat and a dog from fighting?" A solution from a student - "Put cat food all over the dog and dog food all over the cat and they will lick the food off and become friends."

The Role of the Teacher during your practical "hands-on" workshops.

1. **Monitor students'** progress. Do they
   - Work co-operatively?
   - Ask questions?
   - Organise information?
   - Keep trying?
   - Look for all possibilities?
   - Discuss their work?
   - Able to Estimate/Generalise?
   - Concentrate on the task?
   - Use a range of strategies?
   - Is Understanding being encountered?

2. Assist with language and difficult words.

3. Ask "**Open-Ended" questions**.

4. Rather than give or show answer, help them develop reasoning and understanding, I use the **"Guided Discovery Approach".**

5. Maintain class discipline, behavior, if needed. Children stay in pairs.

6. Encourage students to go beyond their expectation, and persevere if possible. **Positive r**e-enforcement.

7. Can take photos, for newsletter, etc. and to display on our wall/notice board. Post on school Facebook.

8. Prepare for “Follow-Up” in the classroom.

9. Enrichment includes students posing questions for new problems.

10. Let the students write the problem for an answer.

Teachers play a vital part in creating this environment in which creativity is valued and utilised.

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9. Problem Solving in mathematics Years 4-7 Alfred S. Posamentier.

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To contact Geoff Todman for a school workshop or P.D. afternoon at your school you can find his details on http://www.funwithmaths.com.au

Answers to Student Problems

Volume 42, Number 2

1. *AAA + AA + A + A + A = 1 000*

Using place value AAA = $100A + 10A + A$

AA = $10A + A$

Now

$$100A + 10A + A + 10A + A + A + A + A = 1\,000$$
$$125A = 1000$$
$$A = \frac{1\,000}{125}$$
$$A = 8$$

2. $\bar{x} = \frac{\sum x}{n}$

$\sum x = \bar{x} \times n$

$\sum x = 54 \times 9$

$\sum x = 486$

New total of set of numbers = $486 - 95$

= 391

New mean = $\frac{\sum x}{n}$

= $\frac{391}{8}$

= 48.875

Percentage reduction in the mean = $\frac{54-48.875}{54} \times 100$

≈ 9.49%

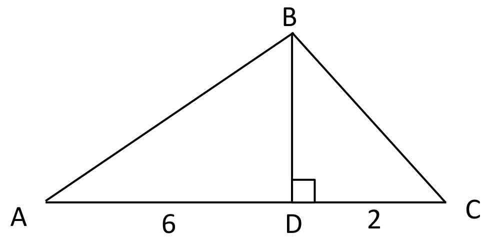
3. Using patterns $2^1 = 2$ $2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$ $2^6 = 64$ $2^7 = 128,$ etc

The pattern is 2, 4, 8, 6 as last digits.

$\frac{2^{50}}{8} = \frac{2^{50}}{2^3} = 2^{47}$ Now $\frac{47}{4} = 11\ remainder\ 3$

The third number in the pattern is an eight. Hence, the last digit of $\frac{2^{50}}{8}$ will be an eight.

4. Given that ΔABC is similar to ΔBDC and $\angle ABC = 90°$



As $\Delta ABC \sim \Delta BDC$ AB corresponds to BD

AC corresponds to BC

BC corresponds to DC

$\frac{8}{BC} = \frac{BC}{2}$

$16 = (BC)^2$

$BC = 4\ units$

5. Let *c* = number of correct questions
i = number of incorrect questions

$c + i = 15$ (1)
$25c - 15i = 215$ (2)

$15 \times (1)$

$15c + 15i = 225$ (3)
$25c - 15i = 215$ (2)

$(3) + (2)$

$40c = 440$

$c = 11$

Xiu got 11 questions correct.

Entries
Solutions for the student Problems were submitted by All Saints Anglican School, Isis District SHS, Mary MacKillop College, Moreton Bay College, St Joseph's College Gregory Terrace, St Laurence's College

Winners
Congratulations are extended to Sarah Luckensmeyer of Moreton Bay College and Alex Dang of Mary MacKillop College.

Prizes are provided by our generous sponsor, The University of Queensland.

Submitting Solutions
Students are invited to submit solutions to the Student Problems.

Please photocopy the problem page and clearly print your name, your school, and your year level.

Write your solutions (with full working) for each question.

Send your solutions to:

"QAMT Student Problems"
C/- Rodney Anderson
Moreton Bay College
PO Box 84
WYNNUM QLD 4178

Closing date is Friday, 27th October, 2017

Student Problems

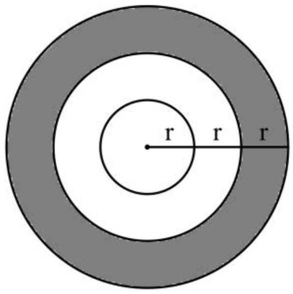
Name: .... **School:** .... **Year:** ....

Write your solutions (with working) next to each question or on a separate sheet of paper or by filling in the appropriate boxes.

A pdf copy of the student problems is on www.qamt.org/resources.

Question 1. In a right-angled triangle $sin\,\theta = k$, what is $tan\,\theta$?

Question 2. What fraction of the total area is the shaded area?



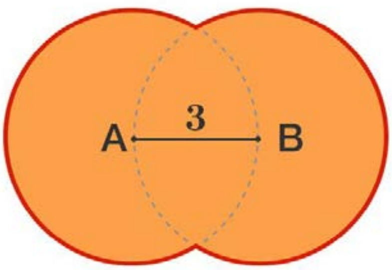
Question 3. A banana and an apple cost a total of 55 cents. An apple and a pear cost a total of 45 cents. An apple costs twice as much as a pear. Algebraically determine what is the total cost of a banana and a pear?

Question 4. Robert increased the length of his fish tank (rectangular prism) by 25% and decreased the width by 40%. By what percent should he increase the height so that the volume of the tank remains the same?

Question 5. Two overlapping circles with centres A and B form the figure to the right.

What is the exact perimeter of the figure?

$\overline{AB}$ is the radius of both circles.



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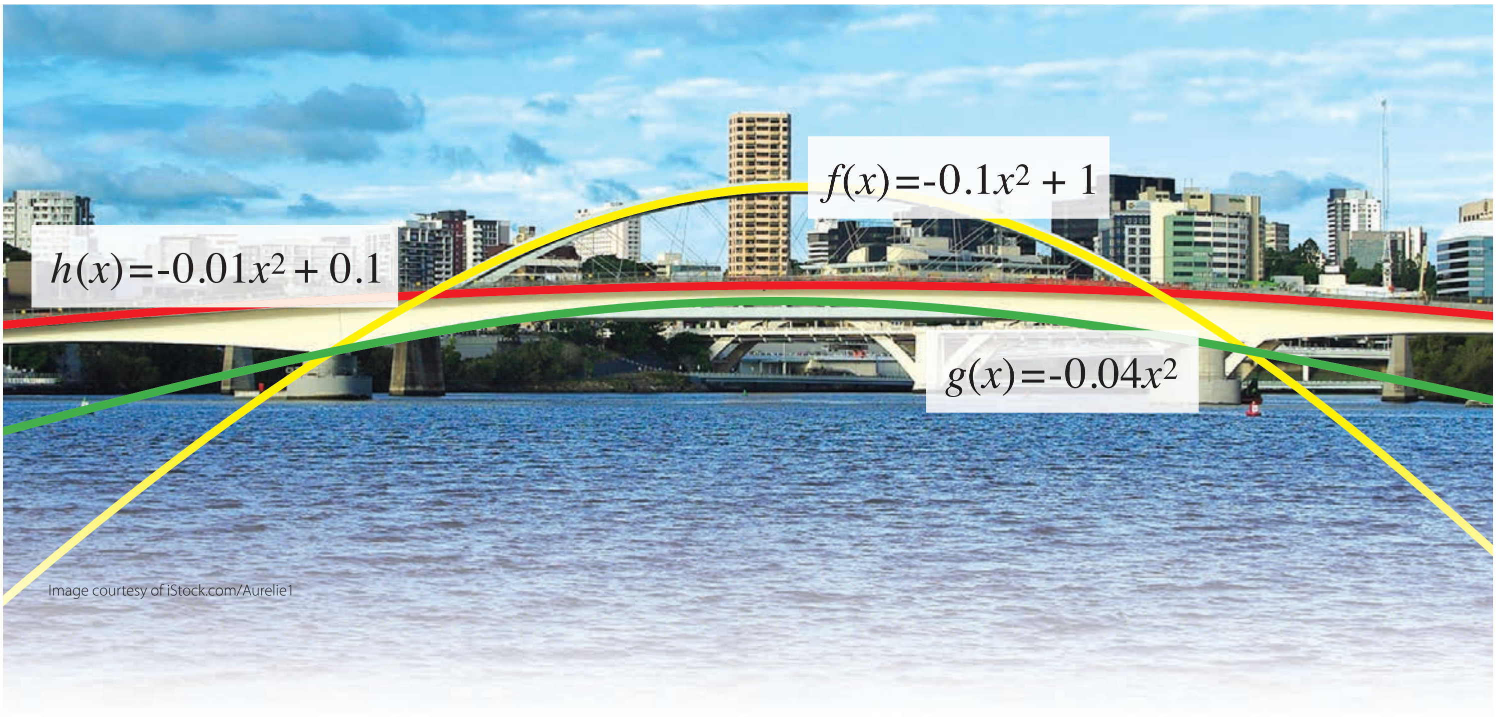

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